# Computer algebra independent integration tests

## 1-Algebraic-functions/1.1-Binomial-products/1.1.4-Improper/1.1.4.2-c-x-^m-a-x^j+b-x^n-^p

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# Contents

# Chapter 1

# Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [ 454 ]. This is test number [ 30 ].

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | solved | Failed |
|---|---|---|
| Rubi | % 100. ( 454 ) | % 0. ( 0 ) |
| Mathematica | % 100. ( 454 ) | % 0. ( 0 ) |
| Maple | % 84.8 ( 385 ) | % 15.2 ( 69 ) |
| Maxima | % 29.96 ( 136 ) | % 70.04 ( 318 ) |
| Fricas | % 56.39 ( 256 ) | % 43.61 ( 198 ) |
| Sympy | % 24.89 ( 113 ) | % 75.11 ( 341 ) |
| Giac | % 51.54 ( 234 ) | % 48.46 ( 220 ) |

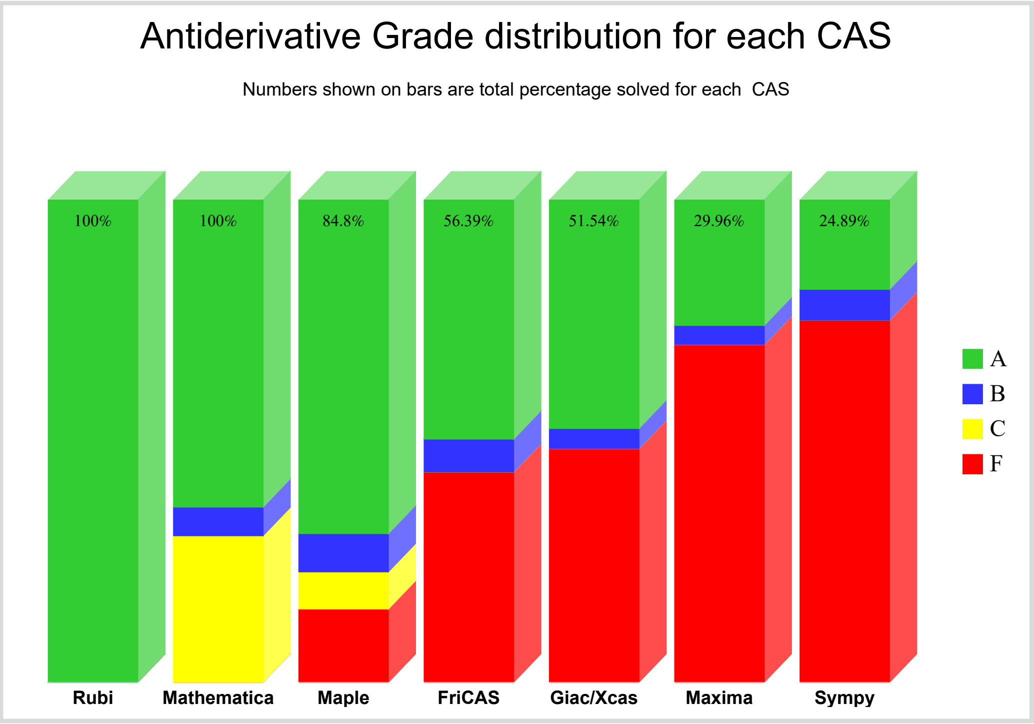
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

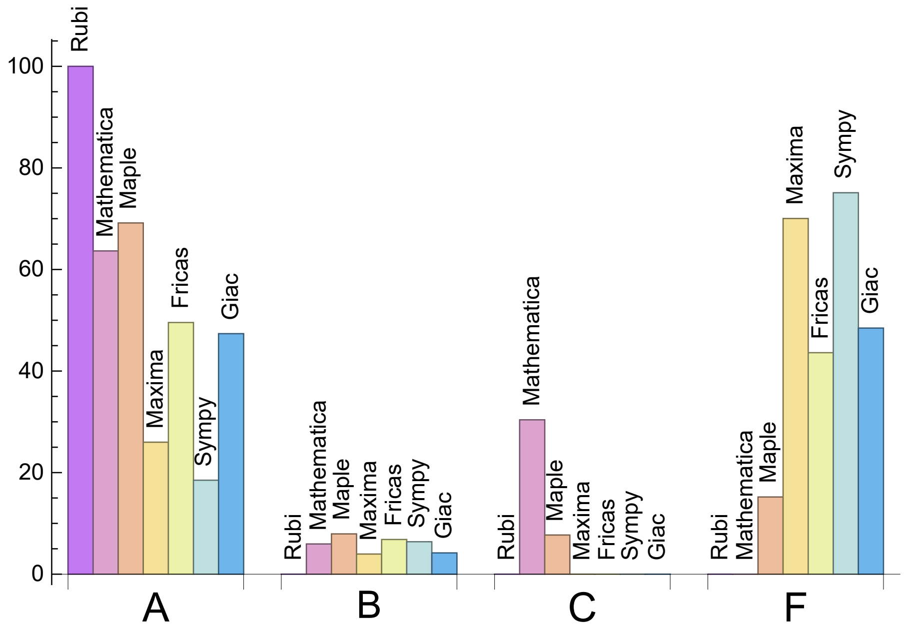
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 100. | 0. | 0. | 0. |
| Mathematica | 63.66 | 5.95 | 30.4 | 0. |
| Maple | 69.16 | 7.93 | 7.71 | 15.2 |
| Maxima | 25.99 | 3.96 | 0. | 70.04 |
| Fricas | 49.56 | 6.83 | 0. | 43.61 |
| Sympy | 18.5 | 6.39 | 0. | 75.11 |
| Giac | 47.36 | 4.19 | 0. | 48.46 |

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



## 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

| System | Mean time (sec) | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|---|
| Rubi | 0.14 | 119.28 | 1. | 74. | 1. |
| Mathematica | 0.04 | 62.66 | 1.08 | 55. | 0.89 |
| Maple | 0.02 | 192.7 | 1.96 | 79. | 0.94 |
| Maxima | 1.07 | 48.33 | 2.02 | 32. | 1.2 |
| Fricas | 1.09 | 184.34 | 4.45 | 128. | 2.8 |
| Sympy | 0.59 | 46.24 | 2.15 | 26. | 0.88 |
| Giac | 1.24 | 91.34 | 1.8 | 61.5 | 1.2 |

## 1.4 list of integrals that has no closed form antiderivative

{}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {347}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

### 1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

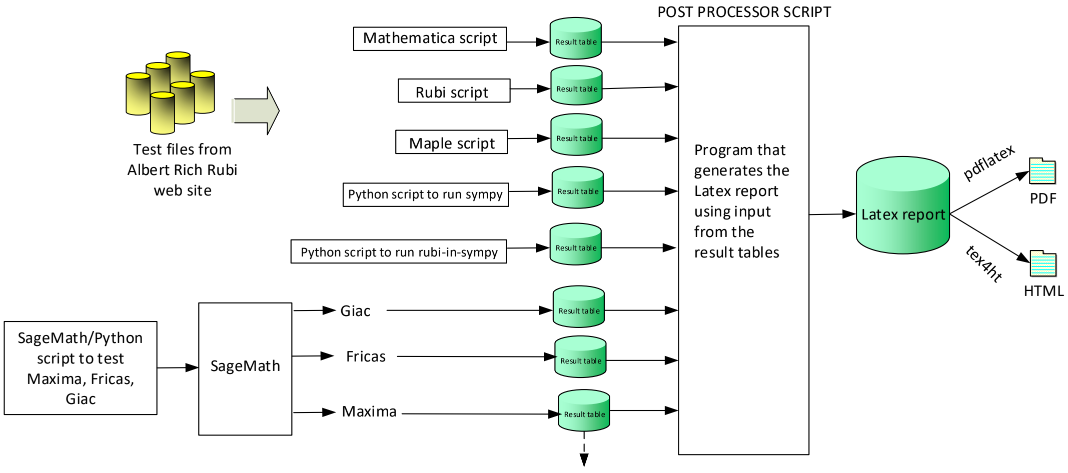
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



**One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)**

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

**High level overview of the CAS independent integration test build system**

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June 22, 2018

# Chapter 2

# detailed summary tables of results

## 2.1 List of integrals sorted by grade for each CAS

### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 89, 91, 92, 93, 94, 95, 96, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129, 130, 167, 168, 169, 170, 171, 176, 177, 178, 179, 180, 185, 186, 187, 188, 189, 190, 194, 195, 196, 197, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 248, 252, 253, 254, 255, 256, 257, 260, 261, 262, 263, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 297, 300, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 331, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 407, 408, 409, 411, 412, 413, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade: { 26, 28, 30, 32, 34, 328, 329, 330, 332, 333, 334, 349, 350, 351, 352, 353, 354, 392, 410, 414, 415, 416, 417, 418, 419, 420, 421 }

C grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 86, 88, 90, 97, 98, 99, 100, 101, 102, 110, 111, 112, 124, 125, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 172, 173, 174, 175, 181, 182, 183, 184, 191, 192, 193, 198, 199, 200, 201, 202, 238, 239, 247, 249, 250, 251, 258, 259, 264, 265, 266, 267, 268, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 304, 305, 307, 402, 403, 404, 405, 406, 437, 438 }

F grade: { }

### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 94, 95, 96, 103, 105, 117, 118, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 275, 276, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 303, 306, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 355, 356, 358, 360, 362, 363, 369, 377, 382, 383, 384, 385, 386, 388, 389, 394, 402, 408, 431, 432, 433, 434, 435, 436, 448, 449, 450 }

B grade: { 28, 32, 86, 104, 110, 111, 112, 119, 120, 124, 125, 126, 127, 272, 313, 328, 329, 330, 331, 332, 333, 334, 335, 336, 348, 349, 350, 351, 352, 353, 354, 357, 359, 361, 387, 412 }

C grade: { 92, 93, 97, 98, 99, 100, 101, 102, 106, 107, 108, 109, 113, 114, 115, 116, 121, 122, 123, 128, 129, 130, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 307, 407, 411 }

F grade: { 277, 278, 279, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 409, 410, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 451, 452, 453, 454 }

### 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 20, 22, 24, 26, 27, 29, 31, 33, 34, 35, 36, 37, 94, 95, 96, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, 252, 253, 254, 255, 260, 261, 262, 263, 284, 285, 286, 303, 306, 308, 309, 319, 320, 321, 322, 323, 324, 325, 326, 338, 339, 340, 344, 345, 346, 347, 349, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 433, 434, 435, 436, 448, 449, 450

B grade: { 28, 30, 32, 283, 327, 328, 329, 330, 332, 333, 334, 337, 341, 342, 343, 350, 351, 353 }

C grade: { }

F grade: { 13, 15, 17, 19, 21, 23, 25, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 236, 237, 238, 239, 246, 247, 248, 249, 250, 251, 256, 257, 258, 259, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 307, 310, 311, 312, 313, 314, 315, 316, 317, 318, 331, 335, 336, 348, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 451, 452, 453, 454 }

### 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 106, 107, 108, 109, 113, 114, 115, 116, 121, 122, 123, 128, 129, 130, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 280, 281, 282, 284, 285, 286, 287, 288, 297, 300, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 338, 339, 340, 344, 345, 346, 355, 356, 357, 358, 359, 360, 361, 362, 363, 369, 377, 382, 383, 384, 385, 386, 387, 388, 389, 394, 402, 407, 408, 409, 410, 411, 412, 413, 414, 416, 419, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade: { 28, 30, 32, 34, 79, 84, 127, 283, 323, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 341, 342, 343, 347, 348, 349, 350, 351, 352, 353, 354 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 97, 98, 99, 100, 101, 102, 103, 104, 105, 110, 111, 112, 117, 118, 119, 120, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 277, 278, 279, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 304, 305, 307, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 415, 417, 418, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443 }

### 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 16, 18, 20, 22, 23, 24, 25, 27, 29, 31, 33, 34, 35, 36, 37, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 308, 309, 319, 320, 321, 322, 324, 325, 326, 327, 338, 339, 340, 344, 345, 346, 347, 355, 356, 357, 358, 359, 360, 362, 369, 377, 394, 433 }

B grade: { 9, 13, 15, 17, 19, 21, 26, 28, 30, 32, 283, 323, 328, 329, 330, 332, 333, 334, 341, 342, 349, 350, 351, 352, 353, 354, 361, 363, 402 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312, 313, 314, 315, 316, 317, 318, 331, 335, 336, 337, 343, 348, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

### 2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 34, 35, 36, 37, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 103, 104, 105, 106, 107, 108, 109, 113, 117, 118, 119, 120, 121, 122, 123, 127, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 255, 256, 262, 263, 269, 270, 271, 272, 273, 274, 275, 276, 283, 287, 288, 297, 300, 303, 306, 308, 309, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 352, 354, 360, 361, 382, 383, 384, 386, 387, 388, 432, 433, 434, 435, 436 }

B grade: { 26, 28, 30, 32, 245, 328, 329, 330, 332, 333, 334, 335, 336, 348, 350, 351, 353, 362, 431 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 97, 98, 99, 100, 101, 102, 110, 111, 112, 114, 115, 116, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 252, 253, 254, 257, 258, 259, 260, 261, 264, 265, 266, 267, 268, 277, 278, 279, 280, 281, 282, 284, 285, 286, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 304, 305, 307, 310, 311, 325, 326, 327, 331, 337, 355, 356, 357, 358, 359, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

| Problem 1 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 14 | 18 | 31 | 12 | 18 |
| normalized size | 1 | 1. | 1. | 0.82 | 1.06 | 1.82 | 0.71 | 1.06 |
| time (sec) | N/A | 0.005 | 0.001 | 0. | 1.06 | 1.184 | 0.053 | 1.125 |

| Problem 2 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 14 | 18 | 31 | 12 | 18 |
| normalized size | 1 | 1. | 1. | 0.82 | 1.06 | 1.82 | 0.71 | 1.06 |
| time (sec) | N/A | 0.004 | 0.001 | 0.001 | 1.079 | 1.22 | 0.053 | 1.181 |

| Problem 3 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 14 | 18 | 31 | 12 | 18 |
| normalized size | 1 | 1. | 1. | 0.82 | 1.06 | 1.82 | 0.71 | 1.06 |
| time (sec) | N/A | 0.003 | 0. | 0.002 | 1.063 | 1.271 | 0.052 | 1.177 |

| Problem 4 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 11 | 14 | 23 | 8 | 14 |
| normalized size | 1 | 1. | 1. | 0.92 | 1.17 | 1.92 | 0.67 | 1.17 |
| time (sec) | N/A | 0.003 | 0. | 0. | 1.041 | 1.345 | 0.052 | 1.239 |

| Problem 5 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 13 | 12 | 15 | 30 | 10 | 19 |
| normalized size | 1 | 1. | 1. | 0.92 | 1.15 | 2.31 | 0.77 | 1.46 |
| time (sec) | N/A | 0.005 | 0.001 | 0.002 | 1.036 | 1.334 | 0.077 | 1.184 |

| Problem 6 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 25 | 32 | 55 | 26 | 32 |
| normalized size | 1 | 1. | 1. | 0.83 | 1.07 | 1.83 | 0.87 | 1.07 |
| time (sec) | N/A | 0.015 | 0.001 | 0.001 | 1.059 | 1.204 | 0.061 | 1.131 |

| Problem 7 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 25 | 32 | 55 | 24 | 32 |
| normalized size | 1 | 1. | 1. | 0.83 | 1.07 | 1.83 | 0.8 | 1.07 |
| time (sec) | N/A | 0.02 | 0.001 | 0.001 | 1.029 | 1.177 | 0.059 | 1.166 |

| Problem 8 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 25 | 32 | 55 | 26 | 32 |
| normalized size | 1 | 1. | 1. | 0.83 | 1.07 | 1.83 | 0.87 | 1.07 |
| time (sec) | N/A | 0.011 | 0.002 | 0.001 | 1.089 | 1.201 | 0.06 | 1.209 |

| Problem 9 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 25 | 32 | 55 | 24 | 32 |
| normalized size | 1 | 1. | 1. | 1.56 | 2. | 3.44 | 1.5 | 2. |
| time (sec) | N/A | 0.007 | 0.002 | 0.002 | 1.103 | 1.342 | 0.068 | 1.17 |

| Problem 10 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 22 | 28 | 47 | 22 | 28 |
| normalized size | 1 | 1. | 1. | 0.88 | 1.12 | 1.88 | 0.88 | 1.12 |
| time (sec) | N/A | 0.012 | 0.001 | 0.002 | 0.996 | 1.319 | 0.06 | 1.22 |

| Problem 11 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 46 | 37 | 49 | 130 | 42 | 49 |
| normalized size | 1 | 1. | 1. | 0.8 | 1.07 | 2.83 | 0.91 | 1.07 |
| time (sec) | N/A | 0.016 | 0.002 | 0.002 | 1.042 | 1.208 | 0.062 | 1.253 |

| Problem 12 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 27 | 24 | 31 | 49 | 20 | 32 |
| normalized size | 1 | 1. | 1. | 0.89 | 1.15 | 1.81 | 0.74 | 1.19 |
| time (sec) | N/A | 0.023 | 0.004 | 0.001 | 1.064 | 1.385 | 0.29 | 1.201 |

| Problem 13 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 31 | 27 | 0 | 165 | 56 | 35 |
| normalized size | 1 | 1. | 1. | 0.87 | 0. | 5.32 | 1.81 | 1.13 |
| time (sec) | N/A | 0.016 | 0.009 | 0.004 | 0. | 1.412 | 0.288 | 1.247 |

| Problem 14 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 14 | 18 | 30 | 10 | 19 |
| normalized size | 1 | 1. | 1. | 0.93 | 1.2 | 2. | 0.67 | 1.27 |
| time (sec) | N/A | 0.007 | 0.002 | 0.002 | 1.052 | 1.367 | 0.101 | 1.271 |

| Problem 15 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 24 | 16 | 0 | 151 | 53 | 20 |
| normalized size | 1 | 1. | 1. | 0.67 | 0. | 6.29 | 2.21 | 0.83 |
| time (sec) | N/A | 0.009 | 0.004 | 0.003 | 0. | 1.41 | 0.123 | 1.162 |

| Problem 16 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 22 | 21 | 27 | 49 | 15 | 32 |
| normalized size | 1 | 1. | 1. | 0.95 | 1.23 | 2.23 | 0.68 | 1.45 |
| time (sec) | N/A | 0.012 | 0.004 | 0.005 | 1.061 | 1.409 | 0.178 | 1.182 |

| Problem 17 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 34 | 30 | 0 | 173 | 65 | 39 |
| normalized size | 1 | 1. | 1. | 0.88 | 0. | 5.09 | 1.91 | 1.15 |
| time (sec) | N/A | 0.017 | 0.012 | 0.005 | 0. | 1.398 | 0.312 | 1.271 |

| Problem 18 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 35 | 32 | 42 | 80 | 31 | 58 |
| normalized size | 1 | 1. | 1. | 0.91 | 1.2 | 2.29 | 0.89 | 1.66 |
| time (sec) | N/A | 0.026 | 0.006 | 0.004 | 1.048 | 1.423 | 0.4 | 1.305 |

| Problem 19 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 43 | 39 | 0 | 234 | 87 | 54 |
| normalized size | 1 | 1. | 1. | 0.91 | 0. | 5.44 | 2.02 | 1.26 |
| time (sec) | N/A | 0.026 | 0.019 | 0.005 | 0. | 1.398 | 0.369 | 1.256 |

| Problem 20 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 49 | 44 | 59 | 108 | 42 | 77 |
| normalized size | 1 | 1. | 1. | 0.9 | 1.2 | 2.2 | 0.86 | 1.57 |
| time (sec) | N/A | 0.033 | 0.007 | 0.006 | 1.07 | 1.438 | 0.453 | 1.215 |

| Problem 21 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 45 | 36 | 0 | 261 | 78 | 47 |
| normalized size | 1 | 1. | 1. | 0.8 | 0. | 5.8 | 1.73 | 1.04 |
| time (sec) | N/A | 0.015 | 0.024 | 0.004 | 0. | 1.425 | 0.345 | 1.263 |

| Problem 22 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 33 | 35 | 46 | 108 | 34 | 63 |
| normalized size | 1 | 1. | 0.87 | 0.92 | 1.21 | 2.84 | 0.89 | 1.66 |
| time (sec) | N/A | 0.028 | 0.013 | 0.009 | 1.008 | 1.408 | 0.415 | 1.202 |

| Problem 23 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 54 | 46 | 0 | 288 | 90 | 63 |
| normalized size | 1 | 1. | 0.95 | 0.81 | 0. | 5.05 | 1.58 | 1.11 |
| time (sec) | N/A | 0.018 | 0.034 | 0.008 | 0. | 1.46 | 0.436 | 1.183 |

| Problem 24 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 41 | 46 | 68 | 157 | 49 | 69 |
| normalized size | 1 | 1. | 0.84 | 0.94 | 1.39 | 3.2 | 1. | 1.41 |
| time (sec) | N/A | 0.039 | 0.035 | 0.012 | 1.11 | 1.422 | 0.515 | 1.195 |

| Problem 25 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 67 | 59 | 0 | 359 | 114 | 80 |
| normalized size | 1 | 1. | 0.99 | 0.87 | 0. | 5.28 | 1.68 | 1.18 |
| time (sec) | N/A | 0.029 | 0.037 | 0.01 | 0. | 1.466 | 0.516 | 1.24 |

| Problem 26 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | A | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 13 | 13 | 29 | 22 | 28 | 65 | 19 | 31 |
| normalized size | 1 | 1. | 2.23 | 1.69 | 2.15 | 5. | 1.46 | 2.38 |
| time (sec) | N/A | 0.011 | 0.004 | 0.003 | 1.067 | 1.505 | 0.094 | 1.197 |

| Problem 27 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 18 | 19 | 24 | 39 | 14 | 20 |
| normalized size | 1 | 1. | 0.9 | 0.95 | 1.2 | 1.95 | 0.7 | 1. |
| time (sec) | N/A | 0.015 | 0.003 | 0.002 | 1.036 | 1.561 | 0.075 | 1.324 |

| Problem 28 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 6 | 6 | 22 | 17 | 22 | 51 | 14 | 24 |
| normalized size | 1 | 1. | 3.67 | 2.83 | 3.67 | 8.5 | 2.33 | 4. |
| time (sec) | N/A | 0.008 | 0.003 | 0.001 | 1.042 | 1.686 | 0.083 | 1.231 |

| Problem 29 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 14 | 18 | 26 | 8 | 20 |
| normalized size | 1 | 1. | 1. | 1.17 | 1.5 | 2.17 | 0.67 | 1.67 |
| time (sec) | N/A | 0.006 | 0.002 | 0.003 | 1.023 | 1.61 | 0.074 | 1.271 |

| Problem 30 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 2 | 2 | 19 | 3 | 18 | 45 | 12 | 20 |
| normalized size | 1 | 1. | 9.5 | 1.5 | 9. | 22.5 | 6. | 10. |
| time (sec) | N/A | 0.004 | 0.002 | 0.001 | 1.074 | 1.608 | 0.083 | 1.236 |

| Problem 31 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 16 | 20 | 38 | 10 | 22 |
| normalized size | 1 | 1. | 1. | 1.07 | 1.33 | 2.53 | 0.67 | 1.47 |
| time (sec) | N/A | 0.008 | 0.002 | 0.006 | 1.028 | 1.445 | 0.088 | 1.221 |

| Problem 32 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 8 | 8 | 24 | 19 | 24 | 55 | 15 | 27 |
| normalized size | 1 | 1. | 3. | 2.38 | 3. | 6.88 | 1.88 | 3.38 |
| time (sec) | N/A | 0.008 | 0.003 | 0.005 | 1.1 | 1.434 | 0.093 | 1.193 |

| Problem 33 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 22 | 21 | 27 | 65 | 17 | 35 |
| normalized size | 1 | 1. | 1. | 0.95 | 1.23 | 2.95 | 0.77 | 1.59 |
| time (sec) | N/A | 0.015 | 0.003 | 0.007 | 1.047 | 1.385 | 0.097 | 1.27 |

| Problem 34 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | A | A | B | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 31 | 24 | 34 | 80 | 24 | 36 |
| normalized size | 1 | 1. | 2.07 | 1.6 | 2.27 | 5.33 | 1.6 | 2.4 |
| time (sec) | N/A | 0.01 | 0.004 | 0.005 | 1.097 | 1.406 | 0.108 | 1.231 |

| Problem 35 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 29 | 26 | 36 | 78 | 22 | 45 |
| normalized size | 1 | 1. | 1. | 0.9 | 1.24 | 2.69 | 0.76 | 1.55 |
| time (sec) | N/A | 0.016 | 0.003 | 0.007 | 1.141 | 1.405 | 0.112 | 1.236 |

| Problem 36 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 14 | 18 | 41 | 12 | 24 |
| normalized size | 1 | 1. | 1. | 0.93 | 1.2 | 2.73 | 0.8 | 1.6 |
| time (sec) | N/A | 0.009 | 0.003 | 0.005 | 1.035 | 1.385 | 0.123 | 1.215 |

| Problem 37 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 16 | 20 | 39 | 12 | 24 |
| normalized size | 1 | 1. | 1. | 0.89 | 1.11 | 2.17 | 0.67 | 1.33 |
| time (sec) | N/A | 0.01 | 0.003 | 0.003 | 1.051 | 1.408 | 0.122 | 1.235 |

| Problem 38 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 163 | 163 | 95 | 168 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.58 | 1.03 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.188 | 0.05 | 0.018 | 0. | 0. | 0. | 0. |

| Problem 39 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 281 | 281 | 80 | 197 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.28 | 0.7 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.264 | 0.032 | 0.011 | 0. | 0. | 0. | 0. |

| Problem 40 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 79 | 146 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.58 | 1.07 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.122 | 0.027 | 0.012 | 0. | 0. | 0. | 0. |

| Problem 41 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 255 | 255 | 51 | 175 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.2 | 0.69 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.173 | 0.011 | 0.011 | 0. | 0. | 0. | 0. |

| Problem 42 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 113 | 48 | 124 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.42 | 1.1 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.086 | 0.011 | 0.012 | 0. | 0. | 0. | 0. |

| Problem 43 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 248 | 248 | 51 | 177 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.21 | 0.71 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.201 | 0.012 | 0.013 | 0. | 0. | 0. | 0. |

| Problem 44 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 53 | 123 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.46 | 1.06 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.088 | 0.014 | 0.005 | 0. | 0. | 0. | 0. |

| Problem 45 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 283 | 283 | 53 | 201 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.19 | 0.71 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.251 | 0.013 | 0.006 | 0. | 0. | 0. | 0. |

| Problem 46 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 186 | 94 | 188 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.51 | 1.01 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.23 | 0.055 | 0.014 | 0. | 0. | 0. | 0. |

| Problem 47 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 304 | 84 | 217 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.28 | 0.71 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.3 | 0.042 | 0.012 | 0. | 0. | 0. | 0. |

| Problem 48 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 83 | 166 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.53 | 1.05 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.134 | 0.035 | 0.011 | 0. | 0. | 0. | 0. |

| Problem 49 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 275 | 52 | 195 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.19 | 0.71 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.225 | 0.012 | 0.011 | 0. | 0. | 0. | 0. |

| Problem 50 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 49 | 144 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.37 | 1.07 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.132 | 0.014 | 0.012 | 0. | 0. | 0. | 0. |

| Problem 51 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 274 | 274 | 52 | 194 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.19 | 0.71 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.233 | 0.014 | 0.016 | 0. | 0. | 0. | 0. |

| Problem 52 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 134 | 54 | 139 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.4 | 1.04 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.131 | 0.015 | 0.017 | 0. | 0. | 0. | 0. |

| Problem 53 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 277 | 277 | 54 | 196 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.19 | 0.71 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.261 | 0.013 | 0.016 | 0. | 0. | 0. | 0. |

| Problem 54 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 54 | 142 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.39 | 1.04 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.134 | 0.016 | 0.015 | 0. | 0. | 0. | 0. |

| Problem 55 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 306 | 306 | 54 | 223 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.18 | 0.73 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.312 | 0.016 | 0.018 | 0. | 0. | 0. | 0. |

| Problem 56 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 163 | 163 | 54 | 169 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.33 | 1.04 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.178 | 0.016 | 0.019 | 0. | 0. | 0. | 0. |

| Problem 57 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 140 | 140 | 80 | 149 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.57 | 1.06 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.137 | 0.032 | 0.013 | 0. | 0. | 0. | 0. |

| Problem 58 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 258 | 258 | 66 | 178 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.26 | 0.69 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.205 | 0.026 | 0.011 | 0. | 0. | 0. | 0. |

| Problem 59 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 64 | 127 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.55 | 1.09 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.091 | 0.025 | 0.013 | 0. | 0. | 0. | 0. |

| Problem 60 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 229 | 53 | 158 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.23 | 0.69 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.15 | 0.013 | 0.011 | 0. | 0. | 0. | 0. |

| Problem 61 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 49 | 108 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.53 | 1.17 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.049 | 0.01 | 0.013 | 0. | 0. | 0. | 0. |

| Problem 62 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 253 | 253 | 48 | 182 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.19 | 0.72 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.2 | 0.012 | 0.015 | 0. | 0. | 0. | 0. |

| Problem 63 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 119 | 119 | 53 | 129 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.45 | 1.08 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.089 | 0.015 | 0.014 | 0. | 0. | 0. | 0. |

| Problem 64 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 286 | 286 | 53 | 204 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.19 | 0.71 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.254 | 0.014 | 0.016 | 0. | 0. | 0. | 0. |

| Problem 65 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 80 | 172 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.5 | 1.07 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.193 | 0.033 | 0.018 | 0. | 0. | 0. | 0. |

| Problem 66 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 279 | 279 | 68 | 200 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.24 | 0.72 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.269 | 0.026 | 0.015 | 0. | 0. | 0. | 0. |

| Problem 67 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 67 | 147 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.49 | 1.07 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.148 | 0.029 | 0.015 | 0. | 0. | 0. | 0. |

| Problem 68 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 253 | 253 | 57 | 182 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.23 | 0.72 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.213 | 0.022 | 0.016 | 0. | 0. | 0. | 0. |

| Problem 69 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 54 | 130 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.47 | 1.13 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.1 | 0.02 | 0.015 | 0. | 0. | 0. | 0. |

| Problem 70 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 254 | 56 | 184 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.22 | 0.72 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.209 | 0.015 | 0.015 | 0. | 0. | 0. | 0. |

| Problem 71 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 114 | 54 | 132 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.47 | 1.16 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.084 | 0.019 | 0.016 | 0. | 0. | 0. | 0. |

| Problem 72 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 273 | 273 | 51 | 206 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.19 | 0.75 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.224 | 0.013 | 0.018 | 0. | 0. | 0. | 0. |

| Problem 73 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 56 | 150 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.4 | 1.08 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.142 | 0.017 | 0.018 | 0. | 0. | 0. | 0. |

| Problem 74 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 306 | 306 | 56 | 228 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.18 | 0.75 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.323 | 0.016 | 0.02 | 0. | 0. | 0. | 0. |

| Problem 75 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 159 | 130 | 212 | 0 | 845 | 0 | 123 |
| normalized size | 1 | 1. | 0.82 | 1.33 | 0. | 5.31 | 0. | 0.77 |
| time (sec) | N/A | 0.248 | 0.181 | 0.04 | 0. | 1.589 | 0. | 1.326 |

| Problem 76 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 77 | 70 | 0 | 234 | 0 | 108 |
| normalized size | 1 | 1. | 0.61 | 0.56 | 0. | 1.86 | 0. | 0.86 |
| time (sec) | N/A | 0.2 | 0.037 | 0.007 | 0. | 1.466 | 0. | 1.281 |

| Problem 77 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 130 | 120 | 198 | 0 | 783 | 0 | 105 |
| normalized size | 1 | 1. | 0.92 | 1.52 | 0. | 6.02 | 0. | 0.81 |
| time (sec) | N/A | 0.206 | 0.253 | 0.022 | 0. | 1.472 | 0. | 1.408 |

| Problem 78 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 66 | 59 | 0 | 207 | 0 | 86 |
| normalized size | 1 | 1. | 0.65 | 0.58 | 0. | 2.05 | 0. | 0.85 |
| time (sec) | N/A | 0.161 | 0.03 | 0.005 | 0. | 1.413 | 0. | 1.271 |

| Problem 79 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 27 | 0 | 130 | 0 | 23 |
| normalized size | 1 | 1. | 1. | 1.08 | 0. | 5.2 | 0. | 0.92 |
| time (sec) | N/A | 0.037 | 0.017 | 0.004 | 0. | 1.429 | 0. | 1.236 |

| Problem 80 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 55 | 48 | 0 | 184 | 0 | 68 |
| normalized size | 1 | 1. | 0.72 | 0.63 | 0. | 2.42 | 0. | 0.89 |
| time (sec) | N/A | 0.119 | 0.026 | 0.005 | 0. | 1.426 | 0. | 1.249 |

| Problem 81 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 44 | 37 | 0 | 159 | 0 | 39 |
| normalized size | 1 | 1. | 0.86 | 0.73 | 0. | 3.12 | 0. | 0.76 |
| time (sec) | N/A | 0.075 | 0.021 | 0.004 | 0. | 1.433 | 0. | 1.337 |

| Problem 82 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 44 | 37 | 0 | 158 | 0 | 45 |
| normalized size | 1 | 1. | 0.86 | 0.73 | 0. | 3.1 | 0. | 0.88 |
| time (sec) | N/A | 0.074 | 0.02 | 0.006 | 0. | 1.592 | 0. | 1.263 |

| Problem 83 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 55 | 48 | 0 | 185 | 0 | 58 |
| normalized size | 1 | 1. | 0.72 | 0.63 | 0. | 2.43 | 0. | 0.76 |
| time (sec) | N/A | 0.113 | 0.021 | 0.007 | 0. | 1.66 | 0. | 1.274 |

| Problem 84 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | B | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 27 | 0 | 132 | 0 | 31 |
| normalized size | 1 | 1. | 1. | 1.08 | 0. | 5.28 | 0. | 1.24 |
| time (sec) | N/A | 0.037 | 0.014 | 0.003 | 0. | 1.709 | 0. | 1.274 |

| Problem 85 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 66 | 59 | 0 | 203 | 0 | 74 |
| normalized size | 1 | 1. | 0.65 | 0.58 | 0. | 2.01 | 0. | 0.73 |
| time (sec) | N/A | 0.157 | 0.023 | 0.005 | 0. | 1.501 | 0. | 1.311 |

| Problem 86 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 130 | 43 | 217 | 0 | 803 | 0 | 154 |
| normalized size | 1 | 1. | 0.33 | 1.67 | 0. | 6.18 | 0. | 1.18 |
| time (sec) | N/A | 0.203 | 0.012 | 0.012 | 0. | 1.295 | 0. | 1.288 |

| Problem 87 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 77 | 70 | 0 | 239 | 0 | 96 |
| normalized size | 1 | 1. | 0.61 | 0.56 | 0. | 1.9 | 0. | 0.76 |
| time (sec) | N/A | 0.192 | 0.03 | 0.004 | 0. | 1.671 | 0. | 1.441 |

| Problem 88 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 159 | 44 | 234 | 0 | 884 | 0 | 142 |
| normalized size | 1 | 1. | 0.28 | 1.47 | 0. | 5.56 | 0. | 0.89 |
| time (sec) | N/A | 0.241 | 0.02 | 0.013 | 0. | 1.377 | 0. | 1.329 |

| Problem 89 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 152 | 88 | 81 | 0 | 265 | 0 | 116 |
| normalized size | 1 | 1. | 0.58 | 0.53 | 0. | 1.74 | 0. | 0.76 |
| time (sec) | N/A | 0.233 | 0.025 | 0.006 | 0. | 1.851 | 0. | 1.38 |

| Problem 90 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 46 | 247 | 0 | 963 | 0 | 165 |
| normalized size | 1 | 1. | 0.24 | 1.31 | 0. | 5.1 | 0. | 0.87 |
| time (sec) | N/A | 0.293 | 0.019 | 0.013 | 0. | 1.426 | 0. | 1.359 |

| Problem 91 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 180 | 99 | 92 | 0 | 298 | 0 | 136 |
| normalized size | 1 | 1. | 0.55 | 0.51 | 0. | 1.66 | 0. | 0.76 |
| time (sec) | N/A | 0.285 | 0.031 | 0.007 | 0. | 2.285 | 0. | 1.402 |

| Problem 92 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 55 | 81 | 997 | 0 | 315 | 0 | 61 |
| normalized size | 1 | 1. | 1.47 | 18.13 | 0. | 5.73 | 0. | 1.11 |
| time (sec) | N/A | 0.07 | 0.033 | 0.032 | 0. | 1.98 | 0. | 1.323 |

| Problem 93 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 61 | 979 | 0 | 225 | 0 | 31 |
| normalized size | 1 | 1. | 1.91 | 30.59 | 0. | 7.03 | 0. | 0.97 |
| time (sec) | N/A | 0.032 | 0.013 | 0.015 | 0. | 1.896 | 0. | 1.309 |

| Problem 94 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 23 | 27 | 35 | 43 | 0 | 19 |
| normalized size | 1 | 1. | 1. | 1.17 | 1.52 | 1.87 | 0. | 0.83 |
| time (sec) | N/A | 0.034 | 0.011 | 0.003 | 1.067 | 1.268 | 0. | 1.197 |

| Problem 95 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 48 | 31 | 35 | 51 | 63 | 0 | 36 |
| normalized size | 1 | 1. | 0.65 | 0.73 | 1.06 | 1.31 | 0. | 0.75 |
| time (sec) | N/A | 0.066 | 0.014 | 0.004 | 1.014 | 1.199 | 0. | 1.2 |

| Problem 96 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 44 | 48 | 68 | 90 | 0 | 58 |
| normalized size | 1 | 1. | 0.59 | 0.65 | 0.92 | 1.22 | 0. | 0.78 |
| time (sec) | N/A | 0.102 | 0.015 | 0.005 | 1.081 | 1.186 | 0. | 1.235 |

| Problem 97 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 224 | 64 | 688 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.29 | 3.07 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.213 | 0.023 | 0.014 | 0. | 0. | 0. | 0. |

| Problem 98 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 197 | 49 | 671 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.25 | 3.41 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.13 | 0.01 | 0.016 | 0. | 0. | 0. | 0. |

| Problem 99 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 225 | 225 | 53 | 696 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.24 | 3.09 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.187 | 0.013 | 0.018 | 0. | 0. | 0. | 0. |

| Problem 100 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 503 | 503 | 66 | 1079 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.13 | 2.15 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.514 | 0.024 | 0.016 | 0. | 0. | 0. | 0. |

| Problem 101 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 474 | 474 | 53 | 1054 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.11 | 2.22 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.39 | 0.012 | 0.015 | 0. | 0. | 0. | 0. |

| Problem 102 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 497 | 497 | 48 | 1083 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.1 | 2.18 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.472 | 0.011 | 0.015 | 0. | 0. | 0. | 0. |

| Problem 103 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 174 | 151 | 223 | 0 | 0 | 0 | 150 |
| normalized size | 1 | 1. | 0.87 | 1.28 | 0. | 0. | 0. | 0.86 |
| time (sec) | N/A | 0.151 | 0.17 | 0.06 | 0. | 0. | 0. | 1.398 |

| Problem 104 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 129 | 181 | 0 | 0 | 0 | 112 |
| normalized size | 1 | 1. | 1.11 | 1.56 | 0. | 0. | 0. | 0.97 |
| time (sec) | N/A | 0.087 | 0.106 | 0.007 | 0. | 0. | 0. | 1.366 |

| Problem 105 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 88 | 83 | 0 | 0 | 0 | 73 |
| normalized size | 1 | 1. | 1.57 | 1.48 | 0. | 0. | 0. | 1.3 |
| time (sec) | N/A | 0.06 | 0.048 | 0.003 | 0. | 0. | 0. | 1.334 |

| Problem 106 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 160 | 0 | 51 | 0 | 34 |
| normalized size | 1 | 1. | 1. | 6.4 | 0. | 2.04 | 0. | 1.36 |
| time (sec) | N/A | 0.038 | 0.008 | 0.011 | 0. | 2.431 | 0. | 1.222 |

| Problem 107 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 48 | 218 | 0 | 103 | 0 | 113 |
| normalized size | 1 | 1. | 0.57 | 2.6 | 0. | 1.23 | 0. | 1.35 |
| time (sec) | N/A | 0.117 | 0.048 | 0.01 | 0. | 2.315 | 0. | 1.252 |

| Problem 108 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 142 | 72 | 262 | 0 | 155 | 0 | 197 |
| normalized size | 1 | 1. | 0.51 | 1.85 | 0. | 1.09 | 0. | 1.39 |
| time (sec) | N/A | 0.203 | 0.053 | 0.013 | 0. | 2.336 | 0. | 1.169 |

| Problem 109 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 200 | 96 | 306 | 0 | 212 | 0 | 281 |
| normalized size | 1 | 1. | 0.48 | 1.53 | 0. | 1.06 | 0. | 1.4 |
| time (sec) | N/A | 0.297 | 0.059 | 0.013 | 0. | 2.414 | 0. | 1.278 |

| Problem 110 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 197 | 64 | 549 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.32 | 2.79 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.175 | 0.06 | 0.012 | 0. | 0. | 0. | 0. |

| Problem 111 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 64 | 503 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.46 | 3.62 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.127 | 0.043 | 0.01 | 0. | 0. | 0. | 0. |

| Problem 112 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 64 | 237 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.83 | 3.08 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.074 | 0.038 | 0.006 | 0. | 0. | 0. | 0. |

| Problem 113 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 404 | 0 | 77 | 0 | 46 |
| normalized size | 1 | 1. | 1. | 16.16 | 0. | 3.08 | 0. | 1.84 |
| time (sec) | N/A | 0.005 | 0.016 | 0.008 | 0. | 2.282 | 0. | 1.147 |

| Problem 114 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 48 | 524 | 0 | 132 | 0 | 0 |
| normalized size | 1 | 1. | 0.61 | 6.63 | 0. | 1.67 | 0. | 0. |
| time (sec) | N/A | 0.122 | 0.052 | 0.011 | 0. | 1.871 | 0. | 0. |

| Problem 115 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 72 | 570 | 0 | 190 | 0 | 0 |
| normalized size | 1 | 1. | 0.53 | 4.16 | 0. | 1.39 | 0. | 0. |
| time (sec) | N/A | 0.202 | 0.059 | 0.01 | 0. | 1.789 | 0. | 0. |

| Problem 116 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 96 | 614 | 0 | 246 | 0 | 0 |
| normalized size | 1 | 1. | 0.49 | 3.15 | 0. | 1.26 | 0. | 0. |
| time (sec) | N/A | 0.3 | 0.063 | 0.012 | 0. | 1.9 | 0. | 0. |

| Problem 117 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 204 | 204 | 164 | 245 | 0 | 0 | 0 | 169 |
| normalized size | 1 | 1. | 0.8 | 1.2 | 0. | 0. | 0. | 0.83 |
| time (sec) | N/A | 0.172 | 0.179 | 0.01 | 0. | 0. | 0. | 1.325 |

| Problem 118 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 146 | 142 | 203 | 0 | 0 | 0 | 131 |
| normalized size | 1 | 1. | 0.97 | 1.39 | 0. | 0. | 0. | 0.9 |
| time (sec) | N/A | 0.12 | 0.137 | 0.006 | 0. | 0. | 0. | 1.311 |

| Problem 119 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 102 | 160 | 0 | 0 | 0 | 93 |
| normalized size | 1 | 1. | 1.17 | 1.84 | 0. | 0. | 0. | 1.07 |
| time (sec) | N/A | 0.079 | 0.08 | 0.007 | 0. | 0. | 0. | 1.346 |

| Problem 120 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 65 | 133 | 0 | 0 | 0 | 50 |
| normalized size | 1 | 1. | 1.91 | 3.91 | 0. | 0. | 0. | 1.47 |
| time (sec) | N/A | 0.049 | 0.042 | 0.009 | 0. | 0. | 0. | 1.291 |

| Problem 121 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 37 | 194 | 0 | 72 | 0 | 72 |
| normalized size | 1 | 1. | 0.69 | 3.59 | 0. | 1.33 | 0. | 1.33 |
| time (sec) | N/A | 0.074 | 0.047 | 0.009 | 0. | 2.305 | 0. | 1.278 |

| Problem 122 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 59 | 240 | 0 | 123 | 0 | 155 |
| normalized size | 1 | 1. | 0.53 | 2.14 | 0. | 1.1 | 0. | 1.38 |
| time (sec) | N/A | 0.154 | 0.05 | 0.008 | 0. | 2.368 | 0. | 1.228 |

| Problem 123 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 170 | 83 | 284 | 0 | 178 | 0 | 239 |
| normalized size | 1 | 1. | 0.49 | 1.67 | 0. | 1.05 | 0. | 1.41 |
| time (sec) | N/A | 0.245 | 0.055 | 0.01 | 0. | 2.135 | 0. | 1.189 |

| Problem 124 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 171 | 62 | 527 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.36 | 3.08 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.147 | 0.052 | 0.009 | 0. | 0. | 0. | 0. |

| Problem 125 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | F(-1) | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 113 | 62 | 440 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.55 | 3.89 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.103 | 0.049 | 0.008 | 0. | 0. | 0. | 0. |

| Problem 126 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 79 | 238 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.32 | 3.97 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.067 | 0.105 | 0.006 | 0. | 0. | 0. | 0. |

| Problem 127 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 45 | 111 | 0 | 109 | 0 | 35 |
| normalized size | 1 | 1. | 1.5 | 3.7 | 0. | 3.63 | 0. | 1.17 |
| time (sec) | N/A | 0.047 | 0.037 | 0.01 | 0. | 2.292 | 0. | 1.217 |

| Problem 128 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 57 | 548 | 0 | 163 | 0 | 0 |
| normalized size | 1 | 1. | 0.53 | 5.12 | 0. | 1.52 | 0. | 0. |
| time (sec) | N/A | 0.156 | 0.051 | 0.011 | 0. | 2.343 | 0. | 0. |

| Problem 129 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 165 | 83 | 592 | 0 | 220 | 0 | 0 |
| normalized size | 1 | 1. | 0.5 | 3.59 | 0. | 1.33 | 0. | 0. |
| time (sec) | N/A | 0.257 | 0.052 | 0.01 | 0. | 2.399 | 0. | 0. |

| Problem 130 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 223 | 107 | 636 | 0 | 278 | 0 | 0 |
| normalized size | 1 | 1. | 0.48 | 2.85 | 0. | 1.25 | 0. | 0. |
| time (sec) | N/A | 0.353 | 0.063 | 0.011 | 0. | 2.363 | 0. | 0. |

| Problem 131 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 301 | 301 | 155 | 264 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.51 | 0.88 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.509 | 0.161 | 0.033 | 0. | 0. | 0. | 0. |

| Problem 132 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 411 | 411 | 136 | 273 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.33 | 0.66 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.578 | 0.114 | 0.011 | 0. | 0. | 0. | 0. |

| Problem 133 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 213 | 118 | 198 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.55 | 0.93 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.299 | 0.099 | 0.013 | 0. | 0. | 0. | 0. |

| Problem 134 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 323 | 323 | 94 | 207 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.29 | 0.64 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.326 | 0.054 | 0.012 | 0. | 0. | 0. | 0. |

| Problem 135 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 123 | 54 | 132 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.44 | 1.07 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.146 | 0.044 | 0.012 | 0. | 0. | 0. | 0. |

| Problem 136 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 325 | 325 | 59 | 213 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.18 | 0.66 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.353 | 0.046 | 0.018 | 0. | 0. | 0. | 0. |

| Problem 137 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 188 | 59 | 179 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.31 | 0.95 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.244 | 0.05 | 0.017 | 0. | 0. | 0. | 0. |

| Problem 138 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 413 | 413 | 59 | 281 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.14 | 0.68 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.544 | 0.047 | 0.018 | 0. | 0. | 0. | 0. |

| Problem 139 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 276 | 276 | 59 | 245 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.21 | 0.89 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.413 | 0.055 | 0.019 | 0. | 0. | 0. | 0. |

| Problem 140 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 298 | 298 | 142 | 197 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.48 | 0.66 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.501 | 0.138 | 0.027 | 0. | 0. | 0. | 0. |

| Problem 141 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 408 | 408 | 123 | 263 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.3 | 0.64 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.554 | 0.1 | 0.018 | 0. | 0. | 0. | 0. |

| Problem 142 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 208 | 106 | 164 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.51 | 0.79 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.271 | 0.081 | 0.017 | 0. | 0. | 0. | 0. |

| Problem 143 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 319 | 319 | 60 | 230 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.19 | 0.72 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.329 | 0.046 | 0.017 | 0. | 0. | 0. | 0. |

| Problem 144 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 144 | 60 | 130 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.42 | 0.9 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.202 | 0.058 | 0.02 | 0. | 0. | 0. | 0. |

| Problem 145 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 350 | 350 | 62 | 339 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.18 | 0.97 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.431 | 0.053 | 0.022 | 0. | 0. | 0. | 0. |

| Problem 146 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 213 | 62 | 168 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.29 | 0.79 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.309 | 0.066 | 0.023 | 0. | 0. | 0. | 0. |

| Problem 147 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 438 | 438 | 62 | 411 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.14 | 0.94 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.64 | 0.055 | 0.026 | 0. | 0. | 0. | 0. |

| Problem 148 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 301 | 301 | 62 | 201 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.21 | 0.67 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.479 | 0.07 | 0.026 | 0. | 0. | 0. | 0. |

| Problem 149 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 304 | 161 | 196 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.53 | 0.64 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.506 | 0.092 | 0.032 | 0. | 0. | 0. | 0. |

| Problem 150 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 414 | 414 | 143 | 261 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.35 | 0.63 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.566 | 0.071 | 0.024 | 0. | 0. | 0. | 0. |

| Problem 151 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 216 | 216 | 124 | 164 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.57 | 0.76 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.319 | 0.067 | 0.007 | 0. | 0. | 0. | 0. |

| Problem 152 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 326 | 326 | 106 | 230 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.33 | 0.71 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.349 | 0.057 | 0.008 | 0. | 0. | 0. | 0. |

| Problem 153 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 80 | 128 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.63 | 1.02 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.118 | 0.036 | 0.005 | 0. | 0. | 0. | 0. |

| Problem 154 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 294 | 294 | 54 | 253 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.18 | 0.86 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.294 | 0.049 | 0.018 | 0. | 0. | 0. | 0. |

| Problem 155 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 163 | 163 | 59 | 142 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.36 | 0.87 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.201 | 0.05 | 0.022 | 0. | 0. | 0. | 0. |

| Problem 156 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 388 | 388 | 59 | 363 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.15 | 0.94 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.476 | 0.05 | 0.025 | 0. | 0. | 0. | 0. |

| Problem 157 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 251 | 251 | 59 | 179 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.24 | 0.71 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.345 | 0.057 | 0.026 | 0. | 0. | 0. | 0. |

| Problem 158 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 437 | 437 | 131 | 384 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.3 | 0.88 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.674 | 0.093 | 0.026 | 0. | 0. | 0. | 0. |

| Problem 159 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 239 | 239 | 124 | 261 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.52 | 1.09 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.377 | 0.083 | 0.024 | 0. | 0. | 0. | 0. |

| Problem 160 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 349 | 349 | 94 | 314 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.27 | 0.9 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.426 | 0.067 | 0.011 | 0. | 0. | 0. | 0. |

| Problem 161 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 149 | 82 | 185 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.55 | 1.24 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.209 | 0.063 | 0.01 | 0. | 0. | 0. | 0. |

| Problem 162 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 296 | 296 | 62 | 245 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.21 | 0.83 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.26 | 0.029 | 0.005 | 0. | 0. | 0. | 0. |

| Problem 163 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 62 | 181 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.39 | 1.15 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.205 | 0.06 | 0.012 | 0. | 0. | 0. | 0. |

| Problem 164 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 383 | 383 | 64 | 341 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.17 | 0.89 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.479 | 0.056 | 0.011 | 0. | 0. | 0. | 0. |

| Problem 165 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 246 | 246 | 64 | 262 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.26 | 1.07 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.365 | 0.056 | 0.014 | 0. | 0. | 0. | 0. |

| Problem 166 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 471 | 471 | 64 | 413 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.14 | 0.88 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.694 | 0.055 | 0.013 | 0. | 0. | 0. | 0. |

| Problem 167 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 371 | 371 | 181 | 156 | 0 | 0 | 0 | 259 |
| normalized size | 1 | 1. | 0.49 | 0.42 | 0. | 0. | 0. | 0.7 |
| time (sec) | N/A | 0.627 | 0.135 | 0.01 | 0. | 0. | 0. | 1.133 |

| Problem 168 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 283 | 283 | 144 | 123 | 0 | 0 | 0 | 203 |
| normalized size | 1 | 1. | 0.51 | 0.43 | 0. | 0. | 0. | 0.72 |
| time (sec) | N/A | 0.441 | 0.095 | 0.004 | 0. | 0. | 0. | 1.126 |

| Problem 169 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 107 | 90 | 0 | 0 | 0 | 146 |
| normalized size | 1 | 1. | 0.55 | 0.46 | 0. | 0. | 0. | 0.75 |
| time (sec) | N/A | 0.273 | 0.069 | 0.002 | 0. | 0. | 0. | 1.131 |

| Problem 170 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 70 | 57 | 0 | 0 | 0 | 89 |
| normalized size | 1 | 1. | 0.64 | 0.52 | 0. | 0. | 0. | 0.82 |
| time (sec) | N/A | 0.137 | 0.04 | 0.003 | 0. | 0. | 0. | 1.119 |

| Problem 171 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 23 | 27 | 0 | 0 | 0 | 31 |
| normalized size | 1 | 1. | 1. | 1.17 | 0. | 0. | 0. | 1.35 |
| time (sec) | N/A | 0.04 | 0.011 | 0.003 | 0. | 0. | 0. | 1.126 |

| Problem 172 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 57 | 80 | 0 | 0 | 0 | 97 |
| normalized size | 1 | 1. | 0.63 | 0.89 | 0. | 0. | 0. | 1.08 |
| time (sec) | N/A | 0.139 | 0.042 | 0.01 | 0. | 0. | 0. | 1.217 |

| Problem 173 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 178 | 57 | 125 | 0 | 0 | 0 | 170 |
| normalized size | 1 | 1. | 0.32 | 0.7 | 0. | 0. | 0. | 0.96 |
| time (sec) | N/A | 0.296 | 0.041 | 0.011 | 0. | 0. | 0. | 1.205 |

| Problem 174 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 266 | 266 | 57 | 167 | 0 | 0 | 0 | 239 |
| normalized size | 1 | 1. | 0.21 | 0.63 | 0. | 0. | 0. | 0.9 |
| time (sec) | N/A | 0.475 | 0.046 | 0.013 | 0. | 0. | 0. | 1.269 |

| Problem 175 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 354 | 354 | 57 | 209 | 0 | 0 | 0 | 308 |
| normalized size | 1 | 1. | 0.16 | 0.59 | 0. | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.66 | 0.049 | 0.015 | 0. | 0. | 0. | 1.399 |

| Problem 176 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 343 | 343 | 172 | 145 | 0 | 0 | 0 | 508 |
| normalized size | 1 | 1. | 0.5 | 0.42 | 0. | 0. | 0. | 1.48 |
| time (sec) | N/A | 0.616 | 0.131 | 0.004 | 0. | 0. | 0. | 1.183 |

| Problem 177 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 255 | 255 | 135 | 112 | 0 | 0 | 0 | 394 |
| normalized size | 1 | 1. | 0.53 | 0.44 | 0. | 0. | 0. | 1.55 |
| time (sec) | N/A | 0.422 | 0.086 | 0.004 | 0. | 0. | 0. | 1.169 |

| Problem 178 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 169 | 98 | 79 | 0 | 0 | 0 | 281 |
| normalized size | 1 | 1. | 0.58 | 0.47 | 0. | 0. | 0. | 1.66 |
| time (sec) | N/A | 0.249 | 0.058 | 0.003 | 0. | 0. | 0. | 1.228 |

| Problem 179 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 63 | 48 | 0 | 0 | 0 | 155 |
| normalized size | 1 | 1. | 0.75 | 0.57 | 0. | 0. | 0. | 1.85 |
| time (sec) | N/A | 0.139 | 0.058 | 0.004 | 0. | 0. | 0. | 1.177 |

| Problem 180 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 88 | 69 | 0 | 0 | 0 | 112 |
| normalized size | 1 | 1. | 1.13 | 0.88 | 0. | 0. | 0. | 1.44 |
| time (sec) | N/A | 0.137 | 0.069 | 0.004 | 0. | 0. | 0. | 1.215 |

| Problem 181 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 113 | 61 | 93 | 0 | 0 | 0 | 124 |
| normalized size | 1 | 1. | 0.54 | 0.82 | 0. | 0. | 0. | 1.1 |
| time (sec) | N/A | 0.184 | 0.053 | 0.01 | 0. | 0. | 0. | 1.245 |

| Problem 182 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 203 | 61 | 139 | 0 | 0 | 0 | 193 |
| normalized size | 1 | 1. | 0.3 | 0.68 | 0. | 0. | 0. | 0.95 |
| time (sec) | N/A | 0.34 | 0.046 | 0.012 | 0. | 0. | 0. | 1.251 |

| Problem 183 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 291 | 291 | 61 | 181 | 0 | 0 | 0 | 262 |
| normalized size | 1 | 1. | 0.21 | 0.62 | 0. | 0. | 0. | 0.9 |
| time (sec) | N/A | 0.522 | 0.05 | 0.014 | 0. | 0. | 0. | 1.372 |

| Problem 184 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 379 | 379 | 61 | 223 | 0 | 0 | 0 | 331 |
| normalized size | 1 | 1. | 0.16 | 0.59 | 0. | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.718 | 0.061 | 0.016 | 0. | 0. | 0. | 1.447 |

| Problem 185 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 401 | 401 | 185 | 167 | 0 | 0 | 0 | 278 |
| normalized size | 1 | 1. | 0.46 | 0.42 | 0. | 0. | 0. | 0.69 |
| time (sec) | N/A | 0.728 | 0.153 | 0.004 | 0. | 0. | 0. | 1.144 |

| Problem 186 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 313 | 313 | 148 | 134 | 0 | 0 | 0 | 221 |
| normalized size | 1 | 1. | 0.47 | 0.43 | 0. | 0. | 0. | 0.71 |
| time (sec) | N/A | 0.531 | 0.115 | 0.003 | 0. | 0. | 0. | 1.157 |

| Problem 187 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 225 | 225 | 111 | 101 | 0 | 0 | 0 | 165 |
| normalized size | 1 | 1. | 0.49 | 0.45 | 0. | 0. | 0. | 0.73 |
| time (sec) | N/A | 0.346 | 0.088 | 0.005 | 0. | 0. | 0. | 1.166 |

| Problem 188 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 74 | 68 | 0 | 0 | 0 | 108 |
| normalized size | 1 | 1. | 0.54 | 0.5 | 0. | 0. | 0. | 0.79 |
| time (sec) | N/A | 0.18 | 0.064 | 0.003 | 0. | 0. | 0. | 1.133 |

| Problem 189 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 36 | 36 | 0 | 0 | 0 | 49 |
| normalized size | 1 | 1. | 0.77 | 0.77 | 0. | 0. | 0. | 1.04 |
| time (sec) | N/A | 0.05 | 0.027 | 0.003 | 0. | 0. | 0. | 1.142 |

| Problem 190 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 90 | 61 | 0 | 0 | 0 | 69 |
| normalized size | 1 | 1. | 1.48 | 1. | 0. | 0. | 0. | 1.13 |
| time (sec) | N/A | 0.093 | 0.087 | 0.007 | 0. | 0. | 0. | 1.156 |

| Problem 191 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 153 | 48 | 126 | 0 | 0 | 0 | 147 |
| normalized size | 1 | 1. | 0.31 | 0.82 | 0. | 0. | 0. | 0.96 |
| time (sec) | N/A | 0.239 | 0.055 | 0.007 | 0. | 0. | 0. | 1.19 |

| Problem 192 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 241 | 241 | 48 | 188 | 0 | 0 | 0 | 216 |
| normalized size | 1 | 1. | 0.2 | 0.78 | 0. | 0. | 0. | 0.9 |
| time (sec) | N/A | 0.408 | 0.056 | 0.006 | 0. | 0. | 0. | 1.269 |

| Problem 193 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 329 | 329 | 48 | 248 | 0 | 0 | 0 | 285 |
| normalized size | 1 | 1. | 0.15 | 0.75 | 0. | 0. | 0. | 0.87 |
| time (sec) | N/A | 0.577 | 0.055 | 0.007 | 0. | 0. | 0. | 1.354 |

| Problem 194 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 336 | 336 | 161 | 143 | 0 | 0 | 0 | 289 |
| normalized size | 1 | 1. | 0.48 | 0.43 | 0. | 0. | 0. | 0.86 |
| time (sec) | N/A | 0.599 | 0.133 | 0.005 | 0. | 0. | 0. | 1.234 |

| Problem 195 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 248 | 248 | 122 | 110 | 0 | 0 | 0 | 220 |
| normalized size | 1 | 1. | 0.49 | 0.44 | 0. | 0. | 0. | 0.89 |
| time (sec) | N/A | 0.414 | 0.089 | 0.006 | 0. | 0. | 0. | 1.215 |

| Problem 196 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 160 | 85 | 77 | 0 | 0 | 0 | 151 |
| normalized size | 1 | 1. | 0.53 | 0.48 | 0. | 0. | 0. | 0.94 |
| time (sec) | N/A | 0.242 | 0.071 | 0.006 | 0. | 0. | 0. | 1.145 |

| Problem 197 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 60 | 45 | 0 | 0 | 0 | 81 |
| normalized size | 1 | 1. | 0.88 | 0.66 | 0. | 0. | 0. | 1.19 |
| time (sec) | N/A | 0.084 | 0.05 | 0.005 | 0. | 0. | 0. | 1.179 |

| Problem 198 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 45 | 55 | 0 | 0 | 0 | 96 |
| normalized size | 1 | 1. | 0.75 | 0.92 | 0. | 0. | 0. | 1.6 |
| time (sec) | N/A | 0.056 | 0.026 | 0.006 | 0. | 0. | 0. | 1.16 |

| Problem 199 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 146 | 48 | 88 | 0 | 0 | 0 | 142 |
| normalized size | 1 | 1. | 0.33 | 0.6 | 0. | 0. | 0. | 0.97 |
| time (sec) | N/A | 0.241 | 0.057 | 0.013 | 0. | 0. | 0. | 1.232 |

| Problem 200 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 236 | 236 | 48 | 126 | 0 | 0 | 0 | 211 |
| normalized size | 1 | 1. | 0.2 | 0.53 | 0. | 0. | 0. | 0.89 |
| time (sec) | N/A | 0.41 | 0.058 | 0.017 | 0. | 0. | 0. | 1.298 |

| Problem 201 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 324 | 324 | 48 | 159 | 0 | 0 | 0 | 279 |
| normalized size | 1 | 1. | 0.15 | 0.49 | 0. | 0. | 0. | 0.86 |
| time (sec) | N/A | 0.597 | 0.057 | 0.018 | 0. | 0. | 0. | 1.366 |

| Problem 202 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F(-1) | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 412 | 412 | 48 | 192 | 0 | 0 | 0 | 348 |
| normalized size | 1 | 1. | 0.12 | 0.47 | 0. | 0. | 0. | 0.84 |
| time (sec) | N/A | 0.84 | 0.059 | 0.025 | 0. | 0. | 0. | 1.519 |

| Problem 203 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 14 | 18 | 31 | 12 | 18 |
| normalized size | 1 | 1. | 1. | 0.82 | 1.06 | 1.82 | 0.71 | 1.06 |
| time (sec) | N/A | 0.009 | 0.001 | 0.001 | 0.994 | 1.061 | 0.117 | 1.148 |

| Problem 204 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 14 | 18 | 31 | 12 | 18 |
| normalized size | 1 | 1. | 1. | 0.82 | 1.06 | 1.82 | 0.71 | 1.06 |
| time (sec) | N/A | 0.008 | 0.001 | 0.001 | 0.989 | 1.024 | 0.142 | 1.104 |

| Problem 205 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 14 | 18 | 31 | 12 | 18 |
| normalized size | 1 | 1. | 1. | 0.82 | 1.06 | 1.82 | 0.71 | 1.06 |
| time (sec) | N/A | 0.003 | 0. | 0. | 0.99 | 1.09 | 0.218 | 1.154 |

| Problem 206 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 14 | 18 | 31 | 12 | 18 |
| normalized size | 1 | 1. | 1. | 0.82 | 1.06 | 1.82 | 0.71 | 1.06 |
| time (sec) | N/A | 0.006 | 0.001 | 0. | 0.985 | 0.958 | 0.081 | 1.198 |

| Problem 207 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 11 | 14 | 23 | 8 | 14 |
| normalized size | 1 | 1. | 1. | 0.92 | 1.17 | 1.92 | 0.67 | 1.17 |
| time (sec) | N/A | 0.004 | 0. | 0.001 | 0.975 | 0.801 | 0.101 | 1.133 |

| Problem 208 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 25 | 32 | 55 | 24 | 32 |
| normalized size | 1 | 1. | 1. | 0.83 | 1.07 | 1.83 | 0.8 | 1.07 |
| time (sec) | N/A | 0.031 | 0.002 | 0. | 0.986 | 0.575 | 0.123 | 1.143 |

| Problem 209 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 25 | 32 | 55 | 26 | 32 |
| normalized size | 1 | 1. | 1. | 0.83 | 1.07 | 1.83 | 0.87 | 1.07 |
| time (sec) | N/A | 0.017 | 0.002 | 0. | 0.98 | 0.617 | 0.242 | 1.133 |

| Problem 210 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 25 | 32 | 55 | 24 | 32 |
| normalized size | 1 | 1. | 1. | 0.83 | 1.07 | 1.83 | 0.8 | 1.07 |
| time (sec) | N/A | 0.013 | 0.002 | 0. | 0.993 | 0.627 | 0.122 | 1.154 |

| Problem 211 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 25 | 32 | 55 | 26 | 32 |
| normalized size | 1 | 1. | 1. | 0.83 | 1.07 | 1.83 | 0.87 | 1.07 |
| time (sec) | N/A | 0.017 | 0.002 | 0.002 | 1.012 | 0.705 | 0.24 | 1.147 |

| Problem 212 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 25 | 32 | 55 | 24 | 32 |
| normalized size | 1 | 1. | 1. | 0.83 | 1.07 | 1.83 | 0.8 | 1.07 |
| time (sec) | N/A | 0.016 | 0.002 | 0. | 0.993 | 0.711 | 0.22 | 1.206 |

| Problem 213 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 57 | 52 | 70 | 117 | 49 | 72 |
| normalized size | 1 | 1. | 1. | 0.91 | 1.23 | 2.05 | 0.86 | 1.26 |
| time (sec) | N/A | 0.036 | 0.005 | 0.003 | 0.961 | 0.735 | 0.722 | 1.146 |

| Problem 214 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 44 | 41 | 57 | 92 | 37 | 58 |
| normalized size | 1 | 1. | 1. | 0.93 | 1.3 | 2.09 | 0.84 | 1.32 |
| time (sec) | N/A | 0.027 | 0.004 | 0.002 | 0.971 | 0.872 | 0.487 | 1.129 |

| Problem 215 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 31 | 30 | 39 | 68 | 26 | 41 |
| normalized size | 1 | 1. | 1. | 0.97 | 1.26 | 2.19 | 0.84 | 1.32 |
| time (sec) | N/A | 0.021 | 0.003 | 0.001 | 0.977 | 0.69 | 0.625 | 1.157 |

| Problem 216 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 19 | 24 | 38 | 14 | 26 |
| normalized size | 1 | 1. | 1. | 1.06 | 1.33 | 2.11 | 0.78 | 1.44 |
| time (sec) | N/A | 0.016 | 0.003 | 0.002 | 0.989 | 0.831 | 0.648 | 1.152 |

| Problem 217 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 10 | 10 | 11 | 14 | 22 | 7 | 15 |
| normalized size | 1 | 1. | 1. | 1.1 | 1.4 | 2.2 | 0.7 | 1.5 |
| time (sec) | N/A | 0.007 | 0.001 | 0.001 | 1.097 | 0.762 | 0.23 | 1.153 |

| Problem 218 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 19 | 24 | 38 | 10 | 27 |
| normalized size | 1 | 1. | 1. | 1.06 | 1.33 | 2.11 | 0.56 | 1.5 |
| time (sec) | N/A | 0.007 | 0.004 | 0.005 | 0.979 | 0.781 | 0.543 | 1.16 |

| Problem 219 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 28 | 29 | 38 | 61 | 19 | 41 |
| normalized size | 1 | 1. | 1. | 1.04 | 1.36 | 2.18 | 0.68 | 1.46 |
| time (sec) | N/A | 0.015 | 0.004 | 0.006 | 1.017 | 0.796 | 0.715 | 1.141 |

| Problem 220 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 42 | 41 | 54 | 103 | 31 | 61 |
| normalized size | 1 | 1. | 1. | 0.98 | 1.29 | 2.45 | 0.74 | 1.45 |
| time (sec) | N/A | 0.021 | 0.004 | 0.007 | 1.041 | 0.781 | 1.032 | 1.154 |

| Problem 221 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 56 | 53 | 69 | 126 | 44 | 76 |
| normalized size | 1 | 1. | 1. | 0.95 | 1.23 | 2.25 | 0.79 | 1.36 |
| time (sec) | N/A | 0.027 | 0.005 | 0.006 | 0.972 | 0.726 | 1.21 | 1.209 |

| Problem 222 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 54 | 57 | 80 | 155 | 54 | 84 |
| normalized size | 1 | 1. | 0.93 | 0.98 | 1.38 | 2.67 | 0.93 | 1.45 |
| time (sec) | N/A | 0.04 | 0.02 | 0.006 | 0.976 | 0.679 | 1.195 | 1.153 |

| Problem 223 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 43 | 45 | 63 | 132 | 44 | 65 |
| normalized size | 1 | 1. | 0.93 | 0.98 | 1.37 | 2.87 | 0.96 | 1.41 |
| time (sec) | N/A | 0.03 | 0.014 | 0.006 | 0.993 | 0.757 | 0.97 | 1.236 |

| Problem 224 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 29 | 34 | 49 | 97 | 31 | 46 |
| normalized size | 1 | 1. | 0.88 | 1.03 | 1.48 | 2.94 | 0.94 | 1.39 |
| time (sec) | N/A | 0.024 | 0.012 | 0.006 | 0.992 | 0.78 | 0.558 | 1.167 |

| Problem 225 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 20 | 24 | 35 | 62 | 20 | 32 |
| normalized size | 1 | 1. | 0.87 | 1.04 | 1.52 | 2.7 | 0.87 | 1.39 |
| time (sec) | N/A | 0.018 | 0.006 | 0.005 | 0.983 | 0.765 | 0.528 | 1.198 |

| Problem 226 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 13 | 18 | 24 | 10 | 16 |
| normalized size | 1 | 1. | 1. | 1.08 | 1.5 | 2. | 0.83 | 1.33 |
| time (sec) | N/A | 0.007 | 0.003 | 0.001 | 0.989 | 0.789 | 0.921 | 1.149 |

| Problem 227 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 24 | 30 | 38 | 89 | 22 | 42 |
| normalized size | 1 | 1. | 0.83 | 1.03 | 1.31 | 3.07 | 0.76 | 1.45 |
| time (sec) | N/A | 0.019 | 0.01 | 0.006 | 1.017 | 0.73 | 1.039 | 1.145 |

| Problem 228 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 35 | 43 | 61 | 138 | 36 | 61 |
| normalized size | 1 | 1. | 0.83 | 1.02 | 1.45 | 3.29 | 0.86 | 1.45 |
| time (sec) | N/A | 0.024 | 0.037 | 0.009 | 1.001 | 0.725 | 1.241 | 1.168 |

| Problem 229 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 53 | 57 | 86 | 177 | 54 | 86 |
| normalized size | 1 | 1. | 0.91 | 0.98 | 1.48 | 3.05 | 0.93 | 1.48 |
| time (sec) | N/A | 0.032 | 0.05 | 0.01 | 1.017 | 0.791 | 0.759 | 1.184 |

| Problem 230 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 66 | 68 | 99 | 204 | 66 | 99 |
| normalized size | 1 | 1. | 0.96 | 0.99 | 1.43 | 2.96 | 0.96 | 1.43 |
| time (sec) | N/A | 0.039 | 0.053 | 0.01 | 0.987 | 0.81 | 1.767 | 1.163 |

| Problem 231 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 79 | 79 | 116 | 231 | 80 | 116 |
| normalized size | 1 | 1. | 0.94 | 0.94 | 1.38 | 2.75 | 0.95 | 1.38 |
| time (sec) | N/A | 0.052 | 0.043 | 0.01 | 1.109 | 0.677 | 1.703 | 1.141 |

| Problem 232 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 53 | 57 | 72 | 134 | 0 | 84 |
| normalized size | 1 | 1. | 0.5 | 0.54 | 0.69 | 1.28 | 0. | 0.8 |
| time (sec) | N/A | 0.12 | 0.028 | 0.004 | 1.004 | 0.82 | 0. | 1.144 |

| Problem 233 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 42 | 46 | 57 | 111 | 0 | 68 |
| normalized size | 1 | 1. | 0.52 | 0.57 | 0.71 | 1.39 | 0. | 0.85 |
| time (sec) | N/A | 0.074 | 0.02 | 0.003 | 1.017 | 0.813 | 0. | 1.151 |

| Problem 234 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 31 | 35 | 41 | 84 | 0 | 51 |
| normalized size | 1 | 1. | 0.6 | 0.67 | 0.79 | 1.62 | 0. | 0.98 |
| time (sec) | N/A | 0.043 | 0.014 | 0.003 | 0.976 | 0.873 | 0. | 1.186 |

| Problem 235 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 23 | 27 | 16 | 55 | 0 | 34 |
| normalized size | 1 | 1. | 0.92 | 1.08 | 0.64 | 2.2 | 0. | 1.36 |
| time (sec) | N/A | 0.036 | 0.009 | 0.001 | 1.008 | 0.824 | 0. | 1.174 |

| Problem 236 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 53 | 52 | 0 | 247 | 0 | 88 |
| normalized size | 1 | 1. | 1.04 | 1.02 | 0. | 4.84 | 0. | 1.73 |
| time (sec) | N/A | 0.049 | 0.028 | 0.006 | 0. | 0.85 | 0. | 1.213 |

| Problem 237 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 48 | 56 | 0 | 282 | 0 | 58 |
| normalized size | 1 | 1. | 0.92 | 1.08 | 0. | 5.42 | 0. | 1.12 |
| time (sec) | N/A | 0.052 | 0.036 | 0.01 | 0. | 0.87 | 0. | 1.206 |

| Problem 238 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 42 | 73 | 0 | 338 | 0 | 92 |
| normalized size | 1 | 1. | 0.5 | 0.87 | 0. | 4.02 | 0. | 1.1 |
| time (sec) | N/A | 0.092 | 0.012 | 0.01 | 0. | 0.866 | 0. | 1.214 |

| Problem 239 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 42 | 89 | 0 | 393 | 0 | 116 |
| normalized size | 1 | 1. | 0.38 | 0.79 | 0. | 3.51 | 0. | 1.04 |
| time (sec) | N/A | 0.138 | 0.012 | 0.012 | 0. | 0.933 | 0. | 1.188 |

| Problem 240 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 161 | 80 | 79 | 116 | 217 | 0 | 242 |
| normalized size | 1 | 1. | 0.5 | 0.49 | 0.72 | 1.35 | 0. | 1.5 |
| time (sec) | N/A | 0.231 | 0.041 | 0.005 | 1.009 | 0.868 | 0. | 1.135 |

| Problem 241 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 136 | 69 | 68 | 101 | 193 | 0 | 211 |
| normalized size | 1 | 1. | 0.51 | 0.5 | 0.74 | 1.42 | 0. | 1.55 |
| time (sec) | N/A | 0.172 | 0.033 | 0.004 | 1.057 | 0.808 | 0. | 1.171 |

| Problem 242 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 108 | 58 | 57 | 86 | 161 | 0 | 177 |
| normalized size | 1 | 1. | 0.54 | 0.53 | 0.8 | 1.49 | 0. | 1.64 |
| time (sec) | N/A | 0.141 | 0.027 | 0.003 | 0.988 | 0.74 | 0. | 1.264 |

| Problem 243 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 47 | 46 | 72 | 134 | 0 | 144 |
| normalized size | 1 | 1. | 0.59 | 0.57 | 0.9 | 1.68 | 0. | 1.8 |
| time (sec) | N/A | 0.133 | 0.022 | 0.004 | 1.008 | 0.829 | 0. | 1.289 |

| Problem 244 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 36 | 35 | 55 | 105 | 0 | 112 |
| normalized size | 1 | 1. | 0.69 | 0.67 | 1.06 | 2.02 | 0. | 2.15 |
| time (sec) | N/A | 0.083 | 0.017 | 0.003 | 1.027 | 0.818 | 0. | 1.275 |

| Problem 245 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 23 | 27 | 38 | 77 | 0 | 70 |
| normalized size | 1 | 1. | 0.92 | 1.08 | 1.52 | 3.08 | 0. | 2.8 |
| time (sec) | N/A | 0.041 | 0.013 | 0.002 | 0.989 | 0.811 | 0. | 1.242 |

| Problem 246 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 74 | 68 | 63 | 0 | 296 | 0 | 115 |
| normalized size | 1 | 1. | 0.92 | 0.85 | 0. | 4. | 0. | 1.55 |
| time (sec) | N/A | 0.096 | 0.043 | 0.007 | 0. | 0.81 | 0. | 1.192 |

| Problem 247 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 40 | 72 | 0 | 304 | 0 | 84 |
| normalized size | 1 | 1. | 0.55 | 0.99 | 0. | 4.16 | 0. | 1.15 |
| time (sec) | N/A | 0.093 | 0.013 | 0.011 | 0. | 0.899 | 0. | 1.456 |

| Problem 248 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 72 | 74 | 0 | 342 | 0 | 95 |
| normalized size | 1 | 1. | 0.89 | 0.91 | 0. | 4.22 | 0. | 1.17 |
| time (sec) | N/A | 0.092 | 0.046 | 0.01 | 0. | 0.928 | 0. | 1.271 |

| Problem 249 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 42 | 87 | 0 | 397 | 0 | 124 |
| normalized size | 1 | 1. | 0.39 | 0.8 | 0. | 3.64 | 0. | 1.14 |
| time (sec) | N/A | 0.134 | 0.016 | 0.012 | 0. | 0.786 | 0. | 1.215 |

| Problem 250 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 42 | 101 | 0 | 443 | 0 | 147 |
| normalized size | 1 | 1. | 0.31 | 0.74 | 0. | 3.23 | 0. | 1.07 |
| time (sec) | N/A | 0.184 | 0.015 | 0.01 | 0. | 0.893 | 0. | 1.237 |

| Problem 251 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 165 | 42 | 113 | 0 | 504 | 0 | 170 |
| normalized size | 1 | 1. | 0.25 | 0.68 | 0. | 3.05 | 0. | 1.03 |
| time (sec) | N/A | 0.236 | 0.015 | 0.013 | 0. | 0.909 | 0. | 1.326 |

| Problem 252 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 53 | 55 | 72 | 109 | 0 | 0 |
| normalized size | 1 | 1. | 0.51 | 0.53 | 0.7 | 1.06 | 0. | 0. |
| time (sec) | N/A | 0.148 | 0.032 | 0.004 | 1.06 | 0.805 | 0. | 0. |

| Problem 253 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 42 | 44 | 57 | 86 | 0 | 0 |
| normalized size | 1 | 1. | 0.56 | 0.59 | 0.76 | 1.15 | 0. | 0. |
| time (sec) | N/A | 0.1 | 0.02 | 0.004 | 1.151 | 0.793 | 0. | 0. |

| Problem 254 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 30 | 33 | 41 | 61 | 0 | 0 |
| normalized size | 1 | 1. | 0.61 | 0.67 | 0.84 | 1.24 | 0. | 0. |
| time (sec) | N/A | 0.055 | 0.015 | 0.003 | 1.034 | 0.858 | 0. | 0. |

| Problem 255 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 21 | 25 | 16 | 39 | 0 | 35 |
| normalized size | 1 | 1. | 0.91 | 1.09 | 0.7 | 1.7 | 0. | 1.52 |
| time (sec) | N/A | 0.01 | 0.007 | 0.001 | 1.169 | 0.798 | 0. | 1.19 |

| Problem 256 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 46 | 39 | 0 | 171 | 0 | 61 |
| normalized size | 1 | 1. | 1.53 | 1.3 | 0. | 5.7 | 0. | 2.03 |
| time (sec) | N/A | 0.011 | 0.009 | 0.004 | 0. | 0.804 | 0. | 1.168 |

| Problem 257 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 66 | 55 | 0 | 289 | 0 | 0 |
| normalized size | 1 | 1. | 1.22 | 1.02 | 0. | 5.35 | 0. | 0. |
| time (sec) | N/A | 0.049 | 0.045 | 0.007 | 0. | 0.829 | 0. | 0. |

| Problem 258 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 40 | 77 | 0 | 347 | 0 | 0 |
| normalized size | 1 | 1. | 0.46 | 0.89 | 0. | 3.99 | 0. | 0. |
| time (sec) | N/A | 0.091 | 0.01 | 0.007 | 0. | 0.901 | 0. | 0. |

| Problem 259 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 40 | 95 | 0 | 402 | 0 | 0 |
| normalized size | 1 | 1. | 0.35 | 0.83 | 0. | 3.5 | 0. | 0. |
| time (sec) | N/A | 0.135 | 0.01 | 0.006 | 0. | 0.809 | 0. | 0. |

| Problem 260 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 50 | 56 | 55 | 122 | 0 | 0 |
| normalized size | 1 | 1. | 0.51 | 0.57 | 0.56 | 1.24 | 0. | 0. |
| time (sec) | N/A | 0.151 | 0.024 | 0.005 | 1.159 | 0.906 | 0. | 0. |

| Problem 261 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 39 | 46 | 41 | 99 | 0 | 0 |
| normalized size | 1 | 1. | 0.54 | 0.64 | 0.57 | 1.38 | 0. | 0. |
| time (sec) | N/A | 0.105 | 0.017 | 0.006 | 1.145 | 0.829 | 0. | 0. |

| Problem 262 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 26 | 34 | 26 | 74 | 0 | 38 |
| normalized size | 1 | 1. | 0.55 | 0.72 | 0.55 | 1.57 | 0. | 0.81 |
| time (sec) | N/A | 0.057 | 0.012 | 0.003 | 1.113 | 0.829 | 0. | 1.134 |

| Problem 263 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 19 | 27 | 16 | 57 | 0 | 50 |
| normalized size | 1 | 1. | 0.9 | 1.29 | 0.76 | 2.71 | 0. | 2.38 |
| time (sec) | N/A | 0.018 | 0.006 | 0.002 | 1.078 | 0.838 | 0. | 1.182 |

| Problem 264 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 35 | 53 | 0 | 333 | 0 | 0 |
| normalized size | 1 | 1. | 0.67 | 1.02 | 0. | 6.4 | 0. | 0. |
| time (sec) | N/A | 0.058 | 0.009 | 0.007 | 0. | 0.801 | 0. | 0. |

| Problem 265 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 36 | 62 | 0 | 402 | 0 | 0 |
| normalized size | 1 | 1. | 0.48 | 0.83 | 0. | 5.36 | 0. | 0. |
| time (sec) | N/A | 0.086 | 0.009 | 0.012 | 0. | 0.804 | 0. | 0. |

| Problem 266 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 110 | 38 | 76 | 0 | 466 | 0 | 0 |
| normalized size | 1 | 1. | 0.35 | 0.69 | 0. | 4.24 | 0. | 0. |
| time (sec) | N/A | 0.105 | 0.007 | 0.012 | 0. | 0.902 | 0. | 0. |

| Problem 267 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 38 | 86 | 0 | 524 | 0 | 0 |
| normalized size | 1 | 1. | 0.28 | 0.62 | 0. | 3.8 | 0. | 0. |
| time (sec) | N/A | 0.186 | 0.009 | 0.013 | 0. | 0.817 | 0. | 0. |

| Problem 268 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 38 | 100 | 0 | 575 | 0 | 0 |
| normalized size | 1 | 1. | 0.23 | 0.6 | 0. | 3.46 | 0. | 0. |
| time (sec) | N/A | 0.232 | 0.01 | 0.014 | 0. | 0.822 | 0. | 0. |

| Problem 269 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 125 | 104 | 103 | 0 | 428 | 0 | 86 |
| normalized size | 1 | 1. | 0.83 | 0.82 | 0. | 3.42 | 0. | 0.69 |
| time (sec) | N/A | 0.169 | 0.109 | 0.006 | 0. | 0.793 | 0. | 1.308 |

| Problem 270 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 95 | 90 | 92 | 0 | 375 | 0 | 70 |
| normalized size | 1 | 1. | 0.95 | 0.97 | 0. | 3.95 | 0. | 0.74 |
| time (sec) | N/A | 0.125 | 0.052 | 0.005 | 0. | 0.83 | 0. | 1.293 |

| Problem 271 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 73 | 79 | 0 | 315 | 0 | 51 |
| normalized size | 1 | 1. | 1.22 | 1.32 | 0. | 5.25 | 0. | 0.85 |
| time (sec) | N/A | 0.083 | 0.039 | 0.005 | 0. | 0.885 | 0. | 1.355 |

| Problem 272 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 34 | 34 | 55 | 58 | 0 | 189 | 0 | 31 |
| normalized size | 1 | 1. | 1.62 | 1.71 | 0. | 5.56 | 0. | 0.91 |
| time (sec) | N/A | 0.042 | 0.016 | 0.005 | 0. | 0.84 | 0. | 1.397 |

| Problem 273 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 23 | 27 | 0 | 49 | 0 | 41 |
| normalized size | 1 | 1. | 0.92 | 1.08 | 0. | 1.96 | 0. | 1.64 |
| time (sec) | N/A | 0.038 | 0.008 | 0.003 | 0. | 0.713 | 0. | 1.268 |

| Problem 274 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 31 | 33 | 0 | 69 | 0 | 74 |
| normalized size | 1 | 1. | 0.55 | 0.59 | 0. | 1.23 | 0. | 1.32 |
| time (sec) | N/A | 0.077 | 0.015 | 0.003 | 0. | 0.891 | 0. | 1.208 |

| Problem 275 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 44 | 46 | 0 | 96 | 0 | 104 |
| normalized size | 1 | 1. | 0.51 | 0.53 | 0. | 1.12 | 0. | 1.21 |
| time (sec) | N/A | 0.118 | 0.015 | 0.004 | 0. | 0.789 | 0. | 1.16 |

| Problem 276 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 55 | 57 | 0 | 117 | 0 | 139 |
| normalized size | 1 | 1. | 0.47 | 0.49 | 0. | 1.01 | 0. | 1.2 |
| time (sec) | N/A | 0.163 | 0.018 | 0.004 | 0. | 0.757 | 0. | 1.269 |

| Problem 277 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 59 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.97 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.039 | 0.018 | 0.296 | 0. | 0. | 0. | 0. |

| Problem 278 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 61 | 59 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1.27 | 1.23 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.037 | 0.012 | 0.252 | 0. | 0. | 0. | 0. |

| Problem 279 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 52 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.96 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.038 | 0.008 | 0.241 | 0. | 0. | 0. | 0. |

| Problem 280 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 30 | 36 | 0 | 77 | 0 | 0 |
| normalized size | 1 | 1. | 0.94 | 1.12 | 0. | 2.41 | 0. | 0. |
| time (sec) | N/A | 0.025 | 0.014 | 0.003 | 0. | 0.946 | 0. | 0. |

| Problem 281 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 44 | 50 | 0 | 136 | 0 | 0 |
| normalized size | 1 | 1. | 0.63 | 0.71 | 0. | 1.94 | 0. | 0. |
| time (sec) | N/A | 0.054 | 0.021 | 0.005 | 0. | 0.812 | 0. | 0. |

| Problem 282 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 72 | 84 | 0 | 217 | 0 | 0 |
| normalized size | 1 | 1. | 0.62 | 0.72 | 0. | 1.87 | 0. | 0. |
| time (sec) | N/A | 0.091 | 0.029 | 0.005 | 0. | 0.915 | 0. | 0. |

| Problem 283 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 24 | 31 | 49 | 73 | 36 | 30 |
| normalized size | 1 | 1. | 1.26 | 1.63 | 2.58 | 3.84 | 1.89 | 1.58 |
| time (sec) | N/A | 0.01 | 0.008 | 0.007 | 1.172 | 0.719 | 1.509 | 1.164 |

| Problem 284 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 46 | 48 | 62 | 89 | 0 | 0 |
| normalized size | 1 | 1. | 0.57 | 0.6 | 0.78 | 1.11 | 0. | 0. |
| time (sec) | N/A | 0.115 | 0.025 | 0.006 | 1.035 | 0.651 | 0. | 0. |

| Problem 285 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 34 | 37 | 46 | 63 | 0 | 0 |
| normalized size | 1 | 1. | 0.65 | 0.71 | 0.88 | 1.21 | 0. | 0. |
| time (sec) | N/A | 0.065 | 0.017 | 0.005 | 1.159 | 0.81 | 0. | 0. |

| Problem 286 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 27 | 19 | 42 | 0 | 0 |
| normalized size | 1 | 1. | 1. | 1.08 | 0.76 | 1.68 | 0. | 0. |
| time (sec) | N/A | 0.017 | 0.008 | 0.004 | 1.106 | 0.796 | 0. | 0. |

| Problem 287 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 54 | 43 | 0 | 180 | 0 | 63 |
| normalized size | 1 | 1. | 1.69 | 1.34 | 0. | 5.62 | 0. | 1.97 |
| time (sec) | N/A | 0.011 | 0.01 | 0.005 | 0. | 0.875 | 0. | 1.164 |

| Problem 288 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 71 | 66 | 0 | 294 | 0 | 77 |
| normalized size | 1 | 1. | 1.2 | 1.12 | 0. | 4.98 | 0. | 1.31 |
| time (sec) | N/A | 0.055 | 0.061 | 0.006 | 0. | 0.81 | 0. | 1.166 |

| Problem 289 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 238 | 238 | 68 | 248 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.29 | 1.04 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.142 | 0.024 | 0.297 | 0. | 0. | 0. | 0. |

| Problem 290 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 212 | 212 | 52 | 231 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.25 | 1.09 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.067 | 0.009 | 0.005 | 0. | 0. | 0. | 0. |

| Problem 291 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 243 | 243 | 55 | 248 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.23 | 1.02 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.129 | 0.013 | 0.005 | 0. | 0. | 0. | 0. |

| Problem 292 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 514 | 514 | 68 | 676 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.13 | 1.32 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.308 | 0.023 | 0.008 | 0. | 0. | 0. | 0. |

| Problem 293 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 484 | 484 | 55 | 394 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.11 | 0.81 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.19 | 0.011 | 0.005 | 0. | 0. | 0. | 0. |

| Problem 294 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 510 | 510 | 50 | 673 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.1 | 1.32 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.273 | 0.011 | 0.009 | 0. | 0. | 0. | 0. |

| Problem 295 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 265 | 265 | 86 | 2017 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.32 | 7.61 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.282 | 0.032 | 0.533 | 0. | 0. | 0. | 0. |

| Problem 296 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 525 | 525 | 70 | 2586 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.13 | 4.93 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.5 | 0.026 | 0.052 | 0. | 0. | 0. | 0. |

| Problem 297 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 65 | 81 | 3347 | 0 | 358 | 0 | 59 |
| normalized size | 1 | 1. | 1.25 | 51.49 | 0. | 5.51 | 0. | 0.91 |
| time (sec) | N/A | 0.09 | 0.037 | 0.051 | 0. | 1.133 | 0. | 1.398 |

| Problem 298 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 237 | 237 | 70 | 1793 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.3 | 7.57 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.225 | 0.025 | 0.032 | 0. | 0. | 0. | 0. |

| Problem 299 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 492 | 492 | 57 | 2374 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.12 | 4.83 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.398 | 0.013 | 0.034 | 0. | 0. | 0. | 0. |

| Problem 300 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 36 | 36 | 59 | 480 | 0 | 247 | 0 | 55 |
| normalized size | 1 | 1. | 1.64 | 13.33 | 0. | 6.86 | 0. | 1.53 |
| time (sec) | N/A | 0.048 | 0.012 | 0.118 | 0. | 1.331 | 0. | 1.289 |

| Problem 301 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 203 | 55 | 437 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.27 | 2.15 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.166 | 0.014 | 0.126 | 0. | 0. | 0. | 0. |

| Problem 302 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 519 | 519 | 55 | 2860 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.11 | 5.51 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.487 | 0.015 | 0.041 | 0. | 0. | 0. | 0. |

| Problem 303 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 27 | 29 | 35 | 51 | 0 | 31 |
| normalized size | 1 | 1. | 1. | 1.07 | 1.3 | 1.89 | 0. | 1.15 |
| time (sec) | N/A | 0.04 | 0.011 | 0.003 | 1.12 | 0.921 | 0. | 1.449 |

| Problem 304 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 235 | 57 | 1795 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.24 | 7.64 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.221 | 0.013 | 0.037 | 0. | 0. | 0. | 0. |

| Problem 305 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 555 | 555 | 57 | 3048 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.1 | 5.49 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.573 | 0.014 | 0.043 | 0. | 0. | 0. | 0. |

| Problem 306 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 35 | 37 | 51 | 73 | 0 | 49 |
| normalized size | 1 | 1. | 0.62 | 0.66 | 0.91 | 1.3 | 0. | 0.88 |
| time (sec) | N/A | 0.083 | 0.017 | 0.006 | 1.104 | 0.823 | 0. | 1.201 |

| Problem 307 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | C | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 265 | 265 | 57 | 2009 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.22 | 7.58 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.284 | 0.014 | 0.041 | 0. | 0. | 0. | 0. |

| Problem 308 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 28 | 29 | 38 | 61 | 19 | 41 |
| normalized size | 1 | 1. | 1. | 1.04 | 1.36 | 2.18 | 0.68 | 1.46 |
| time (sec) | N/A | 0.017 | 0.004 | 0.007 | 1.104 | 0.736 | 0.787 | 1.203 |

| Problem 309 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 42 | 41 | 54 | 103 | 31 | 61 |
| normalized size | 1 | 1. | 1. | 0.98 | 1.29 | 2.45 | 0.74 | 1.45 |
| time (sec) | N/A | 0.019 | 0.005 | 0.006 | 1.069 | 0.793 | 0.529 | 1.305 |

| Problem 310 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 94 | 120 | 0 | 390 | 0 | 0 |
| normalized size | 1 | 1. | 0.84 | 1.07 | 0. | 3.48 | 0. | 0. |
| time (sec) | N/A | 0.179 | 0.145 | 0.004 | 0. | 0.915 | 0. | 0. |

| Problem 311 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 86 | 92 | 98 | 0 | 338 | 0 | 0 |
| normalized size | 1 | 1. | 1.07 | 1.14 | 0. | 3.93 | 0. | 0. |
| time (sec) | N/A | 0.127 | 0.052 | 0.004 | 0. | 0.811 | 0. | 0. |

| Problem 312 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 75 | 78 | 0 | 277 | 0 | 55 |
| normalized size | 1 | 1. | 1.34 | 1.39 | 0. | 4.95 | 0. | 0.98 |
| time (sec) | N/A | 0.082 | 0.041 | 0.005 | 0. | 0.81 | 0. | 1.212 |

| Problem 313 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 59 | 56 | 0 | 173 | 0 | 31 |
| normalized size | 1 | 1. | 1.84 | 1.75 | 0. | 5.41 | 0. | 0.97 |
| time (sec) | N/A | 0.034 | 0.017 | 0.003 | 0. | 0.821 | 0. | 1.399 |

| Problem 314 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 21 | 25 | 0 | 43 | 0 | 36 |
| normalized size | 1 | 1. | 0.91 | 1.09 | 0. | 1.87 | 0. | 1.57 |
| time (sec) | N/A | 0.005 | 0.007 | 0.002 | 0. | 0.623 | 0. | 1.214 |

| Problem 315 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 52 | 29 | 30 | 0 | 63 | 0 | 36 |
| normalized size | 1 | 1. | 0.56 | 0.58 | 0. | 1.21 | 0. | 0.69 |
| time (sec) | N/A | 0.045 | 0.011 | 0.004 | 0. | 0.86 | 0. | 1.206 |

| Problem 316 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 42 | 46 | 0 | 90 | 0 | 58 |
| normalized size | 1 | 1. | 0.52 | 0.57 | 0. | 1.12 | 0. | 0.72 |
| time (sec) | N/A | 0.086 | 0.012 | 0.003 | 0. | 0.764 | 0. | 1.216 |

| Problem 317 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 108 | 53 | 57 | 0 | 112 | 0 | 77 |
| normalized size | 1 | 1. | 0.49 | 0.53 | 0. | 1.04 | 0. | 0.71 |
| time (sec) | N/A | 0.134 | 0.017 | 0.005 | 0. | 0.79 | 0. | 1.234 |

| Problem 318 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 136 | 64 | 68 | 0 | 143 | 0 | 96 |
| normalized size | 1 | 1. | 0.47 | 0.5 | 0. | 1.05 | 0. | 0.71 |
| time (sec) | N/A | 0.174 | 0.019 | 0.003 | 0. | 0.821 | 0. | 1.278 |

| Problem 319 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 26 | 23 | 30 | 72 | 22 | 43 |
| normalized size | 1 | 1. | 1. | 0.88 | 1.15 | 2.77 | 0.85 | 1.65 |
| time (sec) | N/A | 0.018 | 0.005 | 0.005 | 1.152 | 0.847 | 0.522 | 1.235 |

| Problem 320 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 27 | 23 | 30 | 72 | 22 | 43 |
| normalized size | 1 | 1. | 1. | 0.85 | 1.11 | 2.67 | 0.81 | 1.59 |
| time (sec) | N/A | 0.018 | 0.005 | 0.004 | 1.038 | 0.764 | 0.263 | 1.282 |

| Problem 321 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 8 | 8 | 14 | 9 | 19 | 22 | 5 | 20 |
| normalized size | 1 | 1. | 1.75 | 1.12 | 2.38 | 2.75 | 0.62 | 2.5 |
| time (sec) | N/A | 0.003 | 0.003 | 0. | 1.026 | 0.842 | 0.093 | 1.189 |

| Problem 322 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 10 | 10 | 11 | 22 | 38 | 15 | 22 |
| normalized size | 1 | 1. | 1. | 1.1 | 2.2 | 3.8 | 1.5 | 2.2 |
| time (sec) | N/A | 0.003 | 0.003 | 0. | 1.11 | 0.801 | 0.112 | 1.13 |

| Problem 323 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 11 | 22 | 59 | 27 | 22 |
| normalized size | 1 | 1. | 1. | 0.92 | 1.83 | 4.92 | 2.25 | 1.83 |
| time (sec) | N/A | 0.003 | 0.003 | 0.001 | 0.995 | 0.821 | 0.108 | 1.152 |

| Problem 324 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 10 | 10 | 10 | 11 | 14 | 22 | 7 | 14 |
| normalized size | 1 | 1. | 1. | 1.1 | 1.4 | 2.2 | 0.7 | 1.4 |
| time (sec) | N/A | 0.002 | 0.001 | 0. | 1.087 | 0.64 | 0.081 | 1.137 |

| Problem 325 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 19 | 28 | 41 | 32 | 0 |
| normalized size | 1 | 1. | 1. | 0.95 | 1.4 | 2.05 | 1.6 | 0. |
| time (sec) | N/A | 0.006 | 0.004 | 0.002 | 1.071 | 0.873 | 0.839 | 0. |

| Problem 326 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 21 | 54 | 80 | 82 | 0 |
| normalized size | 1 | 1. | 1. | 1.05 | 2.7 | 4. | 4.1 | 0. |
| time (sec) | N/A | 0.008 | 0.003 | 0.001 | 1.053 | 0.917 | 1.384 | 0. |

| Problem 327 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 21 | 72 | 112 | 119 | 0 |
| normalized size | 1 | 1. | 1. | 1.05 | 3.6 | 5.6 | 5.95 | 0. |
| time (sec) | N/A | 0.009 | 0.003 | 0.002 | 1.014 | 0.919 | 1.806 | 0. |

| Problem 328 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 160 | 135 | 181 | 365 | 160 | 181 |
| normalized size | 1 | 1. | 10. | 8.44 | 11.31 | 22.81 | 10. | 11.31 |
| time (sec) | N/A | 0.004 | 0.004 | 0.003 | 1.024 | 0.598 | 0.122 | 1.156 |

| Problem 329 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 160 | 135 | 181 | 367 | 160 | 181 |
| normalized size | 1 | 1. | 10. | 8.44 | 11.31 | 22.94 | 10. | 11.31 |
| time (sec) | N/A | 0.011 | 0.007 | 0.003 | 1.116 | 0.682 | 0.24 | 1.155 |

| Problem 330 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 160 | 135 | 181 | 371 | 160 | 181 |
| normalized size | 1 | 1. | 10. | 8.44 | 11.31 | 23.19 | 10. | 11.31 |
| time (sec) | N/A | 0.012 | 0.008 | 0.001 | 1.056 | 0.761 | 0.234 | 1.119 |

| Problem 331 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 24 | 339 | 0 | 578 | 0 | 0 |
| normalized size | 1 | 1. | 0.89 | 12.56 | 0. | 21.41 | 0. | 0. |
| time (sec) | N/A | 0.015 | 0.013 | 0.106 | 0. | 0.724 | 0. | 0. |

| Problem 332 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 160 | 135 | 181 | 365 | 160 | 181 |
| normalized size | 1 | 1. | 10. | 8.44 | 11.31 | 22.81 | 10. | 11.31 |
| time (sec) | N/A | 0.004 | 0.003 | 0. | 1.104 | 0.597 | 0.222 | 1.158 |

| Problem 333 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 160 | 135 | 181 | 367 | 160 | 181 |
| normalized size | 1 | 1. | 10. | 8.44 | 11.31 | 22.94 | 10. | 11.31 |
| time (sec) | N/A | 0.005 | 0.005 | 0.003 | 1.029 | 0.659 | 0.172 | 1.128 |

| Problem 334 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 160 | 135 | 181 | 371 | 160 | 181 |
| normalized size | 1 | 1. | 10. | 8.44 | 11.31 | 23.19 | 10. | 11.31 |
| time (sec) | N/A | 0.004 | 0.006 | 0.003 | 1.064 | 0.665 | 0.164 | 1.147 |

| Problem 335 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 24 | 287 | 0 | 493 | 0 | 277 |
| normalized size | 1 | 1. | 0.89 | 10.63 | 0. | 18.26 | 0. | 10.26 |
| time (sec) | N/A | 0.008 | 0.004 | 0.041 | 0. | 0.863 | 0. | 1.255 |

| Problem 336 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 27 | 126 | 0 | 204 | 0 | 126 |
| normalized size | 1 | 1. | 1. | 4.67 | 0. | 7.56 | 0. | 4.67 |
| time (sec) | N/A | 0.01 | 0.013 | 0.018 | 0. | 0.851 | 0. | 1.123 |

| Problem 337 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 27 | 27 | 39 | 89 | 162 | 0 | 0 |
| normalized size | 1 | 1. | 1. | 1.44 | 3.3 | 6. | 0. | 0. |
| time (sec) | N/A | 0.014 | 0.019 | 0.02 | 1.104 | 0.893 | 0. | 0. |

| Problem 338 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 14 | 18 | 30 | 10 | 19 |
| normalized size | 1 | 1. | 1. | 0.93 | 1.2 | 2. | 0.67 | 1.27 |
| time (sec) | N/A | 0.005 | 0.003 | 0.002 | 1.266 | 0.651 | 0.114 | 1.121 |

| Problem 339 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 14 | 18 | 30 | 10 | 19 |
| normalized size | 1 | 1. | 1. | 0.93 | 1.2 | 2. | 0.67 | 1.27 |
| time (sec) | N/A | 0.006 | 0.004 | 0.001 | 1.081 | 0.729 | 0.188 | 1.157 |

| Problem 340 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 14 | 18 | 30 | 10 | 19 |
| normalized size | 1 | 1. | 1. | 0.93 | 1.2 | 2. | 0.67 | 1.27 |
| time (sec) | N/A | 0.005 | 0.005 | 0.001 | 1.12 | 0.706 | 0.174 | 1.1 |

| Problem 341 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 24 | 31 | 49 | 73 | 36 | 30 |
| normalized size | 1 | 1. | 1.26 | 1.63 | 2.58 | 3.84 | 1.89 | 1.58 |
| time (sec) | N/A | 0.005 | 0.008 | 0.007 | 1.159 | 0.784 | 0.482 | 1.151 |

| Problem 342 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 24 | 31 | 49 | 76 | 36 | 30 |
| normalized size | 1 | 1. | 1.26 | 1.63 | 2.58 | 4. | 1.89 | 1.58 |
| time (sec) | N/A | 0.006 | 0.009 | 0.004 | 1.057 | 0.631 | 5.422 | 1.158 |

| Problem 343 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | B | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 24 | 31 | 49 | 76 | 0 | 30 |
| normalized size | 1 | 1. | 1.26 | 1.63 | 2.58 | 4. | 0. | 1.58 |
| time (sec) | N/A | 0.006 | 0.011 | 0.008 | 1.109 | 0.82 | 0. | 1.173 |

| Problem 344 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 24 | 23 | 30 | 50 | 19 | 30 |
| normalized size | 1 | 1. | 1. | 0.96 | 1.25 | 2.08 | 0.79 | 1.25 |
| time (sec) | N/A | 0.011 | 0.001 | 0.003 | 1.106 | 0.726 | 0.317 | 1.174 |

| Problem 345 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 40 | 35 | 46 | 85 | 37 | 62 |
| normalized size | 1 | 1. | 1. | 0.88 | 1.15 | 2.12 | 0.92 | 1.55 |
| time (sec) | N/A | 0.023 | 0.007 | 0.007 | 1.154 | 0.77 | 0.419 | 1.112 |

| Problem 346 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 50 | 45 | 59 | 104 | 48 | 61 |
| normalized size | 1 | 1. | 1. | 0.9 | 1.18 | 2.08 | 0.96 | 1.22 |
| time (sec) | N/A | 0.021 | 0.006 | 0.006 | 1.172 | 0.758 | 0.35 | 1.114 |

| Problem 347 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | A | A |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 185 | 185 | 144 | 156 | 167 | 2210 | 36 | 151 |
| normalized size | 1 | 1. | 0.78 | 0.84 | 0.9 | 11.95 | 0.19 | 0.82 |
| time (sec) | N/A | 0.346 | 0.125 | 0.019 | 1.546 | 5.635 | 1.655 | 1.159 |

| Problem 348 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 29 | 363 | 0 | 753 | 0 | 363 |
| normalized size | 1 | 1. | 1. | 12.52 | 0. | 25.97 | 0. | 12.52 |
| time (sec) | N/A | 0.017 | 0.016 | 0.168 | 0. | 0.854 | 0. | 4.317 |

| Problem 349 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 160 | 135 | 19 | 365 | 160 | 19 |
| normalized size | 1 | 1. | 10. | 8.44 | 1.19 | 22.81 | 10. | 1.19 |
| time (sec) | N/A | 0.003 | 0.005 | 0. | 1.026 | 0.612 | 0.122 | 1.212 |

| Problem 350 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 160 | 135 | 181 | 367 | 160 | 181 |
| normalized size | 1 | 1. | 10. | 8.44 | 11.31 | 22.94 | 10. | 11.31 |
| time (sec) | N/A | 0.009 | 0.006 | 0. | 1.054 | 0.62 | 0.152 | 1.179 |

| Problem 351 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 160 | 135 | 181 | 371 | 160 | 181 |
| normalized size | 1 | 1. | 10. | 8.44 | 11.31 | 23.19 | 10. | 11.31 |
| time (sec) | N/A | 0.012 | 0.007 | 0.001 | 1.064 | 0.581 | 0.127 | 1.193 |

| Problem 352 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 160 | 135 | 19 | 367 | 160 | 19 |
| normalized size | 1 | 1. | 10. | 8.44 | 1.19 | 22.94 | 10. | 1.19 |
| time (sec) | N/A | 0.003 | 0.004 | 0.001 | 1.102 | 0.68 | 0.121 | 1.204 |

| Problem 353 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 160 | 135 | 181 | 371 | 160 | 181 |
| normalized size | 1 | 1. | 10. | 8.44 | 11.31 | 23.19 | 10. | 11.31 |
| time (sec) | N/A | 0.008 | 0.006 | 0.002 | 1.016 | 0.644 | 0.145 | 1.177 |

| Problem 354 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | A | B | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 160 | 135 | 19 | 371 | 160 | 19 |
| normalized size | 1 | 1. | 10. | 8.44 | 1.19 | 23.19 | 10. | 1.19 |
| time (sec) | N/A | 0.003 | 0.006 | 0.001 | 0.981 | 0.589 | 0.133 | 1.187 |

| Problem 355 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 23 | 36 | 50 | 55 | 48 | 0 |
| normalized size | 1 | 1. | 1. | 1.57 | 2.17 | 2.39 | 2.09 | 0. |
| time (sec) | N/A | 0.011 | 0.008 | 0.01 | 1.006 | 0.931 | 0.693 | 0. |

| Problem 356 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 22 | 39 | 36 | 66 | 41 | 0 |
| normalized size | 1 | 1. | 0.96 | 1.7 | 1.57 | 2.87 | 1.78 | 0. |
| time (sec) | N/A | 0.013 | 0.006 | 0.008 | 1.038 | 0.877 | 1.712 | 0. |

| Problem 357 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 41 | 26 | 68 | 39 | 0 |
| normalized size | 1 | 1. | 1. | 2.73 | 1.73 | 4.53 | 2.6 | 0. |
| time (sec) | N/A | 0.008 | 0.004 | 0.01 | 1.024 | 0.907 | 1.994 | 0. |

| Problem 358 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 22 | 32 | 22 | 66 | 20 | 0 |
| normalized size | 1 | 1. | 1. | 1.45 | 1. | 3. | 0.91 | 0. |
| time (sec) | N/A | 0.01 | 0.006 | 0.028 | 1.111 | 0.832 | 1.989 | 0. |

| Problem 359 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 15 | 15 | 15 | 34 | 15 | 68 | 22 | 0 |
| normalized size | 1 | 1. | 1. | 2.27 | 1. | 4.53 | 1.47 | 0. |
| time (sec) | N/A | 0.006 | 0.003 | 0.009 | 1.048 | 0.722 | 1.642 | 0. |

| Problem 360 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 12 | 11 | 27 | 8 | 12 |
| normalized size | 1 | 1. | 1. | 1. | 0.92 | 2.25 | 0.67 | 1. |
| time (sec) | N/A | 0.004 | 0.002 | 0.003 | 1.041 | 0.823 | 0.152 | 1.172 |

| Problem 361 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | A | A | B | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 116 | 23 | 30 | 22 | 24 |
| normalized size | 1 | 1. | 1. | 8.29 | 1.64 | 2.14 | 1.57 | 1.71 |
| time (sec) | N/A | 0.005 | 0.002 | 0.096 | 1.035 | 0.788 | 0.385 | 1.177 |

| Problem 362 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 9 | 11 | 30 | 10 | 43 |
| normalized size | 1 | 1. | 1. | 0.75 | 0.92 | 2.5 | 0.83 | 3.58 |
| time (sec) | N/A | 0.004 | 0.002 | 0. | 1.683 | 0.847 | 0.261 | 1.178 |

| Problem 363 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 24 | 39 | 42 | 78 | 76 | 0 |
| normalized size | 1 | 1. | 1. | 1.62 | 1.75 | 3.25 | 3.17 | 0. |
| time (sec) | N/A | 0.016 | 0.021 | 0.02 | 1.613 | 0.774 | 0.681 | 0. |

| Problem 364 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 104 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.39 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.114 | 0.2 | 0.532 | 0. | 0. | 0. | 0. |

| Problem 365 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 109 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.1 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.157 | 0.064 | 0.584 | 0. | 0. | 0. | 0. |

| Problem 366 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 91 | 103 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.13 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.155 | 0.247 | 0.379 | 0. | 0. | 0. | 0. |

| Problem 367 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 99 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.39 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.076 | 0.144 | 0.341 | 0. | 0. | 0. | 0. |

| Problem 368 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 100 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.15 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.138 | 0.251 | 0.383 | 0. | 0. | 0. | 0. |

| Problem 369 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 46 | 39 | 0 | 223 | 78 | 0 |
| normalized size | 1 | 1. | 0.9 | 0.76 | 0. | 4.37 | 1.53 | 0. |
| time (sec) | N/A | 0.029 | 0.013 | 0.002 | 0. | 0.904 | 2.231 | 0. |

| Problem 370 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 85 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.01 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.171 | 0.062 | 0.368 | 0. | 0. | 0. | 0. |

| Problem 371 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 78 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.28 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.083 | 0.053 | 0.34 | 0. | 0. | 0. | 0. |

| Problem 372 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 85 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1. | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.199 | 0.068 | 0.346 | 0. | 0. | 0. | 0. |

| Problem 373 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 141 | 131 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.93 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.225 | 0.233 | 0.61 | 0. | 0. | 0. | 0. |

| Problem 374 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 128 | 126 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.98 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.209 | 0.293 | 0.335 | 0. | 0. | 0. | 0. |

| Problem 375 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 117 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.12 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.133 | 0.158 | 0.324 | 0. | 0. | 0. | 0. |

| Problem 376 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 122 | 120 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.98 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.188 | 0.251 | 0.323 | 0. | 0. | 0. | 0. |

| Problem 377 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 58 | 51 | 0 | 277 | 88 | 0 |
| normalized size | 1 | 1. | 0.79 | 0.7 | 0. | 3.79 | 1.21 | 0. |
| time (sec) | N/A | 0.041 | 0.026 | 0.002 | 0. | 0.968 | 4.757 | 0. |

| Problem 378 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 117 | 97 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.83 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.23 | 0.071 | 0.323 | 0. | 0. | 0. | 0. |

| Problem 379 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 94 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.96 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.172 | 0.068 | 0.322 | 0. | 0. | 0. | 0. |

| Problem 380 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 122 | 100 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.82 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.276 | 0.084 | 0.322 | 0. | 0. | 0. | 0. |

| Problem 381 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 96 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.96 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.212 | 0.072 | 0.327 | 0. | 0. | 0. | 0. |

| Problem 382 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 58 | 47 | 0 | 231 | 0 | 88 |
| normalized size | 1 | 1. | 1.14 | 0.92 | 0. | 4.53 | 0. | 1.73 |
| time (sec) | N/A | 0.077 | 0.026 | 0.007 | 0. | 0.917 | 0. | 1.125 |

| Problem 383 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 62 | 61 | 0 | 259 | 0 | 92 |
| normalized size | 1 | 1. | 1.48 | 1.45 | 0. | 6.17 | 0. | 2.19 |
| time (sec) | N/A | 0.023 | 0.028 | 0.005 | 0. | 0.86 | 0. | 1.248 |

| Problem 384 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 66 | 55 | 0 | 261 | 0 | 93 |
| normalized size | 1 | 1. | 1.29 | 1.08 | 0. | 5.12 | 0. | 1.82 |
| time (sec) | N/A | 0.065 | 0.037 | 0.007 | 0. | 0.847 | 0. | 1.138 |

| Problem 385 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 70 | 74 | 0 | 255 | 0 | 0 |
| normalized size | 1 | 1. | 1.15 | 1.21 | 0. | 4.18 | 0. | 0. |
| time (sec) | N/A | 0.08 | 0.021 | 1.267 | 0. | 0.776 | 0. | 0. |

| Problem 386 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 66 | 56 | 0 | 228 | 0 | 82 |
| normalized size | 1 | 1. | 1.25 | 1.06 | 0. | 4.3 | 0. | 1.55 |
| time (sec) | N/A | 0.079 | 0.03 | 0.01 | 0. | 0.858 | 0. | 1.286 |

| Problem 387 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 68 | 81 | 0 | 259 | 0 | 86 |
| normalized size | 1 | 1. | 1.58 | 1.88 | 0. | 6.02 | 0. | 2. |
| time (sec) | N/A | 0.023 | 0.031 | 0.008 | 0. | 0.757 | 0. | 1.227 |

| Problem 388 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 73 | 73 | 0 | 258 | 0 | 88 |
| normalized size | 1 | 1. | 1.38 | 1.38 | 0. | 4.87 | 0. | 1.66 |
| time (sec) | N/A | 0.068 | 0.04 | 0.019 | 0. | 0.822 | 0. | 1.302 |

| Problem 389 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 78 | 105 | 0 | 252 | 0 | 0 |
| normalized size | 1 | 1. | 1.24 | 1.67 | 0. | 4. | 0. | 0. |
| time (sec) | N/A | 0.08 | 0.029 | 0.721 | 0. | 0.929 | 0. | 0. |

| Problem 390 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 62 | 98 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.58 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.108 | 0.14 | 0.374 | 0. | 0. | 0. | 0. |

| Problem 391 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 89 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.68 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.099 | 0.154 | 0.331 | 0. | 0. | 0. | 0. |

| Problem 392 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 37 | 78 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 2.11 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.015 | 0.068 | 0.326 | 0. | 0. | 0. | 0. |

| Problem 393 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 87 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.71 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.085 | 0.119 | 0.338 | 0. | 0. | 0. | 0. |

| Problem 394 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | A | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 31 | 26 | 0 | 169 | 27 | 0 |
| normalized size | 1 | 1. | 1. | 0.84 | 0. | 5.45 | 0.87 | 0. |
| time (sec) | N/A | 0.02 | 0.006 | 0.004 | 0. | 0.79 | 1.894 | 0. |

| Problem 395 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 68 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.26 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.111 | 0.048 | 0.339 | 0. | 0. | 0. | 0. |

| Problem 396 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 66 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.65 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.074 | 0.047 | 0.319 | 0. | 0. | 0. | 0. |

| Problem 397 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 68 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.26 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.136 | 0.05 | 0.329 | 0. | 0. | 0. | 0. |

| Problem 398 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 117 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.09 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.187 | 0.176 | 0.37 | 0. | 0. | 0. | 0. |

| Problem 399 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 94 | 109 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.16 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.16 | 0.187 | 0.322 | 0. | 0. | 0. | 0. |

| Problem 400 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 91 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.26 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.088 | 0.139 | 0.319 | 0. | 0. | 0. | 0. |

| Problem 401 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 104 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.22 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.138 | 0.187 | 0.335 | 0. | 0. | 0. | 0. |

| Problem 402 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F(-2) | A | B | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 40 | 42 | 0 | 335 | 185 | 0 |
| normalized size | 1 | 1. | 0.74 | 0.78 | 0. | 6.2 | 3.43 | 0. |
| time (sec) | N/A | 0.033 | 0.014 | 0.003 | 0. | 0.847 | 3.496 | 0. |

| Problem 403 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 55 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.61 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.194 | 0.046 | 0.328 | 0. | 0. | 0. | 0. |

| Problem 404 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 51 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.71 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.156 | 0.041 | 0.326 | 0. | 0. | 0. | 0. |

| Problem 405 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 55 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.61 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.217 | 0.052 | 0.327 | 0. | 0. | 0. | 0. |

| Problem 406 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F(-1) | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 51 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.71 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.152 | 0.042 | 0.336 | 0. | 0. | 0. | 0. |

| Problem 407 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 63 | 477 | 0 | 236 | 0 | 0 |
| normalized size | 1 | 1. | 1.97 | 14.91 | 0. | 7.38 | 0. | 0. |
| time (sec) | N/A | 0.014 | 0.024 | 0.014 | 0. | 1.165 | 0. | 0. |

| Problem 408 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F(-1) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 59 | 49 | 0 | 196 | 0 | 0 |
| normalized size | 1 | 1. | 1.84 | 1.53 | 0. | 6.12 | 0. | 0. |
| time (sec) | N/A | 0.016 | 0.021 | 0.051 | 0. | 0.836 | 0. | 0. |

| Problem 409 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 32 | 32 | 63 | 0 | 0 | 244 | 0 | 0 |
| normalized size | 1 | 1. | 1.97 | 0. | 0. | 7.62 | 0. | 0. |
| time (sec) | N/A | 0.016 | 0.024 | 0.026 | 0. | 2.591 | 0. | 0. |

| Problem 410 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 37 | 76 | 0 | 0 | 216 | 0 | 0 |
| normalized size | 1 | 1. | 2.05 | 0. | 0. | 5.84 | 0. | 0. |
| time (sec) | N/A | 0.024 | 0.039 | 0.358 | 0. | 0.912 | 0. | 0. |

| Problem 411 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F(-1) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 66 | 471 | 0 | 240 | 0 | 0 |
| normalized size | 1 | 1. | 2. | 14.27 | 0. | 7.27 | 0. | 0. |
| time (sec) | N/A | 0.014 | 0.026 | 0.566 | 0. | 1.541 | 0. | 0. |

| Problem 412 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | A | F(-1) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 62 | 53 | 0 | 198 | 0 | 0 |
| normalized size | 1 | 1. | 1.88 | 1.61 | 0. | 6. | 0. | 0. |
| time (sec) | N/A | 0.015 | 0.022 | 0.013 | 0. | 1.026 | 0. | 0. |

| Problem 413 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 66 | 0 | 0 | 248 | 0 | 0 |
| normalized size | 1 | 1. | 2. | 0. | 0. | 7.52 | 0. | 0. |
| time (sec) | N/A | 0.016 | 0.025 | 0.031 | 0. | 3.252 | 0. | 0. |

| Problem 414 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 78 | 0 | 0 | 219 | 0 | 0 |
| normalized size | 1 | 1. | 2.05 | 0. | 0. | 5.76 | 0. | 0. |
| time (sec) | N/A | 0.023 | 0.043 | 0.586 | 0. | 1.004 | 0. | 0. |

| Problem 415 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 37 | 78 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 2.11 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.023 | 0.075 | 0.349 | 0. | 0. | 0. | 0. |

| Problem 416 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 37 | 78 | 0 | 0 | 252 | 0 | 0 |
| normalized size | 1 | 1. | 2.11 | 0. | 0. | 6.81 | 0. | 0. |
| time (sec) | N/A | 0.019 | 0.022 | 2.16 | 0. | 1.025 | 0. | 0. |

| Problem 417 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 37 | 37 | 78 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 2.11 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.018 | 0.022 | 0.334 | 0. | 0. | 0. | 0. |

| Problem 418 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 80 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 2.11 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.021 | 0.08 | 0.346 | 0. | 0. | 0. | 0. |

| Problem 419 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 80 | 0 | 0 | 251 | 0 | 0 |
| normalized size | 1 | 1. | 2.11 | 0. | 0. | 6.61 | 0. | 0. |
| time (sec) | N/A | 0.02 | 0.022 | 0.813 | 0. | 1.011 | 0. | 0. |

| Problem 420 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 80 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 2.11 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.019 | 0.023 | 0.345 | 0. | 0. | 0. | 0. |

| Problem 421 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 218 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 2.04 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.102 | 0.355 | 0.692 | 0. | 0. | 0. | 0. |

| Problem 422 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 156 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.56 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.095 | 0.191 | 0.512 | 0. | 0. | 0. | 0. |

| Problem 423 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 106 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.04 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.097 | 0.081 | 0.37 | 0. | 0. | 0. | 0. |

| Problem 424 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 116 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.05 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.108 | 0.128 | 0.372 | 0. | 0. | 0. | 0. |

| Problem 425 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 166 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.5 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.106 | 0.377 | 0.357 | 0. | 0. | 0. | 0. |

| Problem 426 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 97 | 177 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.82 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.055 | 0.194 | 0.398 | 0. | 0. | 0. | 0. |

| Problem 427 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 87 | 134 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.54 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.053 | 0.143 | 0.387 | 0. | 0. | 0. | 0. |

| Problem 428 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 88 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.95 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.051 | 0.051 | 0.357 | 0. | 0. | 0. | 0. |

| Problem 429 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 104 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.03 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.057 | 0.083 | 0.36 | 0. | 0. | 0. | 0. |

| Problem 430 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F(-2) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 185 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.83 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.057 | 0.243 | 0.348 | 0. | 0. | 0. | 0. |

| Problem 431 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 19 | 16 | 0 | 46 | 0 | 68 |
| normalized size | 1 | 1. | 1.06 | 0.89 | 0. | 2.56 | 0. | 3.78 |
| time (sec) | N/A | 0.007 | 0.005 | 0.024 | 0. | 0.921 | 0. | 1.288 |

| Problem 432 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 18 | 0 | 55 | 0 | 15 |
| normalized size | 1 | 1. | 1. | 0.9 | 0. | 2.75 | 0. | 0.75 |
| time (sec) | N/A | 0.003 | 0.01 | 0.009 | 0. | 1.192 | 0. | 1.276 |

| Problem 433 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 8 | 8 | 8 | 7 | 8 | 26 | 7 | 8 |
| normalized size | 1 | 1. | 1. | 0.88 | 1. | 3.25 | 0.88 | 1. |
| time (sec) | N/A | 0.004 | 0.002 | 0.004 | 1.718 | 0.959 | 0.231 | 1.327 |

| Problem 434 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 29 | 19 | 58 | 0 | 36 |
| normalized size | 1 | 1. | 1. | 1.16 | 0.76 | 2.32 | 0. | 1.44 |
| time (sec) | N/A | 0.01 | 0.012 | 0.005 | 1.249 | 0.909 | 0. | 1.458 |

| Problem 435 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 29 | 19 | 58 | 0 | 36 |
| normalized size | 1 | 1. | 1. | 1.16 | 0.76 | 2.32 | 0. | 1.44 |
| time (sec) | N/A | 0.011 | 0.002 | 0.002 | 1.133 | 0.938 | 0. | 1.171 |

| Problem 436 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 25 | 29 | 19 | 61 | 0 | 19 |
| normalized size | 1 | 1. | 1. | 1.16 | 0.76 | 2.44 | 0. | 0.76 |
| time (sec) | N/A | 0.008 | 0.011 | 0.004 | 1.165 | 0.918 | 0. | 1.242 |

| Problem 437 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 988 | 988 | 61 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.06 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 2.245 | 0.019 | 0.203 | 0. | 0. | 0. | 0. |

| Problem 438 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | F | F | F(-1) | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 487 | 487 | 61 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.13 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.974 | 0.016 | 0.202 | 0. | 0. | 0. | 0. |

| Problem 439 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 92 | 92 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1.03 | 1.03 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.068 | 0.104 | 0.582 | 0. | 0. | 0. | 0. |

| Problem 440 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 73 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.06 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.068 | 0.117 | 0.632 | 0. | 0. | 0. | 0. |

| Problem 441 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 74 | 74 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1.12 | 1.12 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.068 | 0.096 | 0.511 | 0. | 0. | 0. | 0. |

| Problem 442 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 82 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.19 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.078 | 0.159 | 0.51 | 0. | 0. | 0. | 0. |

| Problem 443 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 83 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.99 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.081 | 0.132 | 0.528 | 0. | 0. | 0. | 0. |

| Problem 444 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 43 | 0 | 0 | 147 | 0 | 0 |
| normalized size | 1 | 1. | 0.98 | 0. | 0. | 3.34 | 0. | 0. |
| time (sec) | N/A | 0.015 | 0.035 | 0.547 | 0. | 0.755 | 0. | 0. |

| Problem 445 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 43 | 0 | 0 | 131 | 0 | 0 |
| normalized size | 1 | 1. | 0.98 | 0. | 0. | 2.98 | 0. | 0. |
| time (sec) | N/A | 0.015 | 0.003 | 0.388 | 0. | 0.759 | 0. | 0. |

| Problem 446 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 45 | 0 | 0 | 174 | 0 | 0 |
| normalized size | 1 | 1. | 0.98 | 0. | 0. | 3.78 | 0. | 0. |
| time (sec) | N/A | 0.075 | 0.042 | 0.542 | 0. | 0.768 | 0. | 0. |

| Problem 447 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 45 | 0 | 0 | 190 | 0 | 0 |
| normalized size | 1 | 1. | 0.98 | 0. | 0. | 4.13 | 0. | 0. |
| time (sec) | N/A | 0.045 | 0.004 | 0.542 | 0. | 0.997 | 0. | 0. |

| Problem 448 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 36 | 40 | 23 | 88 | 0 | 0 |
| normalized size | 1 | 1. | 0.82 | 0.91 | 0.52 | 2. | 0. | 0. |
| time (sec) | N/A | 0.018 | 0.031 | 0.028 | 1.179 | 0.775 | 0. | 0. |

| Problem 449 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 36 | 40 | 23 | 90 | 0 | 0 |
| normalized size | 1 | 1. | 0.82 | 0.91 | 0.52 | 2.05 | 0. | 0. |
| time (sec) | N/A | 0.017 | 0.03 | 0.019 | 1.19 | 0.679 | 0. | 0. |

| Problem 450 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 36 | 40 | 23 | 90 | 0 | 0 |
| normalized size | 1 | 1. | 0.82 | 0.91 | 0.52 | 2.05 | 0. | 0. |
| time (sec) | N/A | 0.018 | 0.03 | 0.018 | 1.069 | 0.934 | 0. | 0. |

| Problem 451 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 47 | 0 | 0 | 101 | 0 | 0 |
| normalized size | 1 | 1. | 0.82 | 0. | 0. | 1.77 | 0. | 0. |
| time (sec) | N/A | 0.028 | 0.021 | 0.435 | 0. | 0.919 | 0. | 0. |

| Problem 452 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 45 | 0 | 0 | 107 | 0 | 0 |
| normalized size | 1 | 1. | 0.74 | 0. | 0. | 1.75 | 0. | 0. |
| time (sec) | N/A | 0.025 | 0.025 | 0.395 | 0. | 0.908 | 0. | 0. |

| Problem 453 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 40 | 0 | 0 | 155 | 0 | 0 |
| normalized size | 1 | 1. | 1.03 | 0. | 0. | 3.97 | 0. | 0. |
| time (sec) | N/A | 0.05 | 0.024 | 0.53 | 0. | 0.879 | 0. | 0. |

| Problem 454 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | A | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 40 | 38 | 0 | 0 | 135 | 0 | 0 |
| normalized size | 1 | 1. | 0.95 | 0. | 0. | 3.38 | 0. | 0. |
| time (sec) | N/A | 0.074 | 0.04 | 0.47 | 0. | 0.858 | 0. | 0. |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [347] had the largest ratio of [ 0.7778 ]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 2 | 1 | 1. | 13 | 0.077 |
| 2 | A | 2 | 1 | 1. | 11 | 0.091 |
| 3 | A | 1 | 0 | 1. | 9 | 0. |
| 4 | A | 2 | 1 | 1. | 13 | 0.077 |
| 5 | A | 2 | 1 | 1. | 13 | 0.077 |
| 6 | A | 3 | 2 | 1. | 15 | 0.133 |
| 7 | A | 4 | 3 | 1. | 13 | 0.231 |
| 8 | A | 3 | 2 | 1. | 11 | 0.182 |
| 9 | A | 2 | 2 | 1. | 15 | 0.133 |
| 10 | A | 3 | 2 | 1. | 15 | 0.133 |
| 11 | A | 3 | 2 | 1. | 11 | 0.182 |
| 12 | A | 4 | 3 | 1. | 15 | 0.2 |
| 13 | A | 3 | 3 | 1. | 15 | 0.2 |
| 14 | A | 2 | 2 | 1. | 15 | 0.133 |
| 15 | A | 2 | 2 | 1. | 13 | 0.154 |
| 16 | A | 5 | 5 | 1. | 11 | 0.454 |
| 17 | A | 3 | 3 | 1. | 15 | 0.2 |
| 18 | A | 4 | 3 | 1. | 15 | 0.2 |
| 19 | A | 4 | 3 | 1. | 15 | 0.2 |
| 20 | A | 4 | 3 | 1. | 15 | 0.2 |
| 21 | A | 3 | 3 | 1. | 15 | 0.2 |
| 22 | A | 4 | 3 | 1. | 13 | 0.231 |
| 23 | A | 4 | 4 | 1. | 11 | 0.364 |
| 24 | A | 4 | 3 | 1. | 15 | 0.2 |
| 25 | A | 5 | 4 | 1. | 15 | 0.267 |
| 26 | A | 4 | 3 | 1. | 13 | 0.231 |
| 27 | A | 4 | 3 | 1. | 13 | 0.231 |
| 28 | A | 3 | 3 | 1. | 13 | 0.231 |
| 29 | A | 2 | 2 | 1. | 13 | 0.154 |
| 30 | A | 2 | 2 | 1. | 11 | 0.182 |
| 31 | A | 5 | 5 | 1. | 9 | 0.556 |
| 32 | A | 3 | 3 | 1. | 13 | 0.231 |
| 33 | A | 4 | 3 | 1. | 13 | 0.231 |
| 34 | A | 4 | 3 | 1. | 13 | 0.231 |
| 35 | A | 4 | 3 | 1. | 13 | 0.231 |
| 36 | A | 5 | 5 | 1. | 9 | 0.556 |
| 37 | A | 5 | 5 | 1. | 11 | 0.454 |
| 38 | A | 6 | 5 | 1. | 17 | 0.294 |
| 39 | A | 7 | 7 | 1. | 17 | 0.412 |
| 40 | A | 5 | 5 | 1. | 15 | 0.333 |
| 41 | A | 6 | 6 | 1. | 13 | 0.462 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 42 | A | 4 | 4 | 1. | 17 | 0.235 |
| 43 | A | 6 | 6 | 1. | 17 | 0.353 |
| 44 | A | 4 | 4 | 1. | 17 | 0.235 |
| 45 | A | 7 | 7 | 1. | 17 | 0.412 |
| 46 | A | 7 | 5 | 1. | 17 | 0.294 |
| 47 | A | 8 | 7 | 1. | 15 | 0.467 |
| 48 | A | 6 | 6 | 1. | 13 | 0.462 |
| 49 | A | 7 | 7 | 1. | 17 | 0.412 |
| 50 | A | 5 | 4 | 1. | 17 | 0.235 |
| 51 | A | 7 | 7 | 1. | 17 | 0.412 |
| 52 | A | 5 | 5 | 1. | 17 | 0.294 |
| 53 | A | 7 | 6 | 1. | 17 | 0.353 |
| 54 | A | 5 | 4 | 1. | 17 | 0.235 |
| 55 | A | 8 | 7 | 1. | 17 | 0.412 |
| 56 | A | 6 | 5 | 1. | 17 | 0.294 |
| 57 | A | 5 | 4 | 1. | 17 | 0.235 |
| 58 | A | 6 | 6 | 1. | 17 | 0.353 |
| 59 | A | 4 | 4 | 1. | 17 | 0.235 |
| 60 | A | 5 | 5 | 1. | 15 | 0.333 |
| 61 | A | 3 | 3 | 1. | 13 | 0.231 |
| 62 | A | 6 | 6 | 1. | 17 | 0.353 |
| 63 | A | 4 | 4 | 1. | 17 | 0.235 |
| 64 | A | 7 | 6 | 1. | 17 | 0.353 |
| 65 | A | 6 | 5 | 1. | 17 | 0.294 |
| 66 | A | 7 | 7 | 1. | 17 | 0.412 |
| 67 | A | 5 | 5 | 1. | 17 | 0.294 |
| 68 | A | 6 | 6 | 1. | 17 | 0.353 |
| 69 | A | 4 | 4 | 1. | 17 | 0.235 |
| 70 | A | 6 | 6 | 1. | 17 | 0.353 |
| 71 | A | 4 | 4 | 1. | 15 | 0.267 |
| 72 | A | 7 | 7 | 1. | 13 | 0.538 |
| 73 | A | 5 | 5 | 1. | 17 | 0.294 |
| 74 | A | 8 | 7 | 1. | 17 | 0.412 |
| 75 | A | 7 | 4 | 1. | 19 | 0.21 |
| 76 | A | 5 | 2 | 1. | 19 | 0.105 |
| 77 | A | 6 | 3 | 1. | 19 | 0.158 |
| 78 | A | 4 | 2 | 1. | 19 | 0.105 |
| 79 | A | 1 | 1 | 1. | 19 | 0.053 |
| 80 | A | 3 | 2 | 1. | 19 | 0.105 |
| 81 | A | 2 | 2 | 1. | 19 | 0.105 |
| 82 | A | 2 | 2 | 1. | 19 | 0.105 |
| 83 | A | 3 | 2 | 1. | 19 | 0.105 |
| 84 | A | 1 | 1 | 1. | 19 | 0.053 |
| 85 | A | 4 | 2 | 1. | 19 | 0.105 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 86 | A | 6 | 3 | 1. | 19 | 0.158 |
| 87 | A | 5 | 2 | 1. | 19 | 0.105 |
| 88 | A | 7 | 4 | 1. | 19 | 0.21 |
| 89 | A | 6 | 3 | 1. | 19 | 0.158 |
| 90 | A | 8 | 4 | 1. | 19 | 0.21 |
| 91 | A | 7 | 3 | 1. | 19 | 0.158 |
| 92 | A | 3 | 3 | 1. | 17 | 0.176 |
| 93 | A | 2 | 2 | 1. | 15 | 0.133 |
| 94 | A | 1 | 1 | 1. | 17 | 0.059 |
| 95 | A | 2 | 2 | 1. | 17 | 0.118 |
| 96 | A | 3 | 2 | 1. | 17 | 0.118 |
| 97 | A | 4 | 4 | 1. | 17 | 0.235 |
| 98 | A | 3 | 3 | 1. | 13 | 0.231 |
| 99 | A | 4 | 4 | 1. | 17 | 0.235 |
| 100 | A | 6 | 6 | 1. | 17 | 0.353 |
| 101 | A | 5 | 5 | 1. | 17 | 0.294 |
| 102 | A | 6 | 6 | 1. | 17 | 0.353 |
| 103 | A | 8 | 5 | 1. | 19 | 0.263 |
| 104 | A | 6 | 5 | 1. | 17 | 0.294 |
| 105 | A | 4 | 4 | 1. | 15 | 0.267 |
| 106 | A | 1 | 1 | 1. | 19 | 0.053 |
| 107 | A | 3 | 2 | 1. | 19 | 0.105 |
| 108 | A | 5 | 2 | 1. | 19 | 0.105 |
| 109 | A | 7 | 2 | 1. | 19 | 0.105 |
| 110 | A | 9 | 6 | 1. | 19 | 0.316 |
| 111 | A | 7 | 6 | 1. | 19 | 0.316 |
| 112 | A | 5 | 5 | 1. | 17 | 0.294 |
| 113 | A | 1 | 1 | 1. | 15 | 0.067 |
| 114 | A | 3 | 3 | 1. | 19 | 0.158 |
| 115 | A | 5 | 3 | 1. | 19 | 0.158 |
| 116 | A | 7 | 3 | 1. | 19 | 0.158 |
| 117 | A | 9 | 5 | 1. | 21 | 0.238 |
| 118 | A | 7 | 5 | 1. | 21 | 0.238 |
| 119 | A | 5 | 5 | 1. | 21 | 0.238 |
| 120 | A | 3 | 3 | 1. | 21 | 0.143 |
| 121 | A | 2 | 2 | 1. | 21 | 0.095 |
| 122 | A | 4 | 2 | 1. | 21 | 0.095 |
| 123 | A | 6 | 2 | 1. | 21 | 0.095 |
| 124 | A | 8 | 6 | 1. | 21 | 0.286 |
| 125 | A | 6 | 6 | 1. | 21 | 0.286 |
| 126 | A | 4 | 4 | 1. | 21 | 0.19 |
| 127 | A | 2 | 2 | 1. | 21 | 0.095 |
| 128 | A | 4 | 3 | 1. | 21 | 0.143 |
| 129 | A | 6 | 3 | 1. | 21 | 0.143 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 130 | A | 8 | 3 | 1. | 21 | 0.143 |
| 131 | A | 11 | 6 | 1. | 19 | 0.316 |
| 132 | A | 11 | 8 | 1. | 19 | 0.421 |
| 133 | A | 8 | 6 | 1. | 17 | 0.353 |
| 134 | A | 8 | 8 | 1. | 15 | 0.533 |
| 135 | A | 5 | 5 | 1. | 19 | 0.263 |
| 136 | A | 8 | 8 | 1. | 19 | 0.421 |
| 137 | A | 7 | 6 | 1. | 19 | 0.316 |
| 138 | A | 11 | 8 | 1. | 19 | 0.421 |
| 139 | A | 10 | 6 | 1. | 19 | 0.316 |
| 140 | A | 11 | 6 | 1. | 19 | 0.316 |
| 141 | A | 11 | 8 | 1. | 17 | 0.471 |
| 142 | A | 8 | 7 | 1. | 15 | 0.467 |
| 143 | A | 8 | 8 | 1. | 19 | 0.421 |
| 144 | A | 6 | 6 | 1. | 19 | 0.316 |
| 145 | A | 9 | 8 | 1. | 19 | 0.421 |
| 146 | A | 8 | 6 | 1. | 19 | 0.316 |
| 147 | A | 12 | 8 | 1. | 19 | 0.421 |
| 148 | A | 11 | 6 | 1. | 19 | 0.316 |
| 149 | A | 11 | 5 | 1. | 19 | 0.263 |
| 150 | A | 11 | 7 | 1. | 19 | 0.368 |
| 151 | A | 8 | 5 | 1. | 19 | 0.263 |
| 152 | A | 8 | 7 | 1. | 17 | 0.412 |
| 153 | A | 5 | 5 | 1. | 15 | 0.333 |
| 154 | A | 7 | 7 | 1. | 19 | 0.368 |
| 155 | A | 6 | 5 | 1. | 19 | 0.263 |
| 156 | A | 10 | 7 | 1. | 19 | 0.368 |
| 157 | A | 9 | 5 | 1. | 19 | 0.263 |
| 158 | A | 12 | 8 | 1. | 19 | 0.421 |
| 159 | A | 9 | 6 | 1. | 19 | 0.316 |
| 160 | A | 9 | 8 | 1. | 19 | 0.421 |
| 161 | A | 6 | 6 | 1. | 17 | 0.353 |
| 162 | A | 7 | 7 | 1. | 15 | 0.467 |
| 163 | A | 6 | 6 | 1. | 19 | 0.316 |
| 164 | A | 10 | 8 | 1. | 19 | 0.421 |
| 165 | A | 9 | 6 | 1. | 19 | 0.316 |
| 166 | A | 13 | 8 | 1. | 19 | 0.421 |
| 167 | A | 13 | 3 | 1. | 19 | 0.158 |
| 168 | A | 10 | 3 | 1. | 19 | 0.158 |
| 169 | A | 7 | 3 | 1. | 17 | 0.176 |
| 170 | A | 4 | 3 | 1. | 15 | 0.2 |
| 171 | A | 1 | 1 | 1. | 19 | 0.053 |
| 172 | A | 4 | 4 | 1. | 19 | 0.21 |
| 173 | A | 7 | 4 | 1. | 19 | 0.21 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 174 | A | 10 | 4 | 1. | 19 | 0.21 |
| 175 | A | 13 | 4 | 1. | 19 | 0.21 |
| 176 | A | 12 | 3 | 1. | 19 | 0.158 |
| 177 | A | 9 | 3 | 1. | 17 | 0.176 |
| 178 | A | 6 | 3 | 1. | 15 | 0.2 |
| 179 | A | 3 | 2 | 1. | 19 | 0.105 |
| 180 | A | 4 | 3 | 1. | 19 | 0.158 |
| 181 | A | 5 | 4 | 1. | 19 | 0.21 |
| 182 | A | 8 | 4 | 1. | 19 | 0.21 |
| 183 | A | 11 | 4 | 1. | 19 | 0.21 |
| 184 | A | 14 | 4 | 1. | 19 | 0.21 |
| 185 | A | 14 | 3 | 1. | 19 | 0.158 |
| 186 | A | 11 | 3 | 1. | 19 | 0.158 |
| 187 | A | 8 | 3 | 1. | 19 | 0.158 |
| 188 | A | 5 | 3 | 1. | 17 | 0.176 |
| 189 | A | 2 | 2 | 1. | 15 | 0.133 |
| 190 | A | 3 | 3 | 1. | 19 | 0.158 |
| 191 | A | 6 | 3 | 1. | 19 | 0.158 |
| 192 | A | 9 | 3 | 1. | 19 | 0.158 |
| 193 | A | 12 | 3 | 1. | 19 | 0.158 |
| 194 | A | 12 | 4 | 1. | 19 | 0.21 |
| 195 | A | 9 | 4 | 1. | 19 | 0.21 |
| 196 | A | 6 | 4 | 1. | 19 | 0.21 |
| 197 | A | 3 | 3 | 1. | 17 | 0.176 |
| 198 | A | 3 | 3 | 1. | 15 | 0.2 |
| 199 | A | 6 | 4 | 1. | 19 | 0.21 |
| 200 | A | 9 | 4 | 1. | 19 | 0.21 |
| 201 | A | 12 | 4 | 1. | 19 | 0.21 |
| 202 | A | 15 | 4 | 1. | 19 | 0.21 |
| 203 | A | 2 | 1 | 1. | 15 | 0.067 |
| 204 | A | 2 | 1 | 1. | 13 | 0.077 |
| 205 | A | 1 | 0 | 1. | 11 | 0. |
| 206 | A | 2 | 1 | 1. | 15 | 0.067 |
| 207 | A | 2 | 1 | 1. | 15 | 0.067 |
| 208 | A | 3 | 2 | 1. | 17 | 0.118 |
| 209 | A | 3 | 2 | 1. | 15 | 0.133 |
| 210 | A | 3 | 2 | 1. | 13 | 0.154 |
| 211 | A | 3 | 2 | 1. | 17 | 0.118 |
| 212 | A | 3 | 2 | 1. | 17 | 0.118 |
| 213 | A | 3 | 2 | 1. | 17 | 0.118 |
| 214 | A | 3 | 2 | 1. | 17 | 0.118 |
| 215 | A | 3 | 2 | 1. | 17 | 0.118 |
| 216 | A | 3 | 2 | 1. | 17 | 0.118 |
| 217 | A | 2 | 2 | 1. | 17 | 0.118 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 218 | A | 4 | 4 | 1. | 15 | 0.267 |
| 219 | A | 3 | 2 | 1. | 13 | 0.154 |
| 220 | A | 3 | 2 | 1. | 17 | 0.118 |
| 221 | A | 3 | 2 | 1. | 17 | 0.118 |
| 222 | A | 3 | 2 | 1. | 17 | 0.118 |
| 223 | A | 3 | 2 | 1. | 17 | 0.118 |
| 224 | A | 3 | 2 | 1. | 17 | 0.118 |
| 225 | A | 3 | 2 | 1. | 17 | 0.118 |
| 226 | A | 2 | 2 | 1. | 17 | 0.118 |
| 227 | A | 3 | 2 | 1. | 17 | 0.118 |
| 228 | A | 3 | 2 | 1. | 17 | 0.118 |
| 229 | A | 3 | 2 | 1. | 15 | 0.133 |
| 230 | A | 3 | 2 | 1. | 13 | 0.154 |
| 231 | A | 3 | 2 | 1. | 17 | 0.118 |
| 232 | A | 4 | 3 | 1. | 19 | 0.158 |
| 233 | A | 3 | 3 | 1. | 17 | 0.176 |
| 234 | A | 2 | 2 | 1. | 15 | 0.133 |
| 235 | A | 1 | 1 | 1. | 19 | 0.053 |
| 236 | A | 3 | 3 | 1. | 19 | 0.158 |
| 237 | A | 3 | 3 | 1. | 19 | 0.158 |
| 238 | A | 4 | 4 | 1. | 19 | 0.21 |
| 239 | A | 5 | 4 | 1. | 19 | 0.21 |
| 240 | A | 6 | 3 | 1. | 19 | 0.158 |
| 241 | A | 5 | 3 | 1. | 17 | 0.176 |
| 242 | A | 4 | 3 | 1. | 15 | 0.2 |
| 243 | A | 3 | 2 | 1. | 19 | 0.105 |
| 244 | A | 2 | 2 | 1. | 19 | 0.105 |
| 245 | A | 1 | 1 | 1. | 19 | 0.053 |
| 246 | A | 4 | 3 | 1. | 19 | 0.158 |
| 247 | A | 4 | 4 | 1. | 19 | 0.21 |
| 248 | A | 4 | 3 | 1. | 19 | 0.158 |
| 249 | A | 5 | 4 | 1. | 19 | 0.21 |
| 250 | A | 6 | 4 | 1. | 19 | 0.21 |
| 251 | A | 7 | 4 | 1. | 19 | 0.21 |
| 252 | A | 4 | 2 | 1. | 19 | 0.105 |
| 253 | A | 3 | 2 | 1. | 19 | 0.105 |
| 254 | A | 2 | 2 | 1. | 19 | 0.105 |
| 255 | A | 1 | 1 | 1. | 17 | 0.059 |
| 256 | A | 2 | 2 | 1. | 15 | 0.133 |
| 257 | A | 3 | 3 | 1. | 19 | 0.158 |
| 258 | A | 4 | 3 | 1. | 19 | 0.158 |
| 259 | A | 5 | 3 | 1. | 19 | 0.158 |
| 260 | A | 4 | 3 | 1. | 19 | 0.158 |
| 261 | A | 3 | 3 | 1. | 19 | 0.158 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 262 | A | 2 | 2 | 1. | 19 | 0.105 |
| 263 | A | 1 | 1 | 1. | 19 | 0.053 |
| 264 | A | 3 | 3 | 1. | 19 | 0.158 |
| 265 | A | 4 | 4 | 1. | 17 | 0.235 |
| 266 | A | 5 | 4 | 1. | 15 | 0.267 |
| 267 | A | 6 | 4 | 1. | 19 | 0.21 |
| 268 | A | 7 | 4 | 1. | 19 | 0.21 |
| 269 | A | 5 | 3 | 1. | 21 | 0.143 |
| 270 | A | 4 | 3 | 1. | 21 | 0.143 |
| 271 | A | 3 | 3 | 1. | 21 | 0.143 |
| 272 | A | 2 | 2 | 1. | 21 | 0.095 |
| 273 | A | 1 | 1 | 1. | 21 | 0.048 |
| 274 | A | 2 | 2 | 1. | 21 | 0.095 |
| 275 | A | 3 | 2 | 1. | 21 | 0.095 |
| 276 | A | 4 | 2 | 1. | 21 | 0.095 |
| 277 | A | 3 | 3 | 1. | 21 | 0.143 |
| 278 | A | 3 | 3 | 1.27 | 19 | 0.158 |
| 279 | A | 3 | 3 | 1. | 21 | 0.143 |
| 280 | A | 1 | 1 | 1. | 21 | 0.048 |
| 281 | A | 2 | 2 | 1. | 21 | 0.095 |
| 282 | A | 3 | 2 | 1. | 21 | 0.095 |
| 283 | A | 2 | 2 | 1. | 17 | 0.118 |
| 284 | A | 3 | 2 | 1. | 19 | 0.105 |
| 285 | A | 2 | 2 | 1. | 19 | 0.105 |
| 286 | A | 1 | 1 | 1. | 19 | 0.053 |
| 287 | A | 2 | 2 | 1. | 15 | 0.133 |
| 288 | A | 3 | 3 | 1. | 19 | 0.158 |
| 289 | A | 3 | 3 | 1. | 19 | 0.158 |
| 290 | A | 2 | 2 | 1. | 17 | 0.118 |
| 291 | A | 3 | 3 | 1. | 19 | 0.158 |
| 292 | A | 5 | 5 | 1. | 19 | 0.263 |
| 293 | A | 4 | 4 | 1. | 19 | 0.21 |
| 294 | A | 5 | 5 | 1. | 19 | 0.263 |
| 295 | A | 5 | 4 | 1. | 21 | 0.19 |
| 296 | A | 6 | 6 | 1. | 21 | 0.286 |
| 297 | A | 3 | 3 | 1. | 21 | 0.143 |
| 298 | A | 4 | 4 | 1. | 21 | 0.19 |
| 299 | A | 5 | 5 | 1. | 21 | 0.238 |
| 300 | A | 2 | 2 | 1. | 21 | 0.095 |
| 301 | A | 3 | 3 | 1. | 21 | 0.143 |
| 302 | A | 6 | 6 | 1. | 21 | 0.286 |
| 303 | A | 1 | 1 | 1. | 21 | 0.048 |
| 304 | A | 4 | 4 | 1. | 21 | 0.19 |
| 305 | A | 7 | 6 | 1. | 21 | 0.286 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 306 | A | 2 | 2 | 1. | 21 | 0.095 |
| 307 | A | 5 | 4 | 1. | 21 | 0.19 |
| 308 | A | 3 | 2 | 1. | 15 | 0.133 |
| 309 | A | 3 | 2 | 1. | 13 | 0.154 |
| 310 | A | 5 | 3 | 1. | 19 | 0.158 |
| 311 | A | 4 | 3 | 1. | 19 | 0.158 |
| 312 | A | 3 | 3 | 1. | 19 | 0.158 |
| 313 | A | 2 | 2 | 1. | 17 | 0.118 |
| 314 | A | 1 | 1 | 1. | 15 | 0.067 |
| 315 | A | 2 | 2 | 1. | 19 | 0.105 |
| 316 | A | 3 | 2 | 1. | 19 | 0.105 |
| 317 | A | 4 | 2 | 1. | 19 | 0.105 |
| 318 | A | 5 | 2 | 1. | 19 | 0.105 |
| 319 | A | 4 | 3 | 1. | 11 | 0.273 |
| 320 | A | 4 | 3 | 1. | 13 | 0.231 |
| 321 | A | 3 | 3 | 1. | 9 | 0.333 |
| 322 | A | 3 | 3 | 1. | 9 | 0.333 |
| 323 | A | 3 | 3 | 1. | 9 | 0.333 |
| 324 | A | 3 | 3 | 1. | 13 | 0.231 |
| 325 | A | 3 | 3 | 1. | 13 | 0.231 |
| 326 | A | 3 | 3 | 1. | 13 | 0.231 |
| 327 | A | 3 | 3 | 1. | 13 | 0.231 |
| 328 | A | 2 | 2 | 1. | 11 | 0.182 |
| 329 | A | 2 | 2 | 1. | 15 | 0.133 |
| 330 | A | 2 | 2 | 1. | 15 | 0.133 |
| 331 | A | 2 | 2 | 1. | 23 | 0.087 |
| 332 | A | 2 | 2 | 1. | 11 | 0.182 |
| 333 | A | 2 | 2 | 1. | 13 | 0.154 |
| 334 | A | 2 | 2 | 1. | 13 | 0.154 |
| 335 | A | 2 | 2 | 1. | 17 | 0.118 |
| 336 | A | 2 | 2 | 1. | 17 | 0.118 |
| 337 | A | 2 | 2 | 1. | 17 | 0.118 |
| 338 | A | 2 | 2 | 1. | 11 | 0.182 |
| 339 | A | 2 | 2 | 1. | 11 | 0.182 |
| 340 | A | 2 | 2 | 1. | 11 | 0.182 |
| 341 | A | 2 | 2 | 1. | 11 | 0.182 |
| 342 | A | 2 | 2 | 1. | 13 | 0.154 |
| 343 | A | 2 | 2 | 1. | 13 | 0.154 |
| 344 | A | 3 | 2 | 1. | 11 | 0.182 |
| 345 | A | 4 | 3 | 1. | 11 | 0.273 |
| 346 | A | 3 | 2 | 1. | 11 | 0.182 |
| 347 | A | 7 | 7 | 1. | 9 | 0.778 |
| 348 | A | 2 | 2 | 1. | 22 | 0.091 |
| 349 | A | 1 | 1 | 1. | 13 | 0.077 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 350 | A | 2 | 2 | 1. | 15 | 0.133 |
| 351 | A | 2 | 2 | 1. | 17 | 0.118 |
| 352 | A | 1 | 1 | 1. | 13 | 0.077 |
| 353 | A | 2 | 2 | 1. | 15 | 0.133 |
| 354 | A | 1 | 1 | 1. | 13 | 0.077 |
| 355 | A | 2 | 2 | 1. | 11 | 0.182 |
| 356 | A | 5 | 5 | 1. | 13 | 0.385 |
| 357 | A | 2 | 2 | 1. | 15 | 0.133 |
| 358 | A | 5 | 5 | 1. | 13 | 0.385 |
| 359 | A | 2 | 2 | 1. | 15 | 0.133 |
| 360 | A | 2 | 2 | 1. | 11 | 0.182 |
| 361 | A | 2 | 2 | 1. | 11 | 0.182 |
| 362 | A | 2 | 2 | 1. | 9 | 0.222 |
| 363 | A | 5 | 5 | 1. | 11 | 0.454 |
| 364 | A | 3 | 3 | 1. | 25 | 0.12 |
| 365 | A | 4 | 4 | 1. | 27 | 0.148 |
| 366 | A | 4 | 4 | 1. | 23 | 0.174 |
| 367 | A | 4 | 4 | 1. | 22 | 0.182 |
| 368 | A | 4 | 4 | 1. | 21 | 0.19 |
| 369 | A | 5 | 5 | 1. | 18 | 0.278 |
| 370 | A | 4 | 4 | 1. | 23 | 0.174 |
| 371 | A | 3 | 3 | 1. | 15 | 0.2 |
| 372 | A | 4 | 4 | 1. | 23 | 0.174 |
| 373 | A | 5 | 4 | 1. | 27 | 0.148 |
| 374 | A | 5 | 4 | 1. | 23 | 0.174 |
| 375 | A | 5 | 4 | 1. | 22 | 0.182 |
| 376 | A | 5 | 4 | 1. | 21 | 0.19 |
| 377 | A | 6 | 5 | 1. | 18 | 0.278 |
| 378 | A | 5 | 4 | 1. | 23 | 0.174 |
| 379 | A | 5 | 5 | 1. | 22 | 0.227 |
| 380 | A | 5 | 4 | 1. | 23 | 0.174 |
| 381 | A | 5 | 4 | 1. | 22 | 0.182 |
| 382 | A | 5 | 5 | 1. | 13 | 0.385 |
| 383 | A | 5 | 5 | 1. | 15 | 0.333 |
| 384 | A | 4 | 4 | 1. | 15 | 0.267 |
| 385 | A | 4 | 4 | 1. | 15 | 0.267 |
| 386 | A | 5 | 5 | 1. | 15 | 0.333 |
| 387 | A | 5 | 5 | 1. | 17 | 0.294 |
| 388 | A | 4 | 4 | 1. | 17 | 0.235 |
| 389 | A | 4 | 4 | 1. | 17 | 0.235 |
| 390 | A | 3 | 3 | 1. | 27 | 0.111 |
| 391 | A | 3 | 3 | 1. | 23 | 0.13 |
| 392 | A | 2 | 2 | 1. | 15 | 0.133 |
| 393 | A | 3 | 3 | 1. | 21 | 0.143 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 394 | A | 4 | 4 | 1. | 18 | 0.222 |
| 395 | A | 3 | 3 | 1. | 23 | 0.13 |
| 396 | A | 3 | 3 | 1. | 22 | 0.136 |
| 397 | A | 3 | 3 | 1. | 23 | 0.13 |
| 398 | A | 4 | 4 | 1. | 27 | 0.148 |
| 399 | A | 4 | 4 | 1. | 23 | 0.174 |
| 400 | A | 4 | 4 | 1. | 22 | 0.182 |
| 401 | A | 4 | 4 | 1. | 21 | 0.19 |
| 402 | A | 5 | 5 | 1. | 18 | 0.278 |
| 403 | A | 4 | 4 | 1. | 23 | 0.174 |
| 404 | A | 4 | 4 | 1. | 22 | 0.182 |
| 405 | A | 4 | 4 | 1. | 23 | 0.174 |
| 406 | A | 4 | 4 | 1. | 22 | 0.182 |
| 407 | A | 3 | 3 | 1. | 15 | 0.2 |
| 408 | A | 3 | 3 | 1. | 15 | 0.2 |
| 409 | A | 3 | 3 | 1. | 15 | 0.2 |
| 410 | A | 3 | 3 | 1. | 19 | 0.158 |
| 411 | A | 3 | 3 | 1. | 16 | 0.188 |
| 412 | A | 3 | 3 | 1. | 16 | 0.188 |
| 413 | A | 3 | 3 | 1. | 16 | 0.188 |
| 414 | A | 3 | 3 | 1. | 20 | 0.15 |
| 415 | A | 3 | 3 | 1. | 19 | 0.158 |
| 416 | A | 3 | 3 | 1. | 17 | 0.176 |
| 417 | A | 3 | 3 | 1. | 17 | 0.176 |
| 418 | A | 3 | 3 | 1. | 20 | 0.15 |
| 419 | A | 3 | 3 | 1. | 19 | 0.158 |
| 420 | A | 3 | 3 | 1. | 18 | 0.167 |
| 421 | A | 3 | 3 | 1. | 21 | 0.143 |
| 422 | A | 3 | 3 | 1. | 21 | 0.143 |
| 423 | A | 3 | 3 | 1. | 21 | 0.143 |
| 424 | A | 3 | 3 | 1. | 21 | 0.143 |
| 425 | A | 3 | 3 | 1. | 21 | 0.143 |
| 426 | A | 3 | 3 | 1. | 15 | 0.2 |
| 427 | A | 3 | 3 | 1. | 15 | 0.2 |
| 428 | A | 3 | 3 | 1. | 15 | 0.2 |
| 429 | A | 3 | 3 | 1. | 15 | 0.2 |
| 430 | A | 3 | 3 | 1. | 15 | 0.2 |
| 431 | A | 2 | 2 | 1. | 11 | 0.182 |
| 432 | A | 1 | 1 | 1. | 11 | 0.091 |
| 433 | A | 3 | 3 | 1. | 13 | 0.231 |
| 434 | A | 1 | 1 | 1. | 17 | 0.059 |
| 435 | A | 1 | 1 | 1. | 17 | 0.059 |
| 436 | A | 2 | 2 | 1. | 15 | 0.133 |
| 437 | A | 11 | 10 | 1. | 19 | 0.526 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 438 | A | 9 | 8 | 1. | 19 | 0.421 |
| 439 | A | 3 | 3 | 1.03 | 17 | 0.176 |
| 440 | A | 3 | 3 | 1. | 22 | 0.136 |
| 441 | A | 3 | 3 | 1.12 | 22 | 0.136 |
| 442 | A | 3 | 3 | 1. | 27 | 0.111 |
| 443 | A | 3 | 3 | 1. | 27 | 0.111 |
| 444 | A | 1 | 1 | 1. | 18 | 0.056 |
| 445 | A | 2 | 2 | 1. | 17 | 0.118 |
| 446 | A | 2 | 2 | 1. | 22 | 0.091 |
| 447 | A | 1 | 1 | 1. | 23 | 0.043 |
| 448 | A | 2 | 2 | 1. | 19 | 0.105 |
| 449 | A | 2 | 2 | 1. | 19 | 0.105 |
| 450 | A | 2 | 2 | 1. | 19 | 0.105 |
| 451 | A | 2 | 2 | 1. | 19 | 0.105 |
| 452 | A | 2 | 2 | 1. | 19 | 0.105 |
| 453 | A | 1 | 1 | 1. | 25 | 0.04 |
| 454 | A | 2 | 2 | 1. | 28 | 0.071 |

# Chapter 3

# Listing of integrals

## 3.1 $\int x^2\left(ax+bx^3\right)\,dx$

**Optimal**. Leaf size=17

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

---

**Rubi [A]** time = 0.0047563, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax+bx^3\right)\,dx &= \int \left(ax^3+bx^5\right)\,dx\\ &= \frac{ax^4}{4}+\frac{bx^6}{6}\end{aligned}$$

**Mathematica [A]** time = 0.0014992, size = 17, normalized size = 1.

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

---

**Maple [A]** time = 0., size = 14, normalized size = 0.8

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x),x)
```

```
[Out] 1/4*a*x^4+1/6*b*x^6
```

---

**Maxima [A]** time = 1.0604, size = 18, normalized size = 1.06

$$\frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

---

**Fricas [A]** time = 1.18363, size = 31, normalized size = 1.82

$$\frac{1}{6}x^6b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b + 1/4*x^4*a
```

---

**Sympy [A]** time = 0.05323, size = 12, normalized size = 0.71

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x),x)
```

```
[Out] a*x**4/4 + b*x**6/6
```

---

**Giac [A]** time = 1.12549, size = 18, normalized size = 1.06

$$\frac{1}{6}bx^6 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

## 3.2 $\int x\left(ax+bx^3\right)\,dx$

**Optimal**. Leaf size=17

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

---

**Rubi [A]** time = 0.0044615, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3),x]

[Out] (a*x^3)/3 + (b*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(ax+bx^3\right)\,dx &= \int \left(ax^2+bx^4\right)\,dx\\ &= \frac{ax^3}{3}+\frac{bx^5}{5}\end{aligned}$$

**Mathematica [A]** time = 0.0014376, size = 17, normalized size = 1.

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3),x]

[Out] (a*x^3)/3 + (b*x^5)/5
```

---

**Maple [A]** time = 0.001, size = 14, normalized size = 0.8

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x),x)
```

```
[Out] 1/3*a*x^3+1/5*b*x^5
```

---

**Maxima [A]** time = 1.07896, size = 18, normalized size = 1.06

$$\frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

---

**Fricas [A]** time = 1.21982, size = 31, normalized size = 1.82

$$\frac{1}{5}x^5b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b + 1/3*x^3*a
```

---

**Sympy [A]** time = 0.053318, size = 12, normalized size = 0.71

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x),x)
```

```
[Out] a*x**3/3 + b*x**5/5
```

---

**Giac [A]** time = 1.18127, size = 18, normalized size = 1.06

$$\frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

## 3.3 $\int \left(ax+bx^3\right)\,dx$

**Optimal**. Leaf size=17

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

---

**Rubi [A]** time = 0.0027001, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a*x + b*x^3,x]

[Out] (a*x^2)/2 + (b*x^4)/4
```

Rubi steps

$$\int \left(ax+bx^3\right)\,dx=\frac{ax^2}{2}+\frac{bx^4}{4}$$

**Mathematica [A]** time = 0.0000298, size = 17, normalized size = 1.

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*x + b*x^3,x]

[Out] (a*x^2)/2 + (b*x^4)/4
```

---

**Maple [A]** time = 0.002, size = 14, normalized size = 0.8

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^3+a*x,x)

[Out] 1/2*a*x^2+1/4*b*x^4
```

---

**Maxima [A]** time = 1.06282, size = 18, normalized size = 1.06

$$\frac{1}{4}\,bx^{4} + \frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x,x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

---

**Fricas [A]** time = 1.27065, size = 31, normalized size = 1.82

$$\frac{1}{4}x^{4}b + \frac{1}{2}x^{2}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b + 1/2*x^2*a
```

---

**Sympy [A]** time = 0.051718, size = 12, normalized size = 0.71

$$\frac{ax^{2}}{2} + \frac{bx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**3+a*x,x)
```

```
[Out] a*x**2/2 + b*x**4/4
```

---

**Giac [A]** time = 1.17737, size = 18, normalized size = 1.06

$$\frac{1}{4}\,bx^{4} + \frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

## 3.4 $\int \frac{ax+bx^3}{x} \, dx$

**Optimal**. Leaf size=12

$$ax + \frac{bx^3}{3}$$

```
[Out] a*x + (b*x^3)/3
```

---

**Rubi [A]** time = 0.0033423, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)/x,x]
```

```
[Out] a*x + (b*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{ax+bx^3}{x} \, dx &= \int \left(a+bx^2\right) \, dx \\ &= ax + \frac{bx^3}{3} \end{aligned}$$

**Mathematica [A]** time = 0.0004057, size = 12, normalized size = 1.

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)/x,x]
```

```
[Out] a*x + (b*x^3)/3
```

---

**Maple [A]** time = 0., size = 11, normalized size = 0.9

$$ax + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)/x,x)
```

```
[Out] a*x+1/3*b*x^3
```

---

**Maxima [A]** time = 1.04063, size = 14, normalized size = 1.17

$$\frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + a*x
```

---

**Fricas [A]** time = 1.3455, size = 23, normalized size = 1.92

$$\frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x,x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + a*x
```

---

**Sympy [A]** time = 0.051531, size = 8, normalized size = 0.67

$$ax + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)/x,x)
```

```
[Out] a*x + b*x**3/3
```

---

**Giac [A]** time = 1.23919, size = 14, normalized size = 1.17

$$\frac{1}{3}bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + a*x
```

## 3.5 $\int \frac{ax+bx^3}{x^2}\,dx$

**Optimal**. Leaf size=13

$$a\log(x)+\frac{bx^2}{2}$$

```
[Out] (b*x^2)/2 + a*Log[x]
```

---

**Rubi [A]** time = 0.0047781, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$a\log(x)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)/x^2,x]
```

```
[Out] (b*x^2)/2 + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{ax+bx^3}{x^2}\,dx &= \int \left(\frac{a}{x}+bx\right)dx\\ &= \frac{bx^2}{2}+a\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.0009369, size = 13, normalized size = 1.

$$a\log(x)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)/x^2,x]
```

```
[Out] (b*x^2)/2 + a*Log[x]
```

---

**Maple [A]** time = 0.002, size = 12, normalized size = 0.9

$$\frac{bx^2}{2}+a\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)/x^2,x)
```

```
[Out] 1/2*b*x^2+a*ln(x)
```

---

**Maxima [A]** time = 1.03596, size = 15, normalized size = 1.15

$$\frac{1}{2}\,bx^2 + a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*log(x)
```

---

**Fricas [A]** time = 1.33364, size = 30, normalized size = 2.31

$$\frac{1}{2}\,bx^2 + a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*log(x)
```

---

**Sympy [A]** time = 0.077242, size = 10, normalized size = 0.77

$$a\log\left(x\right) + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)/x**2,x)
```

```
[Out] a*log(x) + b*x**2/2
```

---

**Giac [A]** time = 1.18372, size = 19, normalized size = 1.46

$$\frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

## 3.6 $\int x^2\left(ax+bx^3\right)^2\,dx$

**Optimal**. Leaf size=30

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

---

**Rubi [A]** time = 0.0150718, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 270}

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax+bx^3\right)^2\,dx&=\int x^4\left(a+bx^2\right)^2\,dx\\&=\int\left(a^2x^4+2abx^6+b^2x^8\right)dx\\&=\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}\end{aligned}$$

**Mathematica [A]** time = 0.0007609, size = 30, normalized size = 1.

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

**Maple [A]** time = 0.001, size = 25, normalized size = 0.8

$$\frac{a^{2}x^{5}}{5}+\frac{2\,abx^{7}}{7}+\frac{b^{2}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x)^2,x)
```

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

**Maxima [A]** time = 1.0588, size = 32, normalized size = 1.07

$$\frac{1}{9}\,b^{2}x^{9}+\frac{2}{7}\,abx^{7}+\frac{1}{5}\,a^{2}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

**Fricas [A]** time = 1.20442, size = 55, normalized size = 1.83

$$\frac{1}{9}x^{9}b^{2}+\frac{2}{7}x^{7}ba+\frac{1}{5}x^{5}a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^2 + 2/7*x^7*b*a + 1/5*x^5*a^2
```

**Sympy [A]** time = 0.060723, size = 26, normalized size = 0.87

$$\frac{a^{2}x^{5}}{5}+\frac{2abx^{7}}{7}+\frac{b^{2}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9
```

**Giac [A]** time = 1.13094, size = 32, normalized size = 1.07

$$\frac{1}{9}\,b^{2}x^{9}+\frac{2}{7}\,abx^{7}+\frac{1}{5}\,a^{2}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

## 3.7 $\int x\left(ax+bx^3\right)^2\,dx$

**Optimal**. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

---

**Rubi [A]** time = 0.0195837, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 43}

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(ax+bx^3\right)^2\,dx &= \int x^3\left(a+bx^2\right)^2\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)^2\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(a^2x+2abx^2+b^2x^3\right)\,dx,x,x^2\right)\\ &= \frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}\end{aligned}$$

**Mathematica [A]** time = 0.0009825, size = 30, normalized size = 1.

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

---

**Maple [A]** time = 0.001, size = 25, normalized size = 0.8

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x)^2,x)
```

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

---

**Maxima [A]** time = 1.02882, size = 32, normalized size = 1.07

$$\frac{1}{8}\,b^2x^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

---

**Fricas [A]** time = 1.1766, size = 55, normalized size = 1.83

$$\frac{1}{8}x^8b^2 + \frac{1}{3}x^6ba + \frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^2 + 1/3*x^6*b*a + 1/4*x^4*a^2
```

---

**Sympy [A]** time = 0.059262, size = 24, normalized size = 0.8

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

---

**Giac [A]** time = 1.16584, size = 32, normalized size = 1.07

$$\frac{1}{8}\,b^{2}x^{8}+\frac{1}{3}\,abx^{6}+\frac{1}{4}\,a^{2}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

## 3.8 $\int \left(ax+bx^3\right)^2\,dx$

**Optimal**. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

---

**Rubi [A]** time = 0.0106576, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 270}

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(ax+bx^3\right)^2\,dx &= \int x^2\left(a+bx^2\right)^2\,dx\\ &= \int \left(a^2x^2+2abx^4+b^2x^6\right)\,dx\\ &= \frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}\end{aligned}$$

**Mathematica [A]** time = 0.001557, size = 30, normalized size = 1.

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

---

**Maple [A]** time = 0.001, size = 25, normalized size = 0.8

$$\frac{x^3a^2}{3}+\frac{2\,abx^5}{5}+\frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^2,x)
```

```
[Out] 1/3*x^3*a^2+2/5*a*b*x^5+1/7*b^2*x^7
```

---

**Maxima [A]** time = 1.08874, size = 32, normalized size = 1.07

$$\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

---

**Fricas [A]** time = 1.20089, size = 55, normalized size = 1.83

$$\frac{1}{7}x^7b^2+\frac{2}{5}x^5ba+\frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^2 + 2/5*x^5*b*a + 1/3*x^3*a^2
```

---

**Sympy [A]** time = 0.059754, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

---

**Giac [A]** time = 1.20869, size = 32, normalized size = 1.07

$$\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

## 3.9 $\int \frac{\left(ax+bx^3\right)^2}{x}\,dx$

**Optimal**. Leaf size=16

$$\frac{\left(a+bx^2\right)^3}{6b}$$

```
[Out] (a + b*x^2)^3/(6*b)
```

---

**Rubi [A]** time = 0.0070228, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 261}

$$\frac{\left(a+bx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^2/x,x]

[Out] (a + b*x^2)^3/(6*b)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax+bx^3\right)^2}{x}\,dx &= \int x\left(a+bx^2\right)^2\,dx\\ &= \frac{\left(a+bx^2\right)^3}{6b}\end{aligned}$$

**Mathematica [A]** time = 0.0016032, size = 16, normalized size = 1.

$$\frac{\left(a+bx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^2/x,x]
```

```
[Out] (a + b*x^2)^3/(6*b)
```

---

**Maple [A]** time = 0.002, size = 25, normalized size = 1.6

$$\frac{b^2x^6}{6} + \frac{abx^4}{2} + \frac{a^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^2/x,x)
```

```
[Out] 1/6*b^2*x^6+1/2*a*b*x^4+1/2*a^2*x^2
```

---

**Maxima [A]** time = 1.10286, size = 32, normalized size = 2.

$$\frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

---

**Fricas [A]** time = 1.34173, size = 55, normalized size = 3.44

$$\frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

---

**Sympy [B]** time = 0.067713, size = 24, normalized size = 1.5

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**2/x,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

---

**Giac [A]** time = 1.17046, size = 32, normalized size = 2.

$$\frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

## 3.10 $\int \frac{(ax+bx^3)^2}{x^2} \, dx$

**Optimal**. Leaf size=25

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

---

**Rubi [A]** time = 0.011589, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 194}

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(ax+bx^3)^2}{x^2} \, dx &= \int (a+bx^2)^2 \, dx\\ &= \int (a^2+2abx^2+b^2x^4) \, dx\\ &= a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}\end{aligned}$$

**Mathematica [A]** time = 0.0009442, size = 25, normalized size = 1.

$$a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

---

**Maple [A]** time = 0.002, size = 22, normalized size = 0.9

$$a^2x + \frac{2\,abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^2/x^2,x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

---

**Maxima [A]** time = 0.995564, size = 28, normalized size = 1.12

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

---

**Fricas [A]** time = 1.31853, size = 47, normalized size = 1.88

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

---

**Sympy [A]** time = 0.059626, size = 22, normalized size = 0.88

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**2/x**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

---

**Giac [A]** time = 1.22034, size = 28, normalized size = 1.12

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

## 3.11 $\int \left(-4x+3x^3\right)^6 \, dx$

**Optimal**. Leaf size=46

$$\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^9+\frac{4096x^7}{7}$$

```
[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (
5832*x^17)/17 + (729*x^19)/19
```

---

**Rubi [A]** time = 0.0157418, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 270}

$$\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^9+\frac{4096x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(-4*x + 3*x^3)^6,x]
```

```
[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (
5832*x^17)/17 + (729*x^19)/19
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(-4x+3x^3\right)^6 \, dx &= \int x^6 \left(-4+3x^2\right)^6 \, dx\\ &= \int \left(4096x^6-18432x^8+34560x^{10}-34560x^{12}+19440x^{14}-5832x^{16}+729x^{18}\right) dx\\ &= \frac{4096x^7}{7}-2048x^9+\frac{34560x^{11}}{11}-\frac{34560x^{13}}{13}+1296x^{15}-\frac{5832x^{17}}{17}+\frac{729x^{19}}{19}\end{aligned}$$

**Mathematica [A]** time = 0.0018519, size = 46, normalized size = 1.

$$\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^9+\frac{4096x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4*x + 3*x^3)^6,x]
```

```
[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (
5832*x^17)/17 + (729*x^19)/19
```

---

**Maple [A]** time = 0.002, size = 37, normalized size = 0.8

$$\frac{4096\,x^{7}}{7}-2048\,x^{9}+\frac{34560\,x^{11}}{11}-\frac{34560\,x^{13}}{13}+1296\,x^{15}-\frac{5832\,x^{17}}{17}+\frac{729\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-4*x)^6,x)
```

```
[Out] 4096/7*x^7-2048*x^9+34560/11*x^11-34560/13*x^13+1296*x^15-5832/17*x^17+729/
19*x^19
```

---

**Maxima [A]** time = 1.04227, size = 49, normalized size = 1.07

$$\frac{729}{19}\,x^{19}-\frac{5832}{17}\,x^{17}+1296\,x^{15}-\frac{34560}{13}\,x^{13}+\frac{34560}{11}\,x^{11}-2048\,x^{9}+\frac{4096}{7}\,x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x)^6,x, algorithm="maxima")
```

```
[Out] 729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 20
48*x^9 + 4096/7*x^7
```

---

**Fricas [A]** time = 1.20751, size = 130, normalized size = 2.83

$$\frac{729}{19}x^{19}-\frac{5832}{17}x^{17}+1296x^{15}-\frac{34560}{13}x^{13}+\frac{34560}{11}x^{11}-2048x^{9}+\frac{4096}{7}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x)^6,x, algorithm="fricas")
```

```
[Out] 729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 20
48*x^9 + 4096/7*x^7
```

---

**Sympy [A]** time = 0.062123, size = 42, normalized size = 0.91

$$\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^{9}+\frac{4096x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-4*x)**6,x)
```

```
[Out] 729*x**19/19 - 5832*x**17/17 + 1296*x**15 - 34560*x**13/13 + 34560*x**11/11
 - 2048*x**9 + 4096*x**7/7
```

---

**Giac [A]** time = 1.25294, size = 49, normalized size = 1.07

$$\frac{729}{19}\,x^{19}-\frac{5832}{17}\,x^{17}+1296\,x^{15}-\frac{34560}{13}\,x^{13}+\frac{34560}{11}\,x^{11}-2048\,x^{9}+\frac{4096}{7}\,x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x)^6,x, algorithm="giac")
```

```
[Out] 729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 20
48*x^9 + 4096/7*x^7
```

## 3.12 $\int \frac{x^4}{ax+bx^3}\,dx$

**Optimal**. Leaf size=27

$$\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

```
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

---

**Rubi [A]** time = 0.0227478, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1584, 266, 43}

$$\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x + b*x^3),x]
```

```
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{ax+bx^3}\,dx &= \int \frac{x^3}{a+bx^2}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{a+bx}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx,x,x^2\right)\\ &= \frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}\end{aligned}$$

**Mathematica [A]** time = 0.0039764, size = 27, normalized size = 1.

$$\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x + b*x^3),x]
```

```
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

---

**Maple [A]** time = 0.001, size = 24, normalized size = 0.9

$$\frac{x^2}{2\,b}-\frac{a\ln\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x),x)
```

```
[Out] 1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2
```

---

**Maxima [A]** time = 1.06432, size = 31, normalized size = 1.15

$$\frac{x^2}{2\,b}-\frac{a\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

---

**Fricas [A]** time = 1.38497, size = 49, normalized size = 1.81

$$\frac{bx^2-a\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

---

**Sympy [A]** time = 0.289509, size = 20, normalized size = 0.74

$$-\frac{a\log\left(a+bx^2\right)}{2b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x),x)
```

```
[Out] -a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

---

**Giac [A]** time = 1.20129, size = 32, normalized size = 1.19

$$\frac{x^2}{2\,b}-\frac{a\log\left(\left|bx^2+a\right|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2/b - 1/2*a*log(abs(b*x^2 + a))/b^2
```

# 3.13 $\int \frac{x^3}{ax+bx^3}\,dx$

**Optimal**. Leaf size=31

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

---

**Rubi [A]** time = 0.0161081, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1584, 321, 205}

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x + b*x^3),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{ax+bx^3}\,dx &= \int \frac{x^2}{a+bx^2}\,dx\\ &= \frac{x}{b}-\frac{a\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.0085384, size = 31, normalized size = 1.

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x + b*x^3),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

---

**Maple [A]** time = 0.004, size = 27, normalized size = 0.9

$$\frac{x}{b}-\frac{a}{b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x),x)
```

```
[Out] x/b-1/b*a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 1.41214, size = 165, normalized size = 5.32

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)+2\,x}{2\,b},\,-\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
-(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

---

**Sympy [B]** time = 0.287938, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x),x)
```

```
[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b**
3) + x)/2 + x/b
```

---

**Giac [A]** time = 1.24738, size = 35, normalized size = 1.13

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

## 3.14 $\int \frac{x^2}{ax+bx^3}\,dx$

**Optimal**. Leaf size=15

$$\frac{\log\left(a+bx^2\right)}{2b}$$

```
[Out] Log[a + b*x^2]/(2*b)
```

---

**Rubi [A]** time = 0.0074074, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 260}

$$\frac{\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3),x]
```

```
[Out] Log[a + b*x^2]/(2*b)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{ax+bx^3}\,dx &= \int \frac{x}{a+bx^2}\,dx\\ &= \frac{\log\left(a+bx^2\right)}{2b}\end{aligned}$$

**Mathematica [A]** time = 0.0020339, size = 15, normalized size = 1.

$$\frac{\log\left(a+bx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3),x]
```

```
[Out] Log[a + b*x^2]/(2*b)
```

---

**Maple [A]** time = 0.002, size = 14, normalized size = 0.9

$$\frac{\ln\left(b x^{2}+a\right)}{2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x),x)
```

```
[Out] 1/2*ln(b*x^2+a)/b
```

---

**Maxima [A]** time = 1.05242, size = 18, normalized size = 1.2

$$\frac{\log\left(b x^{2}+a\right)}{2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 + a)/b
```

---

**Fricas [A]** time = 1.36721, size = 30, normalized size = 2.

$$\frac{\log\left(b x^{2}+a\right)}{2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 + a)/b
```

---

**Sympy [A]** time = 0.101106, size = 10, normalized size = 0.67

$$\frac{\log\left(a+b x^{2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x),x)
```

```
[Out] log(a + b*x**2)/(2*b)
```

---

**Giac [A]** time = 1.27121, size = 19, normalized size = 1.27

$$\frac{\log\left(\left|b x^{2}+a\right|\right)}{2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x^2 + a))/b
```

## 3.15 $\int \frac{x}{ax+bx^3}\,dx$

**Optimal**. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

---

**Rubi [A]** time = 0.0089429, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1584, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{ax+bx^3}\,dx &= \int \frac{1}{a+bx^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\end{aligned}$$

**Mathematica [A]** time = 0.0038165, size = 24, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

---

**Maple [A]** time = 0.003, size = 16, normalized size = 0.7

$$\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 1.41005, size = 151, normalized size = 6.29

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

---

**Sympy [B]** time = 0.122746, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

---

**Giac [A]** time = 1.16161, size = 20, normalized size = 0.83

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

## 3.16 $\int \frac{1}{ax+bx^3} \, dx$

**Optimal**. Leaf size=22

$$\frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

```
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

---

**Rubi [A]** time = 0.0120126, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(-1),x]
```

```
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{ax+bx^3}\,dx &= \int \frac{1}{x\left(a+bx^2\right)}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2a} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^2\right)}{2a}\\
&= \frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}
\end{aligned}
$$

**Mathematica [A]** time = 0.0042559, size = 22, normalized size = 1.

$$\frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(-1),x]
```

```
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

---

**Maple [A]** time = 0.005, size = 21, normalized size = 1.

$$\frac{\ln(x)}{a} - \frac{\ln\left(bx^2+a\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x),x)
```

```
[Out] ln(x)/a-1/2*ln(b*x^2+a)/a
```

---

**Maxima [A]** time = 1.06132, size = 27, normalized size = 1.23

$$-\frac{\log\left(bx^2+a\right)}{2a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x^2 + a)/a + log(x)/a
```

---

**Fricas [A]** time = 1.40941, size = 49, normalized size = 2.23

$$-\frac{\log\left(bx^2+a\right) - 2\log(x)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*(log(b*x^2 + a) - 2*log(x))/a
```

---

**Sympy [A]** time = 0.178339, size = 15, normalized size = 0.68

$$\frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x),x)
```

```
[Out] log(x)/a - log(a/b + x**2)/(2*a)
```

---

**Giac [A]** time = 1.18213, size = 32, normalized size = 1.45

$$\frac{\log\left(x^2\right)}{2\,a} - \frac{\log\left(\left|bx^2 + a\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a - 1/2*log(abs(b*x^2 + a))/a
```

## 3.17 $\int \frac{1}{x(ax+bx^3)}\,dx$

**Optimal**. Leaf size=34

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

---

**Rubi [A]** time = 0.0167518, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1584, 325, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(ax+bx^3\right)}\,dx &= \int \frac{1}{x^2\left(a+bx^2\right)}\,dx\\ &= -\frac{1}{ax}-\frac{b\int \frac{1}{a+bx^2}\,dx}{a}\\ &= -\frac{1}{ax}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.0120363, size = 34, normalized size = 1.

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

---

**Maple [A]** time = 0.005, size = 30, normalized size = 0.9

$$-\frac{1}{ax}-\frac{b}{a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x),x)
```

```
[Out] -1/a/x-b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 1.39829, size = 173, normalized size = 5.09

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2ax}, -\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a*
x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

---

**Sympy [B]** time = 0.312298, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sqr
t(-b/a**3)/b + x)/2 - 1/(a*x)
```

---

**Giac [A]** time = 1.27111, size = 39, normalized size = 1.15

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

## 3.18 $\int \frac{1}{x^2(ax+bx^3)}\,dx$

**Optimal**. Leaf size=35

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

---

**Rubi [A]** time = 0.0259816, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1584, 266, 44}

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3)),x]
```

```
[Out] -1/(2*a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx&=\int \frac{1}{x^3\left(a+bx^2\right)}\,dx\\&=\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx,x,x^2\right)\\&=-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{2a^2}\end{aligned}$$

**Mathematica [A]** time = 0.0062902, size = 35, normalized size = 1.

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3)),x]
```

```
[Out] -1/(2*a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

---

**Maple [A]** time = 0.004, size = 32, normalized size = 0.9

$$-\frac{1}{2\,ax^2}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(bx^2+a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x),x)
```

```
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

---

**Maxima [A]** time = 1.04789, size = 42, normalized size = 1.2

$$\frac{b\log\left(bx^2+a\right)}{2\,a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + a)/a^2 - b*log(x)/a^2 - 1/2/(a*x^2)
```

---

**Fricas [A]** time = 1.42273, size = 80, normalized size = 2.29

$$\frac{bx^2\log\left(bx^2+a\right)-2\,bx^2\log(x)-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

---

**Sympy [A]** time = 0.39978, size = 31, normalized size = 0.89

$$-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x),x)
```

```
[Out] -1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

---

**Giac [A]** time = 1.30532, size = 58, normalized size = 1.66

$$-\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{b\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2)/a^2 + 1/2*b*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 - a)/(a^2*
x^2)
```

## 3.19 $\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx$

**Optimal**. Leaf size=43

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

---

**Rubi [A]** time = 0.0259111, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1584, 325, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a*x + b*x^3)),x]
```

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx &= \int \frac{1}{x^4\left(a+bx^2\right)}\,dx\\ &= -\frac{1}{3ax^3}-\frac{b\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^2\int \frac{1}{a+bx^2}\,dx}{a^2}\\ &= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

**Mathematica [A]** time = 0.019149, size = 43, normalized size = 1.

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a*x + b*x^3)),x]
```

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

---

**Maple [A]** time = 0.005, size = 39, normalized size = 0.9

$$-\frac{1}{3\,a{x}^{3}}+\frac{b}{{a}^{2}x}+\frac{{b}^{2}}{{a}^{2}}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a*x),x)
```

```
[Out] -1/3/a/x^3+b/a^2/x+1/a^2*b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 1.39757, size = 234, normalized size = 5.44

$$\left[\frac{3\,bx^{3}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+6\,bx^{2}-2\,a}{6\,a^{2}x^{3}},\frac{3\,bx^{3}\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,bx^{2}-a}{3\,a^{2}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*b*
x^2 - a)/(a^2*x^3)]
```

---

**Sympy [B]** time = 0.369346, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{-a+3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a*x),x)
```

```
[Out] -sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5)
*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

---

**Giac [A]** time = 1.25553, size = 54, normalized size = 1.26

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}+\frac{3\,bx^2-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

## 3.20 $\int \frac{1}{x^4(ax+bx^3)}\,dx$

**Optimal**. Leaf size=49

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2*a
^3)
```

---

**Rubi [A]** time = 0.0329381, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1584, 266, 44}

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a*x + b*x^3)),x]
```

```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2*a
^3)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(ax+bx^3\right)}\,dx &= \int \frac{1}{x^5\left(a+bx^2\right)}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log\left(a+bx^2\right)}{2a^3}\end{aligned}$$

**Mathematica [A]** time = 0.0066015, size = 49, normalized size = 1.

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a*x + b*x^3)),x]
```

```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2*a
^3)
```

---

**Maple [A]** time = 0.006, size = 44, normalized size = 0.9

$$-\frac{1}{4\,ax^4}+\frac{b}{2\,a^2x^2}+\frac{b^2\ln\left(x\right)}{a^3}-\frac{b^2\ln\left(bx^2+a\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a*x),x)
```

```
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

---

**Maxima [A]** time = 1.07039, size = 59, normalized size = 1.2

$$-\frac{b^2\log\left(bx^2+a\right)}{2\,a^3}+\frac{b^2\log\left(x\right)}{a^3}+\frac{2\,bx^2-a}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log(b*x^2 + a)/a^3 + b^2*log(x)/a^3 + 1/4*(2*b*x^2 - a)/(a^2*x^4)
```

---

**Fricas [A]** time = 1.43787, size = 108, normalized size = 2.2

$$-\frac{2\,b^2x^4\log\left(bx^2+a\right)-4\,b^2x^4\log\left(x\right)-2\,abx^2+a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*x^4*log(b*x^2 + a) - 4*b^2*x^4*log(x) - 2*a*b*x^2 + a^2)/(a^3*x
^4)
```

---

**Sympy [A]** time = 0.453392, size = 42, normalized size = 0.86

$$\frac{-a+2bx^2}{4a^2x^4}+\frac{b^2\log\left(x\right)}{a^3}-\frac{b^2\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a*x),x)
```

```
[Out] (-a + 2*b*x**2)/(4*a**2*x**4) + b**2*log(x)/a**3 - b**2*log(a/b + x**2)/(2*
a**3)
```

---

**Giac [A]** time = 1.21454, size = 77, normalized size = 1.57

$$\frac{b^{2} \log \left(x^{2}\right)}{2 a^{3}} - \frac{b^{2} \log \left(\left|bx^{2} + a\right|\right)}{2 a^{3}} - \frac{3 b^{2} x^{4} - 2 a b x^{2} + a^{2}}{4 a^{3} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(x^2)/a^3 - 1/2*b^2*log(abs(b*x^2 + a))/a^3 - 1/4*(3*b^2*x^4 - 2
*a*b*x^2 + a^2)/(a^3*x^4)
```

## 3.21 $\int \frac{x^2}{\left(ax+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

---

**Rubi [A]** time = 0.015014, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1584, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3)^2,x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(ax+bx^3\right)^2}\,dx &= \int \frac{1}{\left(a+bx^2\right)^2}\,dx\\ &= \frac{x}{2a\left(a+bx^2\right)}+\frac{\int \frac{1}{a+bx^2}\,dx}{2a}\\ &= \frac{x}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\end{aligned}$$

**Mathematica [A]** time = 0.0237474, size = 45, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3)^2,x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

---

**Maple [A]** time = 0.004, size = 36, normalized size = 0.8

$$\frac{x}{2\,a\left(bx^2+a\right)}+\frac{1}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x)^2,x)
```

```
[Out] 1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 1.42494, size = 261, normalized size = 5.8

$$\left[\frac{2\,abx-\left(bx^2+a\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{4\left(a^2b^2x^2+a^3b\right)},\frac{abx+\left(bx^2+a\right)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\left(a^2b^2x^2+a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arctan
(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

**Sympy [B]** time = 0.344998, size = 78, normalized size = 1.73

$$\frac{x}{2a^2 + 2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x)**2,x)
```

```
[Out] x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) + x
)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

---

**Giac [A]** time = 1.26264, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a} + \frac{x}{2\left(bx^2 + a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

## 3.22 $\int \frac{x}{\left(ax+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=38

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

---

**Rubi [A]** time = 0.0283458, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 44}

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3)^2,x]
```

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(ax+bx^3\right)^2}\,dx &= \int \frac{1}{x\left(a+bx^2\right)^2}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{1}{2a\left(a+bx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0131571, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^2} - \log\left(a+bx^2\right) + 2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3)^2,x]
```

```
[Out] (a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

---

**Maple [A]** time = 0.009, size = 35, normalized size = 0.9

$$\frac{1}{2\,a\left(bx^2+a\right)} + \frac{\ln(x)}{a^2} - \frac{\ln\left(bx^2+a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x)^2,x)
```

```
[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

---

**Maxima [A]** time = 1.0081, size = 46, normalized size = 1.21

$$\frac{1}{2\left(abx^2+a^2\right)} - \frac{\log\left(bx^2+a\right)}{2\,a^2} + \frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + log(x)/a^2
```

---

**Fricas [A]** time = 1.4085, size = 108, normalized size = 2.84

$$-\frac{\left(bx^2+a\right)\log\left(bx^2+a\right) - 2\left(bx^2+a\right)\log(x) - a}{2\left(a^2bx^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 + a
^3)
```

---

**Sympy [A]** time = 0.414961, size = 34, normalized size = 0.89

$$\frac{1}{2a^2+2abx^2} + \frac{\log(x)}{a^2} - \frac{\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x)**2,x)
```

```
[Out] 1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

---

**Giac [A]** time = 1.20168, size = 63, normalized size = 1.66

$$\frac{\log\left(x^2\right)}{2\,a^2}-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bx^2+2\,a}{2\left(bx^2+a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2
+ a)*a^2)
```

## 3.23 $\int \frac{1}{\left(ax+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=57

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```

---

**Rubi [A]** time = 0.0178089, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1593, 290, 325, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(-2),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(ax+bx^3\right)^2}\,dx &= \int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx\\ &= \frac{1}{2ax\left(a+bx^2\right)}+\frac{3\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a}\\ &= -\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{(3b)\int \frac{1}{a+bx^2}\,dx}{2a^2}\\ &= -\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}\end{aligned}$$

**Mathematica [A]** time = 0.0339675, size = 54, normalized size = 0.95

$$-\frac{bx}{2a^2\left(a+bx^2\right)}-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(-2),x]
```

```
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*a^(5/2))
```

---

**Maple [A]** time = 0.008, size = 46, normalized size = 0.8

$$-\frac{1}{a^2x}-\frac{bx}{2\,a^2\left(bx^2+a\right)}-\frac{3\,b}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x)^2,x)
```

```
[Out] -1/a^2/x-1/2*b/a^2*x/(b*x^2+a)-3/2*b/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)
)
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 1.46032, size = 288, normalized size = 5.05

$$\left[-\frac{6\,bx^2 - 3\left(bx^3+ax\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+4\,a}{4\left(a^2bx^3+a^3x\right)}, -\frac{3\,bx^2+3\left(bx^3+ax\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,a}{2\left(a^2bx^3+a^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a*x
)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

---

**Sympy [A]** time = 0.436365, size = 90, normalized size = 1.58

$$\frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4} - \frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4} - \frac{2a+3bx^2}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x)**2,x)
```

```
[Out] 3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**3
*sqrt(-b/a**5)/b + x)/4 - (2*a + 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

---

**Giac [A]** time = 1.1826, size = 63, normalized size = 1.11

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2} - \frac{3\,bx^2+2\,a}{2\left(bx^3+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
+ a*x)*a^2)
```

## 3.24 $\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=49

$$-\frac{b}{2a^2\left(a+bx^2\right)}+\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{1}{2a^2x^2}$$

```
[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^
2])/a^3
```

---

**Rubi [A]** time = 0.0390713, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1584, 266, 44}

$$-\frac{b}{2a^2\left(a+bx^2\right)}+\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3)^2),x]
```

```
[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^
2])/a^3
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx &= \int \frac{1}{x^3\left(a+bx^2\right)^2}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right) dx, x, x^2\right) \\
&= -\frac{1}{2a^2x^2}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(a+bx^2\right)}{a^3}
\end{aligned}
$$

**Mathematica [A]** time = 0.0345072, size = 41, normalized size = 0.84

$$
-\frac{a\left(\frac{b}{a+bx^2}+\frac{1}{x^2}\right)-2b\log\left(a+bx^2\right)+4b\log(x)}{2a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3)^2),x]
```

```
[Out] -(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/(2*a^3)
```

---

**Maple [A]** time = 0.012, size = 46, normalized size = 0.9

$$
-\frac{1}{2\,a^2x^2}-\frac{b}{2\,a^2\left(bx^2+a\right)}-2\,\frac{b\ln(x)}{a^3}+\frac{b\ln\left(bx^2+a\right)}{a^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x)^2,x)
```

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

---

**Maxima [A]** time = 1.1098, size = 68, normalized size = 1.39

$$
-\frac{2\,bx^2+a}{2\left(a^2bx^4+a^3x^2\right)}+\frac{b\log\left(bx^2+a\right)}{a^3}-\frac{2\,b\log(x)}{a^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - 2*b*log(x
)/a^3
```

---

**Fricas [A]** time = 1.42221, size = 157, normalized size = 3.2

$$
-\frac{2\,abx^2+a^2-2\left(b^2x^4+abx^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+abx^2\right)\log(x)}{2\left(a^3bx^4+a^4x^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4 +
 a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

---

**Sympy [A]** time = 0.51499, size = 49, normalized size = 1.

$$-\frac{a+2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log{(x)}}{a^3}+\frac{b\log{\left(\frac{a}{b}+x^2\right)}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x)**2,x)
```

```
[Out] -(a + 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/b
 + x**2)/a**3
```

---

**Giac [A]** time = 1.19464, size = 69, normalized size = 1.41

$$-\frac{b\log\left(x^2\right)}{a^3}+\frac{b\log\left(\left|bx^2+a\right|\right)}{a^3}-\frac{2\,bx^2+a}{2\left(bx^4+ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 + a
*x^2)*a^2)
```

## 3.25 $\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=68

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

---

**Rubi [A]** time = 0.0293549, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1584, 290, 325, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3)^2),x]
```

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx &= \int \frac{1}{x^4\left(a+bx^2\right)^2}\,dx \\
&= \frac{1}{2ax^3\left(a+bx^2\right)} + \frac{5\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{2a} \\
&= -\frac{5}{6a^2x^3} + \frac{1}{2ax^3\left(a+bx^2\right)} - \frac{(5b)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a^2} \\
&= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3\left(a+bx^2\right)} + \frac{\left(5b^2\right)\int \frac{1}{a+bx^2}\,dx}{2a^3} \\
&= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3\left(a+bx^2\right)} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.0368775, size = 67, normalized size = 0.99

$$
\frac{b^2x}{2a^3\left(a+bx^2\right)} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}} + \frac{2b}{a^3x} - \frac{1}{3a^2x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3)^2),x]
```

```
[Out] -1/(3*a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

---

**Maple [A]** time = 0.01, size = 59, normalized size = 0.9

$$
-\frac{1}{3\,x^3a^2} + 2\,\frac{b}{a^3x} + \frac{b^2x}{2\,a^3\left(bx^2+a\right)} + \frac{5\,b^2}{2\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x)^2,x)
```

```
[Out] -1/3/x^3/a^2+2*b/a^3/x+1/2*b^2/a^3*x/(b*x^2+a)+5/2*b^2/a^3/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 1.46605, size = 359, normalized size = 5.28

$$\left[\frac{30\,b^2x^4 + 20\,abx^2 + 15\,\left(b^2x^5 + abx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) - 4\,a^2}{12\,\left(a^3bx^5 + a^4x^3\right)}, \frac{15\,b^2x^4 + 10\,abx^2 + 15\,\left(b^2x^5 + abx^3\right)\sqrt{\frac{b}{a}}\,\mathrm{arcta}}{6\,\left(a^3bx^5 + a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*x
^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1/6
*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sqrt(
b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

---

**Sympy [A]** time = 0.516177, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2} + x\right)}{4} + \frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2} + x\right)}{4} + \frac{-2a^2 + 10abx^2 + 15b^2x^4}{6a^4x^3 + 6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x)**2,x)
```

```
[Out] -5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/a
**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*b*
*2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

---

**Giac [A]** time = 1.23994, size = 80, normalized size = 1.18

$$\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3} + \frac{b^2x}{2\,\left(bx^2 + a\right)a^3} + \frac{6\,bx^2 - a}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3)
 + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

## 3.26 $\int \frac{x^5}{x-x^3} \, dx$

**Optimal**. Leaf size=13

$$-\frac{x^3}{3}-x+\tanh^{-1}(x)$$

```
[Out] -x - x^3/3 + ArcTanh[x]
```

---

**Rubi [A]** time = 0.010937, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 302, 206}

$$-\frac{x^3}{3}-x+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(x - x^3),x]
```

```
[Out] -x - x^3/3 + ArcTanh[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{x-x^3} \, dx &= \int \frac{x^4}{1-x^2} \, dx\\
&= \int \left(-1-x^2+\frac{1}{1-x^2}\right) \, dx\\
&= -x-\frac{x^3}{3}+\int \frac{1}{1-x^2} \, dx\\
&= -x-\frac{x^3}{3}+\tanh^{-1}(x)
\end{aligned}$$

**Mathematica [B]** time = 0.004208, size = 29, normalized size = 2.23

$$-\frac{x^3}{3}-x-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(x - x^3),x]
```

```
[Out] -x - x^3/3 - Log[1 - x]/2 + Log[1 + x]/2
```

---

**Maple [A]** time = 0.003, size = 22, normalized size = 1.7

$$-\frac{x^3}{3}-x-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+x),x)
```

```
[Out] -1/3*x^3-x-1/2*ln(-1+x)+1/2*ln(1+x)
```

---

**Maxima [A]** time = 1.06704, size = 28, normalized size = 2.15

$$-\frac{1}{3}x^3-x+\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

---

**Fricas [A]** time = 1.50523, size = 65, normalized size = 5.

$$-\frac{1}{3}x^3-x+\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

---

**Sympy [B]** time = 0.093696, size = 19, normalized size = 1.46

$$-\frac{x^3}{3}-x-\frac{\log(x-1)}{2}+\frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+x),x)
```

```
[Out] -x**3/3 - x - log(x - 1)/2 + log(x + 1)/2
```

---

**Giac [B]** time = 1.19669, size = 31, normalized size = 2.38

$$-\frac{1}{3}x^3 - x + \frac{1}{2}\log\left(|x+1|\right) - \frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/3*x^3 - x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

## 3.27 $\int \frac{x^4}{x-x^3}\,dx$

**Optimal**. Leaf size=20

$$-\frac{x^2}{2}-\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] -x^2/2 - Log[1 - x^2]/2
```

---

**Rubi [A]** time = 0.0151658, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 43}

$$-\frac{x^2}{2}-\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(x - x^3),x]

[Out] -x^2/2 - Log[1 - x^2]/2
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{x-x^3}\,dx &= \int \frac{x^3}{1-x^2}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{1-x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-1+\frac{1}{1-x}\right)\,dx,x,x^2\right)\\
&= -\frac{x^2}{2}-\frac{1}{2}\log\left(1-x^2\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0027918, size = 18, normalized size = 0.9

$$-\frac{x^2}{2}-\frac{1}{2}\log\left(x^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(x - x^3),x]
```

```
[Out] -x^2/2 - Log[-1 + x^2]/2
```

---

**Maple [A]** time = 0.002, size = 19, normalized size = 1.

$$-\frac{x^2}{2}-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3+x),x)
```

```
[Out] -1/2*x^2-1/2*ln(-1+x)-1/2*ln(1+x)
```

---

**Maxima [A]** time = 1.03579, size = 24, normalized size = 1.2

$$-\frac{1}{2}x^2-\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 - 1/2*log(x + 1) - 1/2*log(x - 1)
```

---

**Fricas [A]** time = 1.56051, size = 39, normalized size = 1.95

$$-\frac{1}{2}x^2-\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 - 1/2*log(x^2 - 1)
```

---

**Sympy [A]** time = 0.075149, size = 14, normalized size = 0.7

$$-\frac{x^2}{2}-\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3+x),x)
```

```
[Out] -x**2/2 - log(x**2 - 1)/2
```

---

**Giac [A]** time = 1.32408, size = 20, normalized size = 1.

$$-\frac{1}{2}x^2 - \frac{1}{2}\log\left(\left|x^2 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/2*x^2 - 1/2*log(abs(x^2 - 1))
```

## 3.28 $\int \frac{x^3}{x-x^3} dx$

**Optimal**. Leaf size=6

$$\tanh^{-1}(x) - x$$

```
[Out] -x + ArcTanh[x]
```

---

**Rubi [A]** time = 0.0080615, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 321, 206}

$$\tanh^{-1}(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(x - x^3),x]

[Out] -x + ArcTanh[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{x-x^3} dx &= \int \frac{x^2}{1-x^2} dx \\
&= -x + \int \frac{1}{1-x^2} dx \\
&= -x + \tanh^{-1}(x)
\end{aligned}$$

**Mathematica [B]** time = 0.0030891, size = 22, normalized size = 3.67

$$-x - \frac{1}{2}\log(1-x) + \frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(x - x^3),x]
```

```
[Out] -x - Log[1 - x]/2 + Log[1 + x]/2
```

---

**Maple [B]** time = 0.001, size = 17, normalized size = 2.8

$$-x-\frac{\ln\left(-1+x\right)}{2}+\frac{\ln\left(1+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+x),x)
```

```
[Out] -x-1/2*ln(-1+x)+1/2*ln(1+x)
```

---

**Maxima [B]** time = 1.0418, size = 22, normalized size = 3.67

$$-x+\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

---

**Fricas [B]** time = 1.68595, size = 51, normalized size = 8.5

$$-x+\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

---

**Sympy [B]** time = 0.083079, size = 14, normalized size = 2.33

$$-x-\frac{\log\left(x-1\right)}{2}+\frac{\log\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+x),x)
```

```
[Out] -x - log(x - 1)/2 + log(x + 1)/2
```

---

**Giac [B]** time = 1.23107, size = 24, normalized size = 4.

$$-x + \frac{1}{2} \log\left(|x+1|\right) - \frac{1}{2} \log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+x),x, algorithm="giac")
```

```
[Out] -x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

## 3.29 $\int \frac{x^2}{x-x^3}\,dx$

**Optimal**. Leaf size=12

$$-\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] -Log[1 - x^2]/2
```

---

**Rubi [A]** time = 0.0063917, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1584, 260}

$$-\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(x - x^3),x]
```

```
[Out] -Log[1 - x^2]/2
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{x-x^3}\,dx &= \int \frac{x}{1-x^2}\,dx\\ &= -\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

**Mathematica [A]** time = 0.0018832, size = 12, normalized size = 1.

$$-\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(x - x^3),x]
```

```
[Out] -Log[1 - x^2]/2
```

---

**Maple [A]** time = 0.003, size = 14, normalized size = 1.2

$$-\frac{\ln(-1+x)}{2}-\frac{\ln(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+x),x)
```

```
[Out] -1/2*ln(-1+x)-1/2*ln(1+x)
```

---

**Maxima [A]** time = 1.0234, size = 18, normalized size = 1.5

$$-\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/2*log(x + 1) - 1/2*log(x - 1)
```

---

**Fricas [A]** time = 1.60973, size = 26, normalized size = 2.17

$$-\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -1/2*log(x^2 - 1)
```

---

**Sympy [A]** time = 0.074027, size = 8, normalized size = 0.67

$$-\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+x),x)
```

```
[Out] -log(x**2 - 1)/2
```

---

**Giac [A]** time = 1.27095, size = 20, normalized size = 1.67

$$-\frac{1}{2}\log(|x+1|)-\frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

## 3.30 $\int \frac{x}{x-x^3} \, dx$

**Optimal**. Leaf size=2

$$\tanh^{-1}(x)$$

```
[Out] ArcTanh[x]
```

---

**Rubi [A]** time = 0.0039866, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1584, 206}

$$\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - x^3),x]

[Out] ArcTanh[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{x-x^3} \, dx &= \int \frac{1}{1-x^2} \, dx\\ &= \tanh^{-1}(x)\end{aligned}$$

**Mathematica [B]** time = 0.0020419, size = 19, normalized size = 9.5

$$\frac{1}{2} \log(x+1) - \frac{1}{2} \log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - x^3),x]

[Out] -Log[1 - x]/2 + Log[1 + x]/2
```

---

**Maple [A]** time = 0.001, size = 3, normalized size = 1.5

$$\operatorname{Artanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+x),x)
```

```
[Out] arctanh(x)
```

---

**Maxima [B]** time = 1.07402, size = 18, normalized size = 9.

$$\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+x),x, algorithm="maxima")
```

```
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

---

**Fricas [B]** time = 1.60764, size = 45, normalized size = 22.5

$$\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+x),x, algorithm="fricas")
```

```
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

---

**Sympy [B]** time = 0.082663, size = 12, normalized size = 6.

$$-\frac{\log\left(x-1\right)}{2}+\frac{\log\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+x),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2
```

---

**Giac [B]** time = 1.23626, size = 20, normalized size = 10.

$$\frac{1}{2}\,\log\left(|x+1|\right)-\frac{1}{2}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

## 3.31 $\int \frac{1}{x-x^3} \, dx$

**Optimal**. Leaf size=15

$$\log(x) - \frac{1}{2}\log\left(1 - x^2\right)$$

```
[Out] Log[x] - Log[1 - x^2]/2
```

---

**Rubi [A]** time = 0.0082264, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1593, 266, 36, 31, 29}

$$\log(x) - \frac{1}{2}\log\left(1 - x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - x^3)^(-1),x]
```

```
[Out] Log[x] - Log[1 - x^2]/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x-x^3}\,dx &= \int \frac{1}{x\left(1-x^2\right)}\,dx\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1-x)x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, x^2\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)\\ &= \log(x) - \frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

**Mathematica [A]** time = 0.0024746, size = 15, normalized size = 1.

$$\log(x) - \frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - x^3)^(-1),x]
```

```
[Out] Log[x] - Log[1 - x^2]/2
```

---

**Maple [A]** time = 0.006, size = 16, normalized size = 1.1

$$\ln(x) - \frac{\ln(1+x)}{2} - \frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+x),x)
```

```
[Out] ln(x)-1/2*ln(1+x)-1/2*ln(-1+x)
```

---

**Maxima [A]** time = 1.02813, size = 20, normalized size = 1.33

$$-\frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

---

**Fricas [A]** time = 1.44473, size = 38, normalized size = 2.53

$$-\frac{1}{2}\log\left(x^2-1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -1/2*log(x^2 - 1) + log(x)
```

---

**Sympy [A]** time = 0.087841, size = 10, normalized size = 0.67

$$\log(x) - \frac{\log\left(x^2 - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+x),x)
```

```
[Out] log(x) - log(x**2 - 1)/2
```

---

**Giac [A]** time = 1.22125, size = 22, normalized size = 1.47

$$\frac{1}{2} \log\left(x^2\right) - \frac{1}{2} \log\left(\left|x^2 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

## 3.32 $\int \frac{1}{x\left(x-x^3\right)} dx$

**Optimal**. Leaf size=8

$$\tanh^{-1}(x)-\frac{1}{x}$$

```
[Out] -x^(-1) + ArcTanh[x]
```

---

**Rubi [A]** time = 0.007941, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 325, 206}

$$\tanh^{-1}(x)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(x - x^3)),x]
```

```
[Out] -x^(-1) + ArcTanh[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(x-x^3\right)} dx &= \int \frac{1}{x^2\left(1-x^2\right)} dx\\ &= -\frac{1}{x}+\int \frac{1}{1-x^2} dx\\ &= -\frac{1}{x}+\tanh^{-1}(x)\end{aligned}$$

**Mathematica [B]** time = 0.0033792, size = 24, normalized size = 3.

$$-\frac{1}{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(x - x^3)),x]
```

```
[Out] -x^(-1) - Log[1 - x]/2 + Log[1 + x]/2
```

---

**Maple [B]** time = 0.005, size = 19, normalized size = 2.4

$$\frac{\ln(1+x)}{2}-\frac{\ln(-1+x)}{2}-x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+x),x)
```

```
[Out] 1/2*ln(1+x)-1/2*ln(-1+x)-1/x
```

---

**Maxima [B]** time = 1.0997, size = 24, normalized size = 3.

$$-\frac{1}{x}+\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

---

**Fricas [B]** time = 1.43392, size = 55, normalized size = 6.88

$$\frac{x\log(x+1)-x\log(x-1)-2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+x),x, algorithm="fricas")
```

```
[Out] 1/2*(x*log(x + 1) - x*log(x - 1) - 2)/x
```

---

**Sympy [B]** time = 0.093039, size = 15, normalized size = 1.88

$$-\frac{\log(x-1)}{2}+\frac{\log(x+1)}{2}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+x),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2 - 1/x
```

---

**Giac [B]** time = 1.19293, size = 27, normalized size = 3.38

$$-\frac{1}{x}+\frac{1}{2}\log\left(|x+1|\right)-\frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

## 3.33 $\int \frac{1}{x^2\left(x-x^3\right)}\,dx$

**Optimal**. Leaf size=22

$$-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

```
[Out] -1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

---

**Rubi [A]** time = 0.0146911, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 44}

$$-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(x - x^3)),x]
```

```
[Out] -1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(x-x^3\right)}\,dx &= \int \frac{1}{x^3\left(1-x^2\right)}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1-x)x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{x^2}+\frac{1}{x}\right)dx,x,x^2\right)\\ &= -\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

**Mathematica [A]** time = 0.0034872, size = 22, normalized size = 1.

$$-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(x - x^3)),x]
```

```
[Out] -1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

---

**Maple [A]** time = 0.007, size = 21, normalized size = 1.

$$-\frac{1}{2\,x^2}+\ln(x)-\frac{\ln(1+x)}{2}-\frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3+x),x)
```

```
[Out] -1/2/x^2+ln(x)-1/2*ln(1+x)-1/2*ln(-1+x)
```

---

**Maxima [A]** time = 1.04656, size = 27, normalized size = 1.23

$$-\frac{1}{2\,x^2}-\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

---

**Fricas [A]** time = 1.385, size = 65, normalized size = 2.95

$$-\frac{x^2\log\left(x^2-1\right)-2\,x^2\log(x)+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2*log(x^2 - 1) - 2*x^2*log(x) + 1)/x^2
```

---

**Sympy [A]** time = 0.096854, size = 17, normalized size = 0.77

$$\log(x)-\frac{\log\left(x^2-1\right)}{2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3+x),x)
```

```
[Out] log(x) - log(x**2 - 1)/2 - 1/(2*x**2)
```

---

**Giac [A]** time = 1.27046, size = 35, normalized size = 1.59

$$-\frac{x^2+1}{2\,x^2}+\frac{1}{2}\,\log\left(x^2\right)-\frac{1}{2}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/2*(x^2 + 1)/x^2 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

## 3.34 $\int \frac{1}{x^3(x-x^3)}\,dx$

**Optimal**. Leaf size=15

$$-\frac{1}{3x^3}-\frac{1}{x}+\tanh^{-1}(x)$$

```
[Out] -1/(3*x^3) - x^(-1) + ArcTanh[x]
```

---

**Rubi [A]** time = 0.0097331, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 325, 206}

$$-\frac{1}{3x^3}-\frac{1}{x}+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(x - x^3)),x]
```

```
[Out] -1/(3*x^3) - x^(-1) + ArcTanh[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(x-x^3\right)}\,dx &= \int \frac{1}{x^4\left(1-x^2\right)}\,dx\\
&= -\frac{1}{3x^3}+\int \frac{1}{x^2\left(1-x^2\right)}\,dx\\
&= -\frac{1}{3x^3}-\frac{1}{x}+\int \frac{1}{1-x^2}\,dx\\
&= -\frac{1}{3x^3}-\frac{1}{x}+\tanh^{-1}(x)
\end{aligned}$$

**Mathematica [B]** time = 0.0036898, size = 31, normalized size = 2.07

$$-\frac{1}{3x^3}-\frac{1}{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(x - x^3)),x]
```

```
[Out] -1/(3*x^3) - x^(-1) - Log[1 - x]/2 + Log[1 + x]/2
```

---

**Maple [A]** time = 0.005, size = 24, normalized size = 1.6

$$-\frac{1}{3\,x^3}-x^{-1}+\frac{\ln(1+x)}{2}-\frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+x),x)
```

```
[Out] -1/3/x^3-1/x+1/2*ln(1+x)-1/2*ln(-1+x)
```

---

**Maxima [A]** time = 1.09703, size = 34, normalized size = 2.27

$$-\frac{3\,x^2+1}{3\,x^3}+\frac{1}{2}\,\log(x+1)-\frac{1}{2}\,\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/3*(3*x^2 + 1)/x^3 + 1/2*log(x + 1) - 1/2*log(x - 1)
```

---

**Fricas [B]** time = 1.40575, size = 80, normalized size = 5.33

$$\frac{3\,x^3\log(x+1)-3\,x^3\log(x-1)-6\,x^2-2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+x),x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^3*log(x + 1) - 3*x^3*log(x - 1) - 6*x^2 - 2)/x^3
```

---

**Sympy [A]** time = 0.108128, size = 24, normalized size = 1.6

$$-\frac{\log(x-1)}{2}+\frac{\log(x+1)}{2}-\frac{3x^2+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+x),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2 - (3*x**2 + 1)/(3*x**3)
```

---

**Giac [A]** time = 1.23086, size = 36, normalized size = 2.4

$$-\frac{3\,x^2+1}{3\,x^3}+\frac{1}{2}\,\log\left(|x+1|\right)-\frac{1}{2}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/3*(3*x^2 + 1)/x^3 + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

## 3.35 $\int \frac{1}{x^4\left(x-x^3\right)}\,dx$

**Optimal**. Leaf size=29

$$-\frac{1}{2x^2}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

```
[Out] -1/(4*x^4) - 1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

---

**Rubi [A]** time = 0.0160911, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 44}

$$-\frac{1}{2x^2}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(x - x^3)),x]
```

```
[Out] -1/(4*x^4) - 1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(x-x^3\right)}\,dx &= \int \frac{1}{x^5\left(1-x^2\right)}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1-x)x^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right)dx,x,x^2\right)\\
&= -\frac{1}{4x^4}-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0034672, size = 29, normalized size = 1.

$$-\frac{1}{2x^2}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(x - x^3)),x]
```

```
[Out] -1/(4*x^4) - 1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

---

**Maple [A]** time = 0.007, size = 26, normalized size = 0.9

$$-\frac{1}{4\,x^4}-\frac{1}{2\,x^2}+\ln(x)-\frac{\ln(1+x)}{2}-\frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+x),x)
```

```
[Out] -1/4/x^4-1/2/x^2+ln(x)-1/2*ln(1+x)-1/2*ln(-1+x)
```

---

**Maxima [A]** time = 1.14142, size = 36, normalized size = 1.24

$$-\frac{2\,x^2+1}{4\,x^4}-\frac{1}{2}\,\log(x+1)-\frac{1}{2}\,\log(x-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^2 + 1)/x^4 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

---

**Fricas [A]** time = 1.40528, size = 78, normalized size = 2.69

$$-\frac{2\,x^4\log\left(x^2-1\right)-4\,x^4\log(x)+2\,x^2+1}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^4*log(x^2 - 1) - 4*x^4*log(x) + 2*x^2 + 1)/x^4
```

---

**Sympy [A]** time = 0.111545, size = 22, normalized size = 0.76

$$\log(x)-\frac{\log\left(x^2-1\right)}{2}-\frac{2x^2+1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+x),x)
```

```
[Out] log(x) - log(x**2 - 1)/2 - (2*x**2 + 1)/(4*x**4)
```

---

**Giac [A]** time = 1.23591, size = 45, normalized size = 1.55

$$-\frac{3\,x^4+2\,x^2+1}{4\,x^4}+\frac{1}{2}\,\log\left(x^2\right)-\frac{1}{2}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/4*(3*x^4 + 2*x^2 + 1)/x^4 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

## 3.36 $\int \frac{1}{x+bx^3}\,dx$

**Optimal**. Leaf size=15

$$\log(x)-\frac{1}{2}\log\left(bx^2+1\right)$$

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

---

**Rubi [A]** time = 0.0094817, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1593, 266, 36, 29, 31}

$$\log(x)-\frac{1}{2}\log\left(bx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + b*x^3)^(-1),x]
```

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x+bx^3}\,dx &= \int \frac{1}{x\left(1+bx^2\right)}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(1+bx)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{1+bx}\,dx, x, x^2\right) \\
&= \log(x) - \frac{1}{2}\log\left(1+bx^2\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.0031799, size = 15, normalized size = 1.

$$\log(x) - \frac{1}{2}\log\left(bx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + b*x^3)^(-1),x]
```

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

---

**Maple [A]** time = 0.005, size = 14, normalized size = 0.9

$$\ln(x) - \frac{\ln\left(bx^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+x),x)
```

```
[Out] ln(x)-1/2*ln(b*x^2+1)
```

---

**Maxima [A]** time = 1.03523, size = 18, normalized size = 1.2

$$-\frac{1}{2}\log\left(bx^2+1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+x),x, algorithm="maxima")
```

```
[Out] -1/2*log(b*x^2 + 1) + log(x)
```

---

**Fricas [A]** time = 1.38484, size = 41, normalized size = 2.73

$$-\frac{1}{2}\log\left(bx^2+1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+x),x, algorithm="fricas")
```

```
[Out] -1/2*log(b*x^2 + 1) + log(x)
```

---

**Sympy [A]** time = 0.123091, size = 12, normalized size = 0.8

$$\log(x) - \frac{\log\left(x^2 + \frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+x),x)
```

```
[Out] log(x) - log(x**2 + 1/b)/2
```

---

**Giac [A]** time = 1.21529, size = 24, normalized size = 1.6

$$\frac{1}{2}\log\left(x^2\right) - \frac{1}{2}\log\left(|bx^2 + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+x),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

## 3.37 $\int \frac{1}{-x+bx^3}\,dx$

**Optimal**. Leaf size=18

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

---

**Rubi [A]** time = 0.0102575, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1593, 266, 36, 29, 31}

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + b*x^3)^(-1),x]

[Out] -Log[x] + Log[1 - b*x^2]/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{-x+bx^3}\,dx &= \int \frac{1}{x\left(-1+bx^2\right)}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(-1+bx)}\,dx, x, x^2\right)\\
&= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)\right) + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{-1+bx}\,dx, x, x^2\right)\\
&= -\log(x) + \frac{1}{2}\log\left(1-bx^2\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.0034299, size = 18, normalized size = 1.

$$\frac{1}{2}\log\left(1-bx^2\right) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + b*x^3)^(-1),x]
```

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

---

**Maple [A]** time = 0.003, size = 16, normalized size = 0.9

$$-\ln(x) + \frac{\ln\left(bx^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3-x),x)
```

```
[Out] -ln(x)+1/2*ln(b*x^2-1)
```

---

**Maxima [A]** time = 1.05053, size = 20, normalized size = 1.11

$$\frac{1}{2}\log\left(bx^2-1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3-x),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x^2 - 1) - log(x)
```

---

**Fricas [A]** time = 1.40847, size = 39, normalized size = 2.17

$$\frac{1}{2}\log\left(bx^2-1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3-x),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 - 1) - log(x)
```

---

**Sympy [A]** time = 0.122316, size = 12, normalized size = 0.67

$$-\log\left(x\right)+\frac{\log\left(x^2-\frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3-x),x)
```

```
[Out] -log(x) + log(x**2 - 1/b)/2
```

---

**Giac [A]** time = 1.23484, size = 24, normalized size = 1.33

$$-\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|bx^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3-x),x, algorithm="giac")
```

```
[Out] -1/2*log(x^2) + 1/2*log(abs(b*x^2 - 1))
```

## 3.38 $\int x^3\sqrt{ax+bx^3}\,dx$

**Optimal**. Leaf size=163

$$\frac{10a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}-\frac{20a^2\sqrt{ax+bx^3}}{231b^2}+\frac{2}{11}x^4\sqrt{ax+bx^3}+\frac{4ax^2\sqrt{ax+}}{77b}$$

```
[Out] (-20*a^2*Sqrt[a*x + b*x^3])/(231*b^2) + (4*a*x^2*Sqrt[a*x + b*x^3])/(77*b)
+ (2*x^4*Sqrt[a*x + b*x^3])/11 + (10*a^(11/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)
*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt
[x])/a^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.188378, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2021, 2024, 2011, 329, 220}

$$-\frac{20a^2\sqrt{ax+bx^3}}{231b^2}+\frac{10a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}+\frac{2}{11}x^4\sqrt{ax+bx^3}+\frac{4ax^2\sqrt{ax+bx^3}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a*x + b*x^3],x]
```

```
[Out] (-20*a^2*Sqrt[a*x + b*x^3])/(231*b^2) + (4*a*x^2*Sqrt[a*x + b*x^3])/(77*b)
+ (2*x^4*Sqrt[a*x + b*x^3])/11 + (10*a^(11/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)
*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt
[x])/a^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[a*x + b*x^3])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int x^3\sqrt{ax+bx^3}\,dx &= \frac{2}{11}x^4\sqrt{ax+bx^3}+\frac{1}{11}(2a)\int\frac{x^4}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{4ax^2\sqrt{ax+bx^3}}{77b}+\frac{2}{11}x^4\sqrt{ax+bx^3}-\frac{\left(10a^2\right)\int\frac{x^2}{\sqrt{ax+bx^3}}\,dx}{77b}\\
&= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2}+\frac{4ax^2\sqrt{ax+bx^3}}{77b}+\frac{2}{11}x^4\sqrt{ax+bx^3}+\frac{\left(10a^3\right)\int\frac{1}{\sqrt{ax+bx^3}}\,dx}{231b^2}\\
&= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2}+\frac{4ax^2\sqrt{ax+bx^3}}{77b}+\frac{2}{11}x^4\sqrt{ax+bx^3}+\frac{\left(10a^3\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{231b^2\sqrt{ax+bx^3}}\\
&= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2}+\frac{4ax^2\sqrt{ax+bx^3}}{77b}+\frac{2}{11}x^4\sqrt{ax+bx^3}+\frac{\left(20a^3\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^4}}\,dx,x,\right.}{231b^2\sqrt{ax+bx^3}}\\
&= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2}+\frac{4ax^2\sqrt{ax+bx^3}}{77b}+\frac{2}{11}x^4\sqrt{ax+bx^3}+\frac{10a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\,\mathrm{ta}\right.}{231b^{9/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0504627, size = 95, normalized size = 0.58

$$
\frac{2\sqrt{x\left(a+bx^2\right)}\left(\sqrt{\frac{bx^2}{a}+1}\left(-5a^2+2abx^2+7b^2x^4\right)+5a^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{77b^2\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*(Sqrt[1 + (b*x^2)/a]*(-5*a^2 + 2*a*b*x^2 + 7*b^2*x^4
) + 5*a^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(77*b^2*Sqrt[1
+ (b*x^2)/a])
```

---

**Maple [A]** time = 0.018, size = 168, normalized size = 1.

$$
\frac{2x^4}{11}\sqrt{bx^3+ax}+\frac{4ax^2}{77b}\sqrt{bx^3+ax}-\frac{20a^2}{231b^2}\sqrt{bx^3+ax}+\frac{10a^3}{231b^3}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a*x)^(1/2),x)
```

```
[Out] 2/11*x^4*(b*x^3+a*x)^(1/2)+4/77*a*x^2*(b*x^3+a*x)^(1/2)/b-20/231*a^2*(b*x^3
+a*x)^(1/2)/b^2+10/231*a^3/b^3*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^
(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1
/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2)
)^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^3 + ax}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x^3, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{bx^3 + ax}\,x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^3, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{x\left(a + bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(x*(a + b*x**2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^3 + ax}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x^3, x)
```

## 3.39 $\int x^2\sqrt{ax+bx^3}\,dx$

**Optimal**. Leaf size=281

$$-\frac{2a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}-\frac{4a^2x\left(a+bx^2\right)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right.}{1}$$

```
[Out] (-4*a^2*x*(a + b*x^2))/(15*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
 + (4*a*x*Sqrt[a*x + b*x^3])/(45*b) + (2*x^3*Sqrt[a*x + b*x^3])/9 + (4*a^(9
/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]
*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a*x
+ b*x^3]) - (2*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt
[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(1
5*b^(7/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.26363, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2021, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{4a^2x\left(a+bx^2\right)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{2a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}+\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+b}{\left(\sqrt{a}+\right.}}}{15b^{7/4}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a*x + b*x^3],x]
```

```
[Out] (-4*a^2*x*(a + b*x^2))/(15*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
 + (4*a*x*Sqrt[a*x + b*x^3])/(45*b) + (2*x^3*Sqrt[a*x + b*x^3])/9 + (4*a^(9
/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]
*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a*x
+ b*x^3]) - (2*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt
[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(1
5*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

### Rubi steps

$$
\begin{aligned}
\int x^2\sqrt{ax+bx^3}\,dx &= \frac{2}{9}x^3\sqrt{ax+bx^3}+\frac{1}{9}(2a)\int\frac{x^3}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{4ax\sqrt{ax+bx^3}}{45b}+\frac{2}{9}x^3\sqrt{ax+bx^3}-\frac{\left(2a^2\right)\int\frac{x}{\sqrt{ax+bx^3}}\,dx}{15b}\\
&= \frac{4ax\sqrt{ax+bx^3}}{45b}+\frac{2}{9}x^3\sqrt{ax+bx^3}-\frac{\left(2a^2\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{15b\sqrt{ax+bx^3}}\\
&= \frac{4ax\sqrt{ax+bx^3}}{45b}+\frac{2}{9}x^3\sqrt{ax+bx^3}-\frac{\left(4a^2\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int\frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{15b\sqrt{ax+bx^3}}\\
&= \frac{4ax\sqrt{ax+bx^3}}{45b}+\frac{2}{9}x^3\sqrt{ax+bx^3}-\frac{\left(4a^{5/2}\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{15b^{3/2}\sqrt{ax+bx^3}}+\\
&= -\frac{4a^2x\left(a+bx^2\right)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{4ax\sqrt{ax+bx^3}}{45b}+\frac{2}{9}x^3\sqrt{ax+bx^3}+
\end{aligned}
$$

**Mathematica [C]** time = 0.0315518, size = 80, normalized size = 0.28

$$\frac{2x\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{9b\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*Sqrt[x*(a + b*x^2)]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hypergeometri
c2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(9*b*Sqrt[1 + (b*x^2)/a])
```

**Maple [A]** time = 0.011, size = 197, normalized size = 0.7

$$\frac{2x^3}{9}\sqrt{bx^3+ax}+\frac{4ax}{45b}\sqrt{bx^3+ax}-\frac{2a^2}{15b^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\Bigg(-2\frac{\sqrt{-ab}}{b}\mathrm{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x)^(1/2),x)
```

```
[Out] 2/9*x^3*(b*x^3+a*x)^(1/2)+4/45*a*x*(b*x^3+a*x)^(1/2)/b-2/15/b^2*a^2*(-a*b)^
(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*
b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a
*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)
)+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2)))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\sqrt{bx^3+ax}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x^2, x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{bx^3+ax}x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^2, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{x\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x*(a + b*x**2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^3+ax}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x^2, x)
```

## 3.40 $\int x\sqrt{ax+bx^3}\,dx$

**Optimal**. Leaf size=137

$$-\frac{2a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{ax+bx^3}}+\frac{2}{7}x^2\sqrt{ax+bx^3}+\frac{4a\sqrt{ax+bx^3}}{21b}$$

```
[Out] (4*a*Sqrt[a*x + b*x^3])/(21*b) + (2*x^2*Sqrt[a*x + b*x^3])/7 - (2*a^(7/4)*S
qrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(21*b^(5/4)*Sqrt[a*x + b*x
^3])
```

---

**Rubi [A]** time = 0.121844, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2021, 2024, 2011, 329, 220}

$$-\frac{2a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{ax+bx^3}}+\frac{2}{7}x^2\sqrt{ax+bx^3}+\frac{4a\sqrt{ax+bx^3}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a*x + b*x^3],x]
```

```
[Out] (4*a*Sqrt[a*x + b*x^3])/(21*b) + (2*x^2*Sqrt[a*x + b*x^3])/7 - (2*a^(7/4)*S
qrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(21*b^(5/4)*Sqrt[a*x + b*x
^3])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int x\sqrt{ax+bx^3}\,dx &= \frac{2}{7}x^2\sqrt{ax+bx^3}+\frac{1}{7}(2a)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{4a\sqrt{ax+bx^3}}{21b}+\frac{2}{7}x^2\sqrt{ax+bx^3}-\frac{\left(2a^2\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{21b}\\
&= \frac{4a\sqrt{ax+bx^3}}{21b}+\frac{2}{7}x^2\sqrt{ax+bx^3}-\frac{\left(2a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{21b\sqrt{ax+bx^3}}\\
&= \frac{4a\sqrt{ax+bx^3}}{21b}+\frac{2}{7}x^2\sqrt{ax+bx^3}-\frac{\left(4a^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{21b\sqrt{ax+bx^3}}\\
&= \frac{4a\sqrt{ax+bx^3}}{21b}+\frac{2}{7}x^2\sqrt{ax+bx^3}-\frac{2a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0268107, size = 79, normalized size = 0.58

$$
\frac{2\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{7b\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hypergeometric2
F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(7*b*Sqrt[1 + (b*x^2)/a])
```

---

**Maple [A]** time = 0.012, size = 146, normalized size = 1.1

$$
\frac{2\,x^2}{7}\sqrt{bx^3+ax}+\frac{4\,a}{21\,b}\sqrt{bx^3+ax}-\frac{2\,a^2}{21\,b^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\text{EllipticF}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x)^(1/2),x)
```

```
[Out] 2/7*x^2*(b*x^3+a*x)^(1/2)+4/21*a*(b*x^3+a*x)^(1/2)/b-2/21/b^2*a^2*(-a*b)^(1
/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/
```

```
(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF((
(x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^3+ax}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{bx^3+ax}x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x\left(a+bx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x*(a + b*x**2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^3+ax}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x, x)
```

## 3.41 $\int \sqrt{ax+bx^3}\,dx$

**Optimal**. Leaf size=255

$$\frac{2a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}}-\frac{4a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{5b^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] (4*a*x*(a + b*x^2))/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) + (
2*x*Sqrt[a*x + b*x^3])/5 - (4*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(
1/4)], 1/2])/(5*b^(3/4)*Sqrt[a*x + b*x^3]) + (2*a^(5/4)*Sqrt[x]*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^
(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.172987, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2004, 2032, 329, 305, 220, 1196}

$$\frac{2a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}}-\frac{4a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}}+\frac{}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3],x]
```

```
[Out] (4*a*x*(a + b*x^2))/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) + (
2*x*Sqrt[a*x + b*x^3])/5 - (4*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(
1/4)], 1/2])/(5*b^(3/4)*Sqrt[a*x + b*x^3]) + (2*a^(5/4)*Sqrt[x]*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^
(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{ax+bx^3}\,dx &= \frac{2}{5}x\sqrt{ax+bx^3}+\frac{1}{5}(2a)\int \frac{x}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{2}{5}x\sqrt{ax+bx^3}+\frac{\left(2a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5\sqrt{ax+bx^3}}\\
&= \frac{2}{5}x\sqrt{ax+bx^3}+\frac{\left(4a\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{ax+bx^3}}\\
&= \frac{2}{5}x\sqrt{ax+bx^3}+\frac{\left(4a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{b}\sqrt{ax+bx^3}}-\frac{\left(4a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1-\frac{\sqrt{b}}{\sqrt{}}}{\sqrt{a+b}}\right.}{5\sqrt{b}\sqrt{ax+bx^3}}\\
&= \frac{4ax\left(a+bx^2\right)}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}-\frac{4a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{5b^{3/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0106898, size = 51, normalized size = 0.2

$$\frac{2x\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)])/(
3*Sqrt[1 + (b*x^2)/a])
```

**Maple [A]** time = 0.011, size = 175, normalized size = 0.7

$$\frac{2x}{5}\sqrt{bx^3+ax}+\frac{2a}{5b}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\Bigg(-2\frac{\sqrt{-ab}}{b}\mathrm{EllipticE}\Bigg(\sqrt{\frac{b}{\sqrt{-a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(1/2),x)
```

```
[Out] 2/5*x*(b*x^3+a*x)^(1/2)+2/5*a/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)
^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(
1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(
1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b
*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\sqrt{bx^3+ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{bx^3+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\sqrt{ax+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x + b*x**3), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int\sqrt{bx^3+ax}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x), x)
```

## 3.42 $\int \frac{\sqrt{ax+bx^3}}{x}\,dx$

**Optimal**. Leaf size=113

$$\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}$$

```
[Out] (2*Sqrt[a*x + b*x^3])/3 + (2*a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1
/4)], 1/2])/(3*b^(1/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.0857818, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2021, 2011, 329, 220}

$$\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3]/x,x]
```

```
[Out] (2*Sqrt[a*x + b*x^3])/3 + (2*a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1
/4)], 1/2])/(3*b^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax+bx^3}}{x}\,dx &= \frac{2}{3}\sqrt{ax+bx^3}+\frac{1}{3}(2a)\int \frac{1}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{2}{3}\sqrt{ax+bx^3}+\frac{\left(2a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{3\sqrt{ax+bx^3}}\\
&= \frac{2}{3}\sqrt{ax+bx^3}+\frac{\left(4a\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{ax+bx^3}}\\
&= \frac{2}{3}\sqrt{ax+bx^3}+\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}}
\end{aligned}$$

**Mathematica [C]** time = 0.0113656, size = 48, normalized size = 0.42

$$\frac{2\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3]/x,x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)])/Sqr
t[1 + (b*x^2)/a]
```

---

**Maple [A]** time = 0.012, size = 124, normalized size = 1.1

$$\frac{2}{3}\sqrt{bx^3+ax}+\frac{2a}{3b}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\mathrm{EllipticF}\Bigg(\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(1/2)/x,x)
```

```
[Out] 2/3*(b*x^3+a*x)^(1/2)+2/3*a/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(
1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/
2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))
^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3+ax}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/x, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\left(a+bx^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2))/x, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3+ax}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x, x)
```

## 3.43 $\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx$

**Optimal**. Leaf size=248

$$\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt{ax+bx^3}}+\frac{4\sqrt{b}x\left(a+bx^2\right)}{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}-$$

```
[Out] (4*Sqrt[b]*x*(a + b*x^2))/((Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*Sq
rt[a*x + b*x^3])/x - (4*a^(1/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a
^(1/4)], 1/2])/Sqrt[a*x + b*x^3] + (2*a^(1/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sq
rt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1
/4)*Sqrt[x])/a^(1/4)], 1/2])/Sqrt[a*x + b*x^3]
```

---

**Rubi [A]** time = 0.201338, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2020, 2032, 329, 305, 220, 1196}

$$\frac{4\sqrt{b}x\left(a+bx^2\right)}{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}+\frac{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{ax+bx^3}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3]/x^2,x]
```

```
[Out] (4*Sqrt[b]*x*(a + b*x^2))/((Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*Sq
rt[a*x + b*x^3])/x - (4*a^(1/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a
^(1/4)], 1/2])/Sqrt[a*x + b*x^3] + (2*a^(1/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sq
rt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1
/4)*Sqrt[x])/a^(1/4)], 1/2])/Sqrt[a*x + b*x^3]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx &= -\frac{2\sqrt{ax+bx^3}}{x}+(2b)\int \frac{x}{\sqrt{ax+bx^3}}\,dx\\
&= -\frac{2\sqrt{ax+bx^3}}{x}+\frac{\left(2b\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{x}+\frac{\left(4b\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{x}+\frac{\left(4\sqrt{a}\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{ax+bx^3}}-\frac{\left(4\sqrt{a}\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}}{\sqrt{ax+b}}\\
&= \frac{4\sqrt{b}x\left(a+bx^2\right)}{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}-\frac{4\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}}{\sqrt{ax+bx^3}}
\end{aligned}$$

**Mathematica [C]** time = 0.0122915, size = 51, normalized size = 0.21

$$-\frac{2\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3]/x^2,x]
```

```
[Out] (-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^2)/a)])/(
x*Sqrt[1 + (b*x^2)/a])
```

---

**Maple [A]** time = 0.013, size = 177, normalized size = 0.7

$$-2\,\frac{bx^2+a}{\sqrt{x\left(bx^2+a\right)}}+2\,\frac{\sqrt{-ab}}{\sqrt{bx^3+ax}}\sqrt{\frac{b}{\sqrt{-ab}}\left(x+\frac{\sqrt{-ab}}{b}\right)}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-2\,\frac{\sqrt{-ab}}{b}\mathrm{EllipticE}\left(\sqrt{\frac{b}{\sqrt{-ab}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(1/2)/x^2,x)
```

```
[Out] -2*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a
*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b
)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b
)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+
1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^3+ax}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^2, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^3+ax}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/x^2, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x\left(a+bx^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2))/x**2, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3 + ax}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^2, x)
```

## 3.44 $\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx$

**Optimal**. Leaf size=116

$$\frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}$$

```
[Out] (-2*Sqrt[a*x + b*x^3])/(3*x^2) + (2*b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.0877961, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2020, 2011, 329, 220}

$$\frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3]/x^3,x]
```

```
[Out] (-2*Sqrt[a*x + b*x^3])/(3*x^2) + (2*b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx &= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{1}{3}(2b)\int \frac{1}{\sqrt{ax+bx^3}}\,dx\\
&= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{\left(2b\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{3\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{\left(4b\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0137491, size = 53, normalized size = 0.46

$$
-\frac{2\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3x^2\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3]/x^3,x]
```

```
[Out] (-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^2)/a)])/(
3*x^2*Sqrt[1 + (b*x^2)/a])
```

---

**Maple [A]** time = 0.005, size = 123, normalized size = 1.1

$$
-\frac{2}{3\,x^2}\sqrt{bx^3+ax}+\frac{2}{3}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\operatorname{EllipticF}\left(\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(1/2)/x^3,x)
```

```
[Out] -2/3*(b*x^3+a*x)^(1/2)/x^2+2/3*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^
(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1
/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2)
)^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{bx^3+ax}}{x^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^3, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/x^3, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\left(a+bx^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2))/x**3, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3+ax}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^3, x)
```

## 3.45 $\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx$

**Optimal**. Leaf size=283

$$\frac{2b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}}-\frac{4b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{5a^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] (4*b^(3/2)*x*(a + b*x^2))/(5*a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (
2*Sqrt[a*x + b*x^3])/(5*x^3) - (4*b*Sqrt[a*x + b*x^3])/(5*a*x) - (4*b^(5/4)
*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a*x + b*
x^3]) + (2*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
+ Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(
3/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.250931, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{2b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}}-\frac{4b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3]/x^4,x]
```

```
[Out] (4*b^(3/2)*x*(a + b*x^2))/(5*a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (
2*Sqrt[a*x + b*x^3])/(5*x^3) - (4*b*Sqrt[a*x + b*x^3])/(5*a*x) - (4*b^(5/4)
*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a*x + b*
x^3]) + (2*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
+ Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(
3/4)*Sqrt[a*x + b*x^3])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

### Rubi steps

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = -\frac{2\sqrt{ax+bx^3}}{5x^3}+\frac{1}{5}(2b)\int \frac{1}{x\sqrt{ax+bx^3}}\,dx$$

$$= -\frac{2\sqrt{ax+bx^3}}{5x^3}-\frac{4b\sqrt{ax+bx^3}}{5ax}+\frac{\left(2b^2\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{5a}$$

$$= -\frac{2\sqrt{ax+bx^3}}{5x^3}-\frac{4b\sqrt{ax+bx^3}}{5ax}+\frac{\left(2b^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5a\sqrt{ax+bx^3}}$$

$$= -\frac{2\sqrt{ax+bx^3}}{5x^3}-\frac{4b\sqrt{ax+bx^3}}{5ax}+\frac{\left(4b^2\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5a\sqrt{ax+bx^3}}$$

$$= -\frac{2\sqrt{ax+bx^3}}{5x^3}-\frac{4b\sqrt{ax+bx^3}}{5ax}+\frac{\left(4b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{a}\sqrt{ax+bx^3}}-\frac{(4b^{3/2}\sqrt{x}\sqrt{a+bx^2}}{5}$$

$$= \frac{4b^{3/2}x\left(a+bx^2\right)}{5a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{5x^3}-\frac{4b\sqrt{ax+bx^3}}{5ax}-\frac{4b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E(2\tan}{5a^{3/4}\sqrt{ax+bx^3}}$$

**Mathematica [C]** time = 0.0125866, size = 53, normalized size = 0.19

$$-\frac{2\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3]/x^4,x]
```

```
[Out] (-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^2)/a)])/
(5*x^3*Sqrt[1 + (b*x^2)/a])
```

---

**Maple [A]** time = 0.006, size = 201, normalized size = 0.7

$$-\frac{2}{5\,x^3}\sqrt{bx^3+ax}-\frac{\left(4\,bx^2+4\,a\right)b}{5\,a}\frac{1}{\sqrt{x\left(bx^2+a\right)}}+\frac{2\,b}{5\,a}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(1/2)/x^4,x)
```

```
[Out] -2/5*(b*x^3+a*x)^(1/2)/x^3-4/5*(b*x^2+a)*b/a/(x*(b*x^2+a))^(1/2)+2/5/a*b*(-
a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1
/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/
b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^3+ax}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^4, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+ax}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/x^4, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\left(a+bx^2\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2))/x**4, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3+ax}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^4, x)
```

## 3.46 $\int x^2 \left(ax+bx^3\right)^{3/2} dx$

**Optimal**. Leaf size=186

$$\frac{4a^{15/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}-\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+$$

```
[Out] (-8*a^3*Sqrt[a*x + b*x^3])/(231*b^2) + (8*a^2*x^2*Sqrt[a*x + b*x^3])/(385*b
) + (4*a*x^4*Sqrt[a*x + b*x^3])/55 + (2*x^3*(a*x + b*x^3)^(3/2))/15 + (4*a^
(15/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[
a*x + b*x^3])
```

---

**Rubi [A]** time = 0.229545, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2021, 2024, 2011, 329, 220}

$$-\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{4a^{15/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3)^(3/2),x]
```

```
[Out] (-8*a^3*Sqrt[a*x + b*x^3])/(231*b^2) + (8*a^2*x^2*Sqrt[a*x + b*x^3])/(385*b
) + (4*a*x^4*Sqrt[a*x + b*x^3])/55 + (2*x^3*(a*x + b*x^3)^(3/2))/15 + (4*a^
(15/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[
a*x + b*x^3])
```

### Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
```

```
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(ax+bx^3\right)^{3/2}dx &= \frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{1}{5}(2a)\int x^3\sqrt{ax+bx^3}\,dx\\
&= \frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{1}{55}\left(4a^2\right)\int\frac{x^4}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\right)\int\frac{x^2}{\sqrt{ax+bx^3}}\,dx}{77b}\\
&= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{\left(4a^4\right)\int\frac{1}{\sqrt{ax+bx^3}}}{231b^2}\\
&= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{\left(4a^4\sqrt{x}\sqrt{a+bx\right.}{231b^2\sqrt{}}\\
&= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{\left(8a^4\sqrt{x}\sqrt{a+bx\right.}{2}\\
&= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{4a^{15/4}\sqrt{x}\left(\sqrt{a}+\right.}{}
\end{aligned}
$$

**Mathematica [C]** time = 0.0545894, size = 94, normalized size = 0.51

$$
\frac{2\sqrt{x\left(a+bx^2\right)}\left(5a^3\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-\left(5a-11bx^2\right)\left(a+bx^2\right)^2\sqrt{\frac{bx^2}{a}+1}\right)}{165b^2\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*(-((5*a - 11*b*x^2)*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a
]) + 5*a^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(165*b^2*Sqrt[
1 + (b*x^2)/a])
```

**Maple [A]** time = 0.014, size = 188, normalized size = 1.

$$\frac{2\,bx^{6}}{15}\sqrt{bx^{3}+ax}+\frac{34\,ax^{4}}{165}\sqrt{bx^{3}+ax}+\frac{8\,a^{2}x^{2}}{385\,b}\sqrt{bx^{3}+ax}-\frac{8\,a^{3}}{231\,b^{2}}\sqrt{bx^{3}+ax}+\frac{4\,a^{4}}{231\,b^{3}}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x)^(3/2),x)
```

```
[Out] 2/15*b*x^6*(b*x^3+a*x)^(1/2)+34/165*a*x^4*(b*x^3+a*x)^(1/2)+8/385*a^2*x^2*(
b*x^3+a*x)^(1/2)/b-8/231*a^3*(b*x^3+a*x)^(1/2)/b^2+4/231*a^4/b^3*(-a*b)^(1/
2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(
-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((
x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(bx^{3}+ax\right)^{\frac{3}{2}}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)*x^2, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^{5}+ax^{3}\right)\sqrt{bx^{3}+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^5 + a*x^3)*sqrt(b*x^3 + a*x), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{2}\left(x\left(a+bx^{2}\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**2*(x*(a + b*x**2))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + ax\right)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)*x^2, x)
```

## 3.47 $\int x\left(ax+bx^3\right)^{3/2}\,dx$

**Optimal**. Leaf size=304

$$-\frac{4a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}}-\frac{8a^3x\left(a+bx^2\right)}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{8a^{13/4}\sqrt{x}\left(\sqrt{a}+\right.}{}$$

```
[Out] (-8*a^3*x*(a + b*x^2))/(65*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
 + (8*a^2*x*Sqrt[a*x + b*x^3])/(195*b) + (4*a*x^3*Sqrt[a*x + b*x^3])/39 + (
2*x^2*(a*x + b*x^3)^(3/2))/13 + (8*a^(13/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a*x + b*x^3]) - (4*a^(13/4)*Sqrt[x]*(Sq
rt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.299659, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2021, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{8a^3x\left(a+bx^2\right)}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{4a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}}+\frac{8a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)}{65b^{7}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3)^(3/2),x]
```

```
[Out] (-8*a^3*x*(a + b*x^2))/(65*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
 + (8*a^2*x*Sqrt[a*x + b*x^3])/(195*b) + (4*a*x^3*Sqrt[a*x + b*x^3])/39 + (
2*x^2*(a*x + b*x^3)^(3/2))/13 + (8*a^(13/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a*x + b*x^3]) - (4*a^(13/4)*Sqrt[x]*(Sq
rt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

### Rubi steps

$$
\begin{aligned}
\int x\left(ax+bx^3\right)^{3/2}\,dx &= \frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}+\frac{1}{13}(6a)\int x^2\sqrt{ax+bx^3}\,dx\\
&= \frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}+\frac{1}{39}\left(4a^2\right)\int\frac{x^3}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\right)\int\frac{x}{\sqrt{ax+bx^3}}\,dx}{65b}\\
&= \frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{65b\sqrt{ax+bx^3}}\\
&= \frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}-\frac{\left(8a^3\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int\frac{x^2}{\sqrt{a+bx^4}}\,d\right.}{65b\sqrt{ax+bx^3}}\\
&= \frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}-\frac{\left(8a^{7/2}\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{a+bx^4}}\right.}{65b^{3/2}\sqrt{ax+bx^3}}\\
&= -\frac{8a^3x\left(a+bx^2\right)}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}+\frac{8a}{}
\end{aligned}
$$

**Mathematica [C]** time = 0.0415998, size = 84, normalized size = 0.28

$$\frac{2x\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)^2\sqrt{\frac{bx^2}{a}+1}-a^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{13b\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*x*Sqrt[x*(a + b*x^2)]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hypergeom
etric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)]))/(13*b*Sqrt[1 + (b*x^2)/a])
```

---

**Maple [A]** time = 0.012, size = 217, normalized size = 0.7

$$\frac{2\,bx^5}{13}\sqrt{bx^3+ax}+\frac{10\,ax^3}{39}\sqrt{bx^3+ax}+\frac{8\,a^2x}{195\,b}\sqrt{bx^3+ax}-\frac{4\,a^3}{65\,b^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x)^(3/2),x)
```

```
[Out] 2/13*b*x^5*(b*x^3+a*x)^(1/2)+10/39*a*x^3*(b*x^3+a*x)^(1/2)+8/195*a^2*x*(b*x
^3+a*x)^(1/2)/b-4/65/b^2*a^3*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1
/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2
))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2
))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-
a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+ax\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)*x, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^4+ax^2\right)\sqrt{bx^3+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a*x^2)*sqrt(b*x^3 + a*x), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x*(x*(a + b*x**2))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+ax\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)*x, x)
```

## 3.48 $\int \left(ax+bx^3\right)^{3/2}\,dx$

**Optimal**. Leaf size=158

$$-\frac{4a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{77b^{5/4}\sqrt{ax+bx^3}}+\frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\,(ax+b$$

```
[Out] (8*a^2*Sqrt[a*x + b*x^3])/(77*b) + (12*a*x^2*Sqrt[a*x + b*x^3])/77 + (2*x*(
a*x + b*x^3)^(3/2))/11 - (4*a^(11/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1
/4)], 1/2])/(77*b^(5/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.133583, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2004, 2021, 2024, 2011, 329, 220}

$$-\frac{4a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{ax+bx^3}}+\frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2),x]
```

```
[Out] (8*a^2*Sqrt[a*x + b*x^3])/(77*b) + (12*a*x^2*Sqrt[a*x + b*x^3])/77 + (2*x*(
a*x + b*x^3)^(3/2))/11 - (4*a^(11/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1
/4)], 1/2])/(77*b^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \left(ax+bx^3\right)^{3/2}\,dx &= \frac{2}{11}x\left(ax+bx^3\right)^{3/2}+\frac{1}{11}(6a)\int x\sqrt{ax+bx^3}\,dx\\
&= \frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}+\frac{1}{77}\left(12a^2\right)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{77b}\\
&= \frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{77b\sqrt{ax+bx^3}}\\
&= \frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{\left(8a^3\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\right.}{77b\sqrt{ax+bx^3}}\\
&= \frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{4a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan\right.}{77b^{5/4}\sqrt{ax+bx^3}}
\end{aligned}$$

**Mathematica [C]** time = 0.0354283, size = 83, normalized size = 0.53

$$\frac{2\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)^2\sqrt{\frac{bx^2}{a}+1}-a^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{11b\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hypergeomet
ric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(11*b*Sqrt[1 + (b*x^2)/a])
```

---

**Maple [A]** time = 0.011, size = 166, normalized size = 1.1

$$\frac{2\,bx^4}{11}\sqrt{bx^3+ax}+\frac{26\,ax^2}{77}\sqrt{bx^3+ax}+\frac{8\,a^2}{77\,b}\sqrt{bx^3+ax}-\frac{4\,a^3}{77\,b^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2),x)
```

```
[Out] 2/11*b*x^4*(b*x^3+a*x)^(1/2)+26/77*a*x^2*(b*x^3+a*x)^(1/2)+8/77*a^2*(b*x^3+
a*x)^(1/2)/b-4/77/b^2*a^3*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2)
)^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^
(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+ax\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^3+ax\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a*x)^(3/2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax+bx^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**3)**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+ax\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2), x)
```

## 3.49 $\int \frac{\left(ax+bx^3\right)^{3/2}}{x}\,dx$

**Optimal**. Leaf size=275

$$\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}}-\frac{8a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{15b^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] (8*a^2*x*(a + b*x^2))/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
+ (4*a*x*Sqrt[a*x + b*x^3])/15 + (2*(a*x + b*x^3)^(3/2))/9 - (8*a^(9/4)*Sqr
t[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipt
icE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a*x + b*x^3
]) + (4*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(3/
4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.224599, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2021, 2004, 2032, 329, 305, 220, 1196}

$$\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}}-\frac{8a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x,x]
```

```
[Out] (8*a^2*x*(a + b*x^2))/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
+ (4*a*x*Sqrt[a*x + b*x^3])/15 + (2*(a*x + b*x^3)^(3/2))/9 - (8*a^(9/4)*Sqr
t[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipt
icE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a*x + b*x^3
]) + (4*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(3/
4)*Sqrt[a*x + b*x^3])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax+bx^3\right)^{3/2}}{x}\,dx &= \frac{2}{9}\left(ax+bx^3\right)^{3/2}+\frac{1}{3}(2a)\int \sqrt{ax+bx^3}\,dx\\
&= \frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}+\frac{1}{15}\left(4a^2\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}+\frac{\left(4a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{15\sqrt{ax+bx^3}}\\
&= \frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}+\frac{\left(8a^2\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{15\sqrt{ax+bx^3}}\\
&= \frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}+\frac{\left(8a^{5/2}\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{15\sqrt{b}\sqrt{ax+bx^3}}-\frac{\left(8a^{5/2}\sqrt{x}\sqrt{}\right.}{}\\
&= \frac{8a^2x\left(a+bx^2\right)}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}-\frac{8a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx}{\left(\sqrt{a}+\sqrt{}\right.}}}{15b^{3/4}\sqrt{ax}}
\end{aligned}$$

**Mathematica [C]** time = 0.0122047, size = 52, normalized size = 0.19

$$\frac{2ax\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x,x]
```

```
[Out] (2*a*x*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)])
/(3*Sqrt[1 + (b*x^2)/a])
```

---

**Maple [A]** time = 0.011, size = 195, normalized size = 0.7

$$\frac{2\,bx^3}{9}\sqrt{bx^3+ax}+\frac{22\,ax}{45}\sqrt{bx^3+ax}+\frac{4\,a^2}{15\,b}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\left(-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x,x)
```

```
[Out] 2/9*b*x^3*(b*x^3+a*x)^(1/2)+22/45*a*x*(b*x^3+a*x)^(1/2)+4/15*a^2/b*(-a*b)^(
1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b
/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*
b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{bx^3+ax}\left(bx^2+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x, x)
```

## 3.50 $\int \frac{\left(ax+bx^3\right)^{3/2}}{x^2}\,dx$

**Optimal**. Leaf size=134

$$\frac{4a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{ax+bx^3}}+\frac{4}{7}a\sqrt{ax+bx^3}+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}$$

```
[Out] (4*a*Sqrt[a*x + b*x^3])/7 + (2*(a*x + b*x^3)^(3/2))/(7*x) + (4*a^(7/4)*Sqrt
[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(7*b^(1/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.131994, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2021, 2011, 329, 220}

$$\frac{4a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{ax+bx^3}}+\frac{4}{7}a\sqrt{ax+bx^3}+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (4*a*Sqrt[a*x + b*x^3])/7 + (2*(a*x + b*x^3)^(3/2))/(7*x) + (4*a^(7/4)*Sqrt
[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(7*b^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax+bx^3\right)^{3/2}}{x^2}\,dx &= \frac{2\left(ax+bx^3\right)^{3/2}}{7x}+\frac{1}{7}(6a)\int \frac{\sqrt{ax+bx^3}}{x}\,dx\\
&= \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}+\frac{1}{7}\left(4a^2\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}+\frac{\left(4a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{7\sqrt{ax+bx^3}}\\
&= \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}+\frac{\left(8a^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{7\sqrt{ax+bx^3}}\\
&= \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}+\frac{4a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0144247, size = 49, normalized size = 0.37

$$\frac{2a\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)])/S
qrt[1 + (b*x^2)/a]
```

---

**Maple [A]** time = 0.012, size = 144, normalized size = 1.1

$$\frac{2\,bx^2}{7}\sqrt{bx^3+ax}+\frac{6\,a}{7}\sqrt{bx^3+ax}+\frac{4\,a^2}{7\,b}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\text{EllipticF}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^2,x)
```

```
[Out] 2/7*b*x^2*(b*x^3+a*x)^(1/2)+6/7*a*(b*x^3+a*x)^(1/2)+4/7*a^2/b*(-a*b)^(1/2)*
((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*
b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1
/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^2, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\left(bx^2+a\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**2,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**2, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^2, x)
```

## 3.51 $\int \frac{(ax+bx^3)^{3/2}}{x^3}\,dx$

**Optimal**. Leaf size=274

$$\frac{12a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}-\frac{24a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{5\sqrt{ax+bx^3}}$$

```
[Out] (24*a*Sqrt[b]*x*(a + b*x^2))/(5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) +
(12*b*x*Sqrt[a*x + b*x^3])/5 - (2*(a*x + b*x^3)^(3/2))/x^2 - (24*a^(5/4)*b^
(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b*x^3
]) + (12*a^(5/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sq
rt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/
(5*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.232554, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2020, 2004, 2032, 329, 305, 220, 1196}

$$\frac{12a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}-\frac{24a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{5\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (24*a*Sqrt[b]*x*(a + b*x^2))/(5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) +
(12*b*x*Sqrt[a*x + b*x^3])/5 - (2*(a*x + b*x^3)^(3/2))/x^2 - (24*a^(5/4)*b^
(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b*x^3
]) + (12*a^(5/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sq
rt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/
(5*Sqrt[a*x + b*x^3])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x^3}\,dx = -\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}+(6b)\int \sqrt{ax+bx^3}\,dx$$

$$= \frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}+\frac{1}{5}(12ab)\int \frac{x}{\sqrt{ax+bx^3}}\,dx$$

$$= \frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}+\frac{\left(12ab\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5\sqrt{ax+bx^3}}$$

$$= \frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}+\frac{\left(24ab\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{ax+bx^3}}$$

$$= \frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}+\frac{\left(24a^{3/2}\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{ax+bx^3}}-$$

$$= \frac{24a\sqrt{b}x\left(a+bx^2\right)}{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}-$$

**Mathematica [C]** time = 0.014335, size = 52, normalized size = 0.19

$$-\frac{2a\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^2)/a)])
/(x*Sqrt[1 + (b*x^2)/a])
```

---

**Maple [A]** time = 0.016, size = 194, normalized size = 0.7

$$-2\,\frac{\left(bx^2+a\right)a}{\sqrt{x\left(bx^2+a\right)}}+\frac{2\,bx}{5}\sqrt{bx^3+ax}+\frac{12\,a}{5}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\left(-2\,\frac{\sqrt{-ab}}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^3,x)
```

```
[Out] -2*(b*x^2+a)*a/(x*(b*x^2+a))^(1/2)+2/5*b*x*(b*x^3+a*x)^(1/2)+12/5*a*(-a*b)^
(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*
b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a
*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)
)+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^3, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\left(bx^2+a\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^2, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**3,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**3, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^3, x)
```

## 3.52 $\int \frac{\left(ax+bx^3\right)^{3/2}}{x^4}\,dx$

**Optimal**. Leaf size=134

$$\frac{4a^{3/4}b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt{ax+bx^3}}-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}+\frac{4}{3}b\sqrt{ax+bx^3}$$

```
[Out] (4*b*Sqrt[a*x + b*x^3])/3 - (2*(a*x + b*x^3)^(3/2))/(3*x^3) + (4*a^(3/4)*b^
(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*Sqrt[a*x + b*x^3
])
```

---

**Rubi [A]** time = 0.131137, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2020, 2021, 2011, 329, 220}

$$\frac{4a^{3/4}b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt{ax+bx^3}}-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}+\frac{4}{3}b\sqrt{ax+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (4*b*Sqrt[a*x + b*x^3])/3 - (2*(a*x + b*x^3)^(3/2))/(3*x^3) + (4*a^(3/4)*b^
(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*Sqrt[a*x + b*x^3
])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax+bx^3\right)^{3/2}}{x^4}\,dx &= -\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}+(2b)\int \frac{\sqrt{ax+bx^3}}{x}\,dx\\
&= \frac{4}{3}b\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}+\frac{1}{3}(4ab)\int \frac{1}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{4}{3}b\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}+\frac{\left(4ab\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{3\sqrt{ax+bx^3}}\\
&= \frac{4}{3}b\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}+\frac{\left(8ab\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{ax+bx^3}}\\
&= \frac{4}{3}b\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}+\frac{4a^{3/4}b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0153651, size = 54, normalized size = 0.4

$$
-\frac{2a\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{bx^2}{a}\right)}{3x^2\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^2)/a)])
/(3*x^2*Sqrt[1 + (b*x^2)/a])
```

---

**Maple [A]** time = 0.017, size = 139, normalized size = 1.

$$
-\frac{2a}{3x^2}\sqrt{bx^3+ax}+\frac{2b}{3}\sqrt{bx^3+ax}+\frac{4a}{3}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\mathrm{EllipticF}\Bigg(
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^4,x)
```

```
[Out] -2/3*a*(b*x^3+a*x)^(1/2)/x^2+2/3*b*(b*x^3+a*x)^(1/2)+4/3*a*(-a*b)^(1/2)*((x
+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^
```

```
(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*
(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^4, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\left(bx^2+a\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^3, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**4,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**4, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^4, x)
```

## 3.53 $\int \frac{\left(ax+bx^3\right)^{3/2}}{x^5}\,dx$

**Optimal**. Leaf size=277

$$\frac{12\sqrt[4]{a}b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}+\frac{24b^{3/2}x\left(a+bx^2\right)}{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{24\sqrt[4]{a}b^{5/4}\sqrt{x}\left(\sqrt{a}+\right.}{}$$

```
[Out] (24*b^(3/2)*x*(a + b*x^2))/(5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (1
2*b*Sqrt[a*x + b*x^3])/(5*x) - (2*(a*x + b*x^3)^(3/2))/(5*x^4) - (24*a^(1/4
)*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b
*x^3]) + (12*a^(1/4)*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/
2])/(5*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.260832, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2020, 2032, 329, 305, 220, 1196}

$$\frac{24b^{3/2}x\left(a+bx^2\right)}{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{12\sqrt[4]{a}b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}-\frac{24\sqrt[4]{a}b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{}}{5\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (24*b^(3/2)*x*(a + b*x^2))/(5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (1
2*b*Sqrt[a*x + b*x^3])/(5*x) - (2*(a*x + b*x^3)^(3/2))/(5*x^4) - (24*a^(1/4
)*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b
*x^3]) + (12*a^(1/4)*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/
2])/(5*Sqrt[a*x + b*x^3])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax+bx^3\right)^{3/2}}{x^5}\,dx &= -\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}+\frac{1}{5}(6b)\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx\\
&= -\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}+\frac{1}{5}\left(12b^2\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx\\
&= -\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}+\frac{\left(12b^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5\sqrt{ax+bx^3}}\\
&= -\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}+\frac{\left(24b^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{ax+bx^3}}\\
&= -\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}+\frac{\left(24\sqrt{a}b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{ax+bx^3}}-\frac{\left(24\sqrt{a}\right.}{}\\
&= \frac{24b^{3/2}x\left(a+bx^2\right)}{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}-\frac{24\sqrt[4]{a}b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right.}}}{5\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.013007, size = 54, normalized size = 0.19

$$
-\frac{2a\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^3\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^2)/a)]
)/(5*x^3*Sqrt[1 + (b*x^2)/a])
```

---

**Maple [A]** time = 0.016, size = 196, normalized size = 0.7

$$-\frac{2a}{5x^3}\sqrt{bx^3+ax}-\frac{\left(14bx^2+14a\right)b}{5}\frac{1}{\sqrt{x\left(bx^2+a\right)}}+\frac{12b}{5}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^5,x)
```

```
[Out] -2/5*a*(b*x^3+a*x)^(1/2)/x^3-14/5*(b*x^2+a)*b/(x*(b*x^2+a))^(1/2)+12/5*b*(-
a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1
/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/
b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^5, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^3+ax}\left(bx^2+a\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^4, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**5,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**5, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + ax\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^5, x)
```

## 3.54 $\int \frac{\left(ax+bx^3\right)^{3/2}}{x^6}\,dx$

**Optimal**. Leaf size=137

$$\frac{4b^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{ax+bx^3}}-\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}$$

```
[Out] (-4*b*Sqrt[a*x + b*x^3])/(7*x^2) - (2*(a*x + b*x^3)^(3/2))/(7*x^5) + (4*b^(
7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2
]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(7*a^(1/4)*Sqrt[a*x
+ b*x^3])
```

---

**Rubi [A]** time = 0.134207, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2020, 2011, 329, 220}

$$\frac{4b^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{ax+bx^3}}-\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^6,x]
```

```
[Out] (-4*b*Sqrt[a*x + b*x^3])/(7*x^2) - (2*(a*x + b*x^3)^(3/2))/(7*x^5) + (4*b^(
7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2
]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(7*a^(1/4)*Sqrt[a*x
+ b*x^3])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax+bx^3\right)^{3/2}}{x^6}\,dx &= -\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}+\frac{1}{7}(6b)\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx\\
&= -\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}+\frac{1}{7}\left(4b^2\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx\\
&= -\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}+\frac{\left(4b^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{7\sqrt{ax+bx^3}}\\
&= -\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}+\frac{\left(8b^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{7\sqrt{ax+bx^3}}\\
&= -\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}+\frac{4b^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.016064, size = 54, normalized size = 0.39

$$
-\frac{2a\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{7}{4},-\frac{3}{2};-\frac{3}{4};-\frac{bx^2}{a}\right)}{7x^4\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^6,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^2)/a)]
)/(7*x^4*Sqrt[1 + (b*x^2)/a])
```

**Maple [A]** time = 0.015, size = 142, normalized size = 1.

$$
-\frac{2\,a}{7\,x^4}\sqrt{bx^3+ax}-\frac{6\,b}{7\,x^2}\sqrt{bx^3+ax}+\frac{4\,b}{7}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\,\text{EllipticF}\left(\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^6,x)
```

```
[Out] -2/7*a*(b*x^3+a*x)^(1/2)/x^4-6/7*b*(b*x^3+a*x)^(1/2)/x^2+4/7*b*(-a*b)^(1/2)
*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a
*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+
1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + ax\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^6, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}\left(bx^2 + a\right)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^5, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x\left(a + bx^2\right)\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**6,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**6, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + ax\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^6, x)
```

## 3.55 $\int \frac{\left(ax+bx^3\right)^{3/2}}{x^7}\,dx$

**Optimal**. Leaf size=306

$$\frac{4b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}}-\frac{8b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{15a^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] (8*b^(5/2)*x*(a + b*x^2))/(15*a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) -
(4*b*Sqrt[a*x + b*x^3])/(15*x^3) - (8*b^2*Sqrt[a*x + b*x^3])/(15*a*x) - (2*
(a*x + b*x^3)^(3/2))/(9*x^6) - (8*b^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])
/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a*x + b*x^3]) + (4*b^(9/4)*Sqrt[x]*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTa
n[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.311964, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{4b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}}-\frac{8b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}}+\frac{}{15a\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^7,x]
```

```
[Out] (8*b^(5/2)*x*(a + b*x^2))/(15*a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) -
(4*b*Sqrt[a*x + b*x^3])/(15*x^3) - (8*b^2*Sqrt[a*x + b*x^3])/(15*a*x) - (2*
(a*x + b*x^3)^(3/2))/(9*x^6) - (8*b^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])
/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a*x + b*x^3]) + (4*b^(9/4)*Sqrt[x]*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTa
n[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

### Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax+bx^3\right)^{3/2}}{x^7}\,dx &= -\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}+\frac{1}{3}(2b)\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx \\
&= -\frac{4b\sqrt{ax+bx^3}}{15x^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}+\frac{1}{15}\left(4b^2\right)\int \frac{1}{x\sqrt{ax+bx^3}}\,dx \\
&= -\frac{4b\sqrt{ax+bx^3}}{15x^3}-\frac{8b^2\sqrt{ax+bx^3}}{15ax}-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}+\frac{\left(4b^3\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{15a} \\
&= -\frac{4b\sqrt{ax+bx^3}}{15x^3}-\frac{8b^2\sqrt{ax+bx^3}}{15ax}-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}+\frac{\left(4b^3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{15a\sqrt{ax+bx^3}} \\
&= -\frac{4b\sqrt{ax+bx^3}}{15x^3}-\frac{8b^2\sqrt{ax+bx^3}}{15ax}-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}+\frac{\left(8b^3\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{15a\sqrt{ax+bx^3}} \\
&= -\frac{4b\sqrt{ax+bx^3}}{15x^3}-\frac{8b^2\sqrt{ax+bx^3}}{15ax}-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}+\frac{\left(8b^{5/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{\right.}{15\sqrt{a}\sqrt{ax+bx^3}} \\
&= \frac{8b^{5/2}x\left(a+bx^2\right)}{15a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{4b\sqrt{ax+bx^3}}{15x^3}-\frac{8b^2\sqrt{ax+bx^3}}{15ax}-\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6}-\frac{8b^{9/4}\sqrt{x}\left(\sqrt{a}+\right.}{}
\end{aligned}
$$

**Mathematica [C]** time = 0.0158659, size = 54, normalized size = 0.18

$$
-\frac{2a\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{bx^2}{a}\right)}{9x^5\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^7,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^2)/a)]
)/(9*x^5*Sqrt[1 + (b*x^2)/a])
```

---

**Maple [A]** time = 0.018, size = 223, normalized size = 0.7

$$
-\frac{2\,a}{9\,x^5}\sqrt{bx^3+ax}-\frac{22\,b}{45\,x^3}\sqrt{bx^3+ax}-\frac{\left(8\,bx^2+8\,a\right)b^2}{15\,a}\frac{1}{\sqrt{x\left(bx^2+a\right)}}+\frac{4\,b^2}{15\,a}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^7,x)
```

```
[Out] -2/9*a*(b*x^3+a*x)^(1/2)/x^5-22/45*b*(b*x^3+a*x)^(1/2)/x^3-8/15*(b*x^2+a)/a
*b^2/(x*(b*x^2+a))^(1/2)+4/15/a*b^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-
a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*
b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*
b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x
+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^7, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\left(bx^2+a\right)}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^6, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**7,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**7, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^7, x)
```

## 3.56 $\int \frac{\left(ax+bx^3\right)^{3/2}}{x^8}\,dx$

**Optimal**. Leaf size=163

$$-\frac{4b^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{77a^{5/4}\sqrt{ax+bx^3}}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}$$

```
[Out] (-12*b*Sqrt[a*x + b*x^3])/(77*x^4) - (8*b^2*Sqrt[a*x + b*x^3])/(77*a*x^2) -
 (2*(a*x + b*x^3)^(3/2))/(11*x^7) - (4*b^(11/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*
x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sq
rt[x])/a^(1/4)], 1/2])/(77*a^(5/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.178191, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2020, 2025, 2011, 329, 220}

$$-\frac{4b^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{77a^{5/4}\sqrt{ax+bx^3}}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^8,x]
```

```
[Out] (-12*b*Sqrt[a*x + b*x^3])/(77*x^4) - (8*b^2*Sqrt[a*x + b*x^3])/(77*a*x^2) -
 (2*(a*x + b*x^3)^(3/2))/(11*x^7) - (4*b^(11/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*
x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sq
rt[x])/a^(1/4)], 1/2])/(77*a^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax+bx^3\right)^{3/2}}{x^8}\,dx &= -\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}+\frac{1}{11}(6b)\int \frac{\sqrt{ax+bx^3}}{x^5}\,dx\\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}+\frac{1}{77}\left(12b^2\right)\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx\\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}-\frac{\left(4b^3\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{77a}\\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}-\frac{\left(4b^3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{77a\sqrt{ax+bx^3}}\\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}-\frac{\left(8b^3\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\Bigg(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,}{77a\sqrt{ax+bx^3}}\\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}-\frac{4b^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\Bigg(2\tan}{77a^{5/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0159306, size = 54, normalized size = 0.33

$$
-\frac{2a\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{11}{4},-\frac{3}{2};-\frac{7}{4};-\frac{bx^2}{a}\right)}{11x^6\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^8,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*x^2)/a)
])/(11*x^6*Sqrt[1 + (b*x^2)/a])
```

---

**Maple [A]** time = 0.019, size = 169, normalized size = 1.

$$
-\frac{2\,a}{11\,x^6}\sqrt{bx^3+ax}-\frac{26\,b}{77\,x^4}\sqrt{bx^3+ax}-\frac{8\,b^2}{77\,ax^2}\sqrt{bx^3+ax}-\frac{4\,b^2}{77\,a}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^8,x)
```

```
[Out] -2/11*a*(b*x^3+a*x)^(1/2)/x^6-26/77*b*(b*x^3+a*x)^(1/2)/x^4-8/77*b^2*(b*x^3
+a*x)^(1/2)/a/x^2-4/77/a*b^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1
/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2
))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^
(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^8, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\left(bx^2+a\right)}{x^7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^7, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**8,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**8, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^8, x)
```

## 3.57 $\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx$

**Optimal**. Leaf size=140

$$\frac{5a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{ax+bx^3}}-\frac{10a\sqrt{ax+bx^3}}{21b^2}+\frac{2x^2\sqrt{ax+bx^3}}{7b}$$

```
[Out] (-10*a*Sqrt[a*x + b*x^3])/(21*b^2) + (2*x^2*Sqrt[a*x + b*x^3])/(7*b) + (5*a
^(7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(21*b^(9/4)*Sqrt[a
*x + b*x^3])
```

---

**Rubi [A]** time = 0.13725, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2024, 2011, 329, 220}

$$\frac{5a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{ax+bx^3}}-\frac{10a\sqrt{ax+bx^3}}{21b^2}+\frac{2x^2\sqrt{ax+bx^3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x + b*x^3],x]
```

```
[Out] (-10*a*Sqrt[a*x + b*x^3])/(21*b^2) + (2*x^2*Sqrt[a*x + b*x^3])/(7*b) + (5*a
^(7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(21*b^(9/4)*Sqrt[a
*x + b*x^3])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx &= \frac{2x^2\sqrt{ax+bx^3}}{7b}-\frac{(5a)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx}{7b}\\
&= -\frac{10a\sqrt{ax+bx^3}}{21b^2}+\frac{2x^2\sqrt{ax+bx^3}}{7b}+\frac{\left(5a^2\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{21b^2}\\
&= -\frac{10a\sqrt{ax+bx^3}}{21b^2}+\frac{2x^2\sqrt{ax+bx^3}}{7b}+\frac{\left(5a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{21b^2\sqrt{ax+bx^3}}\\
&= -\frac{10a\sqrt{ax+bx^3}}{21b^2}+\frac{2x^2\sqrt{ax+bx^3}}{7b}+\frac{\left(10a^2\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{21b^2\sqrt{ax+bx^3}}\\
&= -\frac{10a\sqrt{ax+bx^3}}{21b^2}+\frac{2x^2\sqrt{ax+bx^3}}{7b}+\frac{5a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{ax+bx^3}}
\end{aligned}$$

**Mathematica [C]** time = 0.0316225, size = 80, normalized size = 0.57

$$\frac{2x\left(5a^2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-5a^2-2abx^2+3b^2x^4\right)}{21b^2\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*(-5*a^2 - 2*a*b*x^2 + 3*b^2*x^4 + 5*a^2*Sqrt[1 + (b*x^2)/a]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(21*b^2*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.013, size = 149, normalized size = 1.1

$$\frac{2\,x^2}{7\,b}\sqrt{bx^3+ax}-\frac{10\,a}{21\,b^2}\sqrt{bx^3+ax}+\frac{5\,a^2}{21\,b^3}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\mathrm{Elliptic}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x)^(1/2),x)
```

```
[Out] 2/7*x^2*(b*x^3+a*x)^(1/2)/b-10/21*a*(b*x^3+a*x)^(1/2)/b^2+5/21*a^2/b^3*(-a*
b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2
))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*Ellipt
icF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^3 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(b*x^3 + a*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}\,x^3}{bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^3/(b*x^2 + a), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{x\left(a + bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x*(a + b*x**2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^3 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(b*x^3 + a*x), x)
```

## 3.58 $\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx$

**Optimal**. Leaf size=258

$$-\frac{3a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}}+\frac{6a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{}}{\sqrt[4]{a}}\right.\right.}{5b^{7/4}\sqrt{ax+bx^3}}$$

```
[Out] (-6*a*x*(a + b*x^2))/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) +
(2*x*Sqrt[a*x + b*x^3])/(5*b) + (6*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sq
rt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x]
)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a*x + b*x^3]) - (3*a^(5/4)*Sqrt[x]*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTa
n[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.205405, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2024, 2032, 329, 305, 220, 1196}

$$-\frac{3a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}}+\frac{6a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x + b*x^3],x]
```

```
[Out] (-6*a*x*(a + b*x^2))/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) +
(2*x*Sqrt[a*x + b*x^3])/(5*b) + (6*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sq
rt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x]
)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a*x + b*x^3]) - (3*a^(5/4)*Sqrt[x]*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTa
n[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx &= \frac{2x\sqrt{ax+bx^3}}{5b}-\frac{(3a)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{5b}\\
&= \frac{2x\sqrt{ax+bx^3}}{5b}-\frac{\left(3a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5b\sqrt{ax+bx^3}}\\
&= \frac{2x\sqrt{ax+bx^3}}{5b}-\frac{\left(6a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5b\sqrt{ax+bx^3}}\\
&= \frac{2x\sqrt{ax+bx^3}}{5b}-\frac{\left(6a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5b^{3/2}\sqrt{ax+bx^3}}+\frac{\left(6a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\Bigg(\int \frac{1-\frac{\sqrt{b}}{\sqrt{}}}{\sqrt{a+b}}}{5b^{3/2}\sqrt{ax+bx^3}}\\
&= -\frac{6ax\left(a+bx^2\right)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{2x\sqrt{ax+bx^3}}{5b}+\frac{6a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right.}{5b^{7/4}\sqrt{ax+bx^3}}
\end{aligned}$$

**Mathematica [C]** time = 0.0261342, size = 66, normalized size = 0.26

$$\frac{2x^2\left(-a\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+a+bx^2\right)}{5b\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x^2*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4,
-((b*x^2)/a)]))/(5*b*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.011, size = 178, normalized size = 0.7

$$\frac{2x}{5b}\sqrt{bx^3+ax}-\frac{3a}{5b^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\Bigg(-2\frac{\sqrt{-ab}}{b}\mathrm{EllipticE}\Bigg(\sqrt{\frac{b}{\sqrt{-a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x)^(1/2),x)
```

```
[Out] 2/5*x*(b*x^3+a*x)^(1/2)/b-3/5/b^2*a*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-
a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*
b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*
b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x
+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(b*x^3 + a*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^3+ax}x^2}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^2/(b*x^2 + a), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x*(a + b*x**2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^3 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(b*x^3 + a*x), x)
```

## 3.59 $\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx$

**Optimal**. Leaf size=116

$$\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}$$

```
[Out] (2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(
a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^
(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.091224, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2024, 2011, 329, 220}

$$\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(
a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^
(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx &= \frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{3b}\\
&= \frac{2\sqrt{ax+bx^3}}{3b}-\frac{\left(a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{3b\sqrt{ax+bx^3}}\\
&= \frac{2\sqrt{ax+bx^3}}{3b}-\frac{\left(2a\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3b\sqrt{ax+bx^3}}\\
&= \frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0254226, size = 64, normalized size = 0.55

$$\frac{2x\left(-a\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+a+bx^2\right)}{3b\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(
(b*x^2)/a)]))/(3*b*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.013, size = 127, normalized size = 1.1

$$\frac{2}{3\,b}\sqrt{bx^3+ax}-\frac{a}{3\,b^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\operatorname{EllipticF}\left(\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x)^(1/2),x)
```

```
[Out] 2/3*(b*x^3+a*x)^(1/2)/b-1/3/b^2*a*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*
b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)
^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1
/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*x^3 + a*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}x}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x/(b*x^2 + a), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x*(a + b*x**2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*x^3 + a*x), x)
```

## 3.60 $\int \frac{x}{\sqrt{ax+bx^3}}\,dx$

**Optimal**. Leaf size=229

$$\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}-\frac{2\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}-$$

```
[Out] (2*x*(a + b*x^2))/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*a^
(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a*x +
 b*x^3]) + (a^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(3
/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.149676, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2032, 329, 305, 220, 1196}

$$\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}-\frac{2\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}+\frac{}{\sqrt{b}\left(\sqrt{\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*(a + b*x^2))/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*a^
(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a*x +
 b*x^3]) + (a^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(3
/4)*Sqrt[a*x + b*x^3])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{ax+bx^3}}\,dx &= \frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{\sqrt{ax+bx^3}}\\
&= \frac{\left(2\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{ax+bx^3}}\\
&= \frac{\left(2\sqrt{a}\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{b}\sqrt{ax+bx^3}} - \frac{\left(2\sqrt{a}\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{b}\sqrt{ax+bx^3}}\\
&= \frac{2x\left(a+bx^2\right)}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}} + \sqrt[4]{a}\sqrt{x}\,(\sqrt{a}
\end{aligned}$$

**Mathematica [C]** time = 0.0129953, size = 53, normalized size = 0.23

$$\frac{2x^2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^2)/a)])/
(3*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.011, size = 158, normalized size = 0.7

$$\frac{1}{b}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\left(-2\frac{\sqrt{-ab}}{b}\operatorname{EllipticE}\left(\sqrt{\frac{b}{\sqrt{-ab}}\left(x+\frac{\sqrt{-ab}}{b}\right)},1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x)^(1/2),x)
```

```
[Out] 1/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a
*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2
)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),
1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*x^3 + a*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^3+ax}}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b*x^2 + a), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(a + b*x**2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*x^3 + a*x), x)
```

## 3.61 $\int \frac{1}{\sqrt{ax+bx^3}}\,dx$

**Optimal**. Leaf size=92

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax+bx^3}}$$

```
[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(1/4)*b^(1/4)*Sqrt[a*
x + b*x^3])
```

---

**Rubi [A]** time = 0.0488648, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2011, 329, 220}

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x + b*x^3],x]
```

```
[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(1/4)*b^(1/4)*Sqrt[a*
x + b*x^3])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{ax+bx^3}}\,dx &= \frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{\sqrt{ax+bx^3}} \\
&= \frac{\left(2\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}} \\
&= \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0095926, size = 49, normalized size = 0.53

$$
\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{x\left(a+bx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/Sq
rt[x*(a + b*x^2)]
```

---

**Maple [A]** time = 0.013, size = 108, normalized size = 1.2

$$
\frac{1}{b}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\operatorname{EllipticF}\left(\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\frac{1}{\sqrt{bx^3+a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x)^(1/2),x)
```

```
[Out] 1/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a
*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2
)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{\sqrt{bx^3+ax}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^3 + a*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{bx^3+ax}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*x^3 + a*x), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**3), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*x^3 + a*x), x)
```

## 3.62 $\int \frac{1}{x\sqrt{ax+bx^3}}\,dx$

**Optimal**. Leaf size=253

$$\frac{\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}}-\frac{2\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}}-$$

```
[Out] (2*Sqrt[b]*x*(a + b*x^2))/(a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*
Sqrt[a*x + b*x^3])/(a*x) - (2*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(
1/4)], 1/2])/(a^(3/4)*Sqrt[a*x + b*x^3]) + (b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4
)*Sqrt[x])/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.200493, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2025, 2032, 329, 305, 220, 1196}

$$\frac{\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}}-\frac{2\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}}+\frac{2}{a\left(\sqrt{a}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (2*Sqrt[b]*x*(a + b*x^2))/(a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*
Sqrt[a*x + b*x^3])/(a*x) - (2*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(
1/4)], 1/2])/(a^(3/4)*Sqrt[a*x + b*x^3]) + (b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4
)*Sqrt[x])/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{x\sqrt{ax+bx^3}}\,dx = -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{b\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{a}$$

$$= -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{\left(b\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{a\sqrt{ax+bx^3}}$$

$$= -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{\left(2b\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax+bx^3}}$$

$$= -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{\left(2\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{a}\sqrt{ax+bx^3}}-\frac{\left(2\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\Bigg(\int}{\sqrt{a}\sqrt{ax+bx^3}}$$

$$= \frac{2\sqrt{b}x\left(a+bx^2\right)}{a\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}-\frac{2\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\Bigg(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}}{a^{3/4}\sqrt{ax+bx^3}}$$

**Mathematica [C]** time = 0.0122931, size = 48, normalized size = 0.19

$$-\frac{2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^2)/a)])/Sq
rt[x*(a + b*x^2)]
```

---

**Maple [A]** time = 0.015, size = 182, normalized size = 0.7

$$-2\,\frac{bx^2+a}{a\sqrt{x\left(bx^2+a\right)}}+\frac{1}{a}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\Bigg(-2\,\frac{\sqrt{-ab}}{b}\mathrm{EllipticE}\Bigg(\sqrt{\frac{b}{\sqrt{-ab}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x)^(1/2),x)
```

```
[Out] -2*(b*x^2+a)/a/(x*(b*x^2+a))^(1/2)+1/a*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b
/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(
-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(
-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(
((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+axx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^3+ax}}{bx^4+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b*x^4 + a*x^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x*(a + b*x**2))), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + axx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x), x)
```

## 3.63 $\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx$

**Optimal**. Leaf size=119

$$-\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}$$

```
[Out] (-2*Sqrt[a*x + b*x^3])/(3*a*x^2) - (b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.0893417, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2025, 2011, 329, 220}

$$-\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^3])/(3*a*x^2) - (b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx &= -\frac{2\sqrt{ax+bx^3}}{3ax^2}-\frac{b\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{3a}\\
&= -\frac{2\sqrt{ax+bx^3}}{3ax^2}-\frac{\left(b\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{3a\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{3ax^2}-\frac{\left(2b\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3a\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{3ax^2}-\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0150918, size = 53, normalized size = 0.45

$$
-\frac{2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3x\sqrt{x\left(a+bx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^2)/a)])/(3
*x*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.014, size = 129, normalized size = 1.1

$$
-\frac{2}{3\,ax^2}\sqrt{bx^3+ax}-\frac{1}{3\,a}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\mathrm{EllipticF}\Bigg(\sqrt{b\Big(x+\frac{1}{b}\sqrt{-a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x)^(1/2),x)
```

```
[Out] -2/3*(b*x^3+a*x)^(1/2)/a/x^2-1/3/a*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a
*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b
)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(
1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{\sqrt{bx^3+axx^2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^3+ax}}{bx^5+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b*x^5 + a*x^3), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{x\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x*(a + b*x**2))), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+ax}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^2), x)
```

## 3.64 $\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx$

**Optimal**. Leaf size=286

$$-\frac{3b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}}-\frac{6b^{3/2}x\left(a+bx^2\right)}{5a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{6b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right.}{}$$

```
[Out] (-6*b^(3/2)*x*(a + b*x^2))/(5*a^2*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
- (2*Sqrt[a*x + b*x^3])/(5*a*x^3) + (6*b*Sqrt[a*x + b*x^3])/(5*a^2*x) + (6*
b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[a
*x + b*x^3]) - (3*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])
/(5*a^(7/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.254296, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2025, 2032, 329, 305, 220, 1196}

$$-\frac{6b^{3/2}x\left(a+bx^2\right)}{5a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{3b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}}+\frac{6b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+}{(\sqrt{a}+}}}{5a^{7/4}\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-6*b^(3/2)*x*(a + b*x^2))/(5*a^2*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
- (2*Sqrt[a*x + b*x^3])/(5*a*x^3) + (6*b*Sqrt[a*x + b*x^3])/(5*a^2*x) + (6*
b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[a
*x + b*x^3]) - (3*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])
/(5*a^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx &= -\frac{2\sqrt{ax+bx^3}}{5ax^3}-\frac{(3b)\int \frac{1}{x\sqrt{ax+bx^3}}\,dx}{5a}\\
&= -\frac{2\sqrt{ax+bx^3}}{5ax^3}+\frac{6b\sqrt{ax+bx^3}}{5a^2x}-\frac{\left(3b^2\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{5a^2}\\
&= -\frac{2\sqrt{ax+bx^3}}{5ax^3}+\frac{6b\sqrt{ax+bx^3}}{5a^2x}-\frac{\left(3b^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5a^2\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{5ax^3}+\frac{6b\sqrt{ax+bx^3}}{5a^2x}-\frac{\left(6b^2\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5a^2\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{5ax^3}+\frac{6b\sqrt{ax+bx^3}}{5a^2x}-\frac{\left(6b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5a^{3/2}\sqrt{ax+bx^3}}+\frac{\left(6b^{3/2}\sqrt{x}\sqrt{a+b\right.}{}\\
&= -\frac{6b^{3/2}x\left(a+bx^2\right)}{5a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{5ax^3}+\frac{6b\sqrt{ax+bx^3}}{5a^2x}+\frac{6b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(\right.}{5a^{7/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0138129, size = 53, normalized size = 0.19

$$
-\frac{2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^2\sqrt{x\left(a+bx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^2)/a)])/(
5*x^2*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.016, size = 204, normalized size = 0.7

$$-\frac{2}{5\,ax^3}\sqrt{bx^3+ax}+\frac{\left(6\,bx^2+6\,a\right)b}{5\,a^2}\frac{1}{\sqrt{x\left(bx^2+a\right)}}-\frac{3\,b}{5\,a^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a*x)^(1/2),x)
```

```
[Out] -2/5*(b*x^3+a*x)^(1/2)/a/x^3+6/5*(b*x^2+a)*b/a^2/(x*(b*x^2+a))^(1/2)-3/5/a^
2*b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a
*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2
)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),
1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+ax}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^3), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{bx^6+ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b*x^6 + a*x^4), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{x\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x*(a + b*x**2))), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + ax}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^3), x)
```

## 3.65 $\int \frac{x^7}{\left(ax+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=161

$$\frac{15a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{14b^{13/4}\sqrt{ax+bx^3}}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}-\frac{15a\sqrt{ax+bx^3}}{7b^3}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

```
[Out] -(x^5/(b*Sqrt[a*x + b*x^3])) - (15*a*Sqrt[a*x + b*x^3])/(7*b^3) + (9*x^2*Sq
rt[a*x + b*x^3])/(7*b^2) + (15*a^(7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(
a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^
(1/4)], 1/2])/(14*b^(13/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.193408, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2022, 2024, 2011, 329, 220}

$$\frac{15a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{14b^{13/4}\sqrt{ax+bx^3}}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}-\frac{15a\sqrt{ax+bx^3}}{7b^3}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^5/(b*Sqrt[a*x + b*x^3])) - (15*a*Sqrt[a*x + b*x^3])/(7*b^3) + (9*x^2*Sq
rt[a*x + b*x^3])/(7*b^2) + (15*a^(7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(
a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^
(1/4)], 1/2])/(14*b^(13/4)*Sqrt[a*x + b*x^3])
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
```

```
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{\left(ax+bx^3\right)^{3/2}}\,dx &= -\frac{x^5}{b\sqrt{ax+bx^3}}+\frac{9\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx}{2b}\\
&= -\frac{x^5}{b\sqrt{ax+bx^3}}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}-\frac{(45a)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx}{14b^2}\\
&= -\frac{x^5}{b\sqrt{ax+bx^3}}-\frac{15a\sqrt{ax+bx^3}}{7b^3}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}+\frac{\left(15a^2\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{14b^3}\\
&= -\frac{x^5}{b\sqrt{ax+bx^3}}-\frac{15a\sqrt{ax+bx^3}}{7b^3}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}+\frac{\left(15a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{14b^3\sqrt{ax+bx^3}}\\
&= -\frac{x^5}{b\sqrt{ax+bx^3}}-\frac{15a\sqrt{ax+bx^3}}{7b^3}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}+\frac{\left(15a^2\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{7b^3\sqrt{ax+bx^3}}\\
&= -\frac{x^5}{b\sqrt{ax+bx^3}}-\frac{15a\sqrt{ax+bx^3}}{7b^3}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}+\frac{15a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\right.\right.}{14b^{13/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0332699, size = 80, normalized size = 0.5

$$
\frac{x\left(15a^2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-15a^2-6abx^2+2b^2x^4\right)}{7b^3\sqrt{x\left(a+bx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (x*(-15*a^2 - 6*a*b*x^2 + 2*b^2*x^4 + 15*a^2*Sqrt[1 + (b*x^2)/a]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(7*b^3*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.018, size = 172, normalized size = 1.1

$$
-\frac{a^2x}{b^3}\frac{1}{\sqrt{\left(\frac{a}{b}+x^2\right)bx}}+\frac{2x^2}{7b^2}\sqrt{bx^3+ax}-\frac{8a}{7b^3}\sqrt{bx^3+ax}+\frac{15a^2}{14b^4}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a*x)^(3/2),x)
```

```
[Out] -1/b^3*x*a^2/((1/b*a+x^2)*b*x)^(1/2)+2/7*x^2*(b*x^3+a*x)^(1/2)/b^2-8/7*a*(b
*x^3+a*x)^(1/2)/b^3+15/14*a^2/b^4*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*
b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)
^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1
/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(bx^3 + ax\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/(b*x^3 + a*x)^(3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^3 + ax}\, x^5}{b^2x^4 + 2\, abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^5/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(x\left(a + bx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**7/(x*(a + b*x**2))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(bx^3 + ax\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/(b*x^3 + a*x)^(3/2), x)
```

## 3.66 $\int \frac{x^6}{\left(ax+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=279

$$-\frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10b^{11/4}\sqrt{ax+bx^3}}+\frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\Bigg(2\tan^{-1}\Bigg(}{5b^{11/4}\sqrt{ax+bx^3}}$$

```
[Out] -(x^4/(b*Sqrt[a*x + b*x^3])) - (21*a*x*(a + b*x^2))/(5*b^(5/2)*(Sqrt[a] + S
qrt[b]*x)*Sqrt[a*x + b*x^3]) + (7*x*Sqrt[a*x + b*x^3])/(5*b^2) + (21*a^(5/4
)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*E
llipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(11/4)*Sqrt[a*x +
b*x^3]) - (21*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[
a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(10
*b^(11/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.268844, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2022, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{ax+bx^3}}+\frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^4/(b*Sqrt[a*x + b*x^3])) - (21*a*x*(a + b*x^2))/(5*b^(5/2)*(Sqrt[a] + S
qrt[b]*x)*Sqrt[a*x + b*x^3]) + (7*x*Sqrt[a*x + b*x^3])/(5*b^2) + (21*a^(5/4
)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*E
llipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(11/4)*Sqrt[a*x +
b*x^3]) - (21*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[
a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(10
*b^(11/4)*Sqrt[a*x + b*x^3])
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\left(ax+bx^3\right)^{3/2}}\,dx &= -\frac{x^4}{b\sqrt{ax+bx^3}}+\frac{7\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx}{2b}\\
&= -\frac{x^4}{b\sqrt{ax+bx^3}}+\frac{7x\sqrt{ax+bx^3}}{5b^2}-\frac{(21a)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{10b^2}\\
&= -\frac{x^4}{b\sqrt{ax+bx^3}}+\frac{7x\sqrt{ax+bx^3}}{5b^2}-\frac{\left(21a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{10b^2\sqrt{ax+bx^3}}\\
&= -\frac{x^4}{b\sqrt{ax+bx^3}}+\frac{7x\sqrt{ax+bx^3}}{5b^2}-\frac{\left(21a\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5b^2\sqrt{ax+bx^3}}\\
&= -\frac{x^4}{b\sqrt{ax+bx^3}}+\frac{7x\sqrt{ax+bx^3}}{5b^2}-\frac{\left(21a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5b^{5/2}\sqrt{ax+bx^3}}+\frac{\left(21a^{3/2}\sqrt{x}\sqrt{a\ }\right.}{}\\
&= -\frac{x^4}{b\sqrt{ax+bx^3}}-\frac{21ax\left(a+bx^2\right)}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{7x\sqrt{ax+bx^3}}{5b^2}+\frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}}{5b^{11/4}\sqrt{ax+b}}
\end{aligned}$$

**Mathematica [C]** time = 0.0262226, size = 68, normalized size = 0.24

$$\frac{2x^2\left(7a\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)-7a+bx^2\right)}{5b^2\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*x^2*(-7*a + b*x^2 + 7*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2,
7/4, -((b*x^2)/a)]))/(5*b^2*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.015, size = 200, normalized size = 0.7

$$\frac{ax^2}{b^2}\frac{1}{\sqrt{\left(\frac{a}{b}+x^2\right)bx}}+\frac{2\,x}{5\,b^2}\sqrt{bx^3+ax}-\frac{21\,a}{10\,b^3}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\Bigg(-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a*x)^(3/2),x)
```

```
[Out] 1/b^2*x^2*a/((1/b*a+x^2)*b*x)^(1/2)+2/5*x*(b*x^3+a*x)^(1/2)/b^2-21/10/b^3*a
*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)
^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(
-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^
(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^3 + a*x)^(3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^3+ax}x^4}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**6/(x*(a + b*x**2))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^3 + a*x)^(3/2), x)
```

## 3.67 $\int \frac{x^5}{\left(ax+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=137

$$-\frac{5a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{9/4}\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

```
[Out] -(x^3/(b*Sqrt[a*x + b*x^3])) + (5*Sqrt[a*x + b*x^3])/(3*b^2) - (5*a^(3/4)*S
qrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(6*b^(9/4)*Sqrt[a*x + b*x^
3])
```

---

**Rubi [A]** time = 0.147964, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2022, 2024, 2011, 329, 220}

$$-\frac{5a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^3/(b*Sqrt[a*x + b*x^3])) + (5*Sqrt[a*x + b*x^3])/(3*b^2) - (5*a^(3/4)*S
qrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(6*b^(9/4)*Sqrt[a*x + b*x^
3])
```

### Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

### Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
```

```
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(ax+bx^3\right)^{3/2}}\,dx &= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx}{2b}\\
&= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{(5a)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{6b^2}\\
&= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{\left(5a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{6b^2\sqrt{ax+bx^3}}\\
&= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{\left(5a\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3b^2\sqrt{ax+bx^3}}\\
&= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{5a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{ax+bx^3}}
\end{aligned}$$

**Mathematica [C]** time = 0.0287506, size = 67, normalized size = 0.49

$$\frac{x\left(-5a\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+5a+2bx^2\right)}{3b^2\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (x*(5*a + 2*b*x^2 - 5*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4
, -((b*x^2)/a)]))/(3*b^2*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.015, size = 147, normalized size = 1.1

$$\frac{ax}{b^2}\frac{1}{\sqrt{\left(\frac{a}{b}+x^2\right)bx}}+\frac{2}{3\,b^2}\sqrt{bx^3+ax}-\frac{5\,a}{6\,b^3}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\operatorname{EllipticF}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a*x)^(3/2),x)
```

```
[Out] 1/b^2*x*a/((1/b*a+x^2)*b*x)^(1/2)+2/3*(b*x^3+a*x)^(1/2)/b^2-5/6/b^3*a*(-a*b
)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2)
)*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*Ellipti
cF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(b*x^3 + a*x)^(3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}x^3}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^3/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**5/(x*(a + b*x**2))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*x^3 + a*x)^(3/2), x)
```

## 3.68 $\int \frac{x^4}{\left(ax+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=253

$$\frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{7/4}\sqrt{ax+bx^3}}+\frac{3x\left(a+bx^2\right)}{b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)}{b}$$

```
[Out] -(x^2/(b*Sqrt[a*x + b*x^3])) + (3*x*(a + b*x^2))/(b^(3/2)*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[a*x + b*x^3]) - (3*a^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1
/4)], 1/2])/(b^(7/4)*Sqrt[a*x + b*x^3]) + (3*a^(1/4)*Sqrt[x]*(Sqrt[a] + Sqr
t[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/
4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.213289, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2022, 2032, 329, 305, 220, 1196}

$$\frac{3x\left(a+bx^2\right)}{b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{ax+bx^3}}-\frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\right.}}}{b^{7/4}\sqrt{ax+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^2/(b*Sqrt[a*x + b*x^3])) + (3*x*(a + b*x^2))/(b^(3/2)*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[a*x + b*x^3]) - (3*a^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1
/4)], 1/2])/(b^(7/4)*Sqrt[a*x + b*x^3]) + (3*a^(1/4)*Sqrt[x]*(Sqrt[a] + Sqr
t[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/
4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

## Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

## Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

## Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

## Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\left(ax+bx^3\right)^{3/2}}\,dx &= -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{3\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{2b}\\
&= -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{\left(3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{2b\sqrt{ax+bx^3}}\\
&= -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{\left(3\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{b\sqrt{ax+bx^3}}\\
&= -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{\left(3\sqrt{a}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{b^{3/2}\sqrt{ax+bx^3}}-\frac{\left(3\sqrt{a}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1-}{\sqrt{a+}}\right.}{b^{3/2}\sqrt{ax+bx^3}}\\
&= -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{3x\left(a+bx^2\right)}{b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right.}{b^{7/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0216261, size = 57, normalized size = 0.23

$$
-\frac{2x^2\left(\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)-1\right)}{b\sqrt{x\left(a+bx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^2*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^2
)/a)]))/(b*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.016, size = 182, normalized size = 0.7

$$-\frac{x^2}{b}\frac{1}{\sqrt{\left(\frac{a}{b}+x^2\right)bx}}+\frac{3}{2\,b^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\left(-2\,\frac{\sqrt{-ab}}{b}\mathrm{EllipticE}\left(\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x)^(3/2),x)
```

```
[Out] -1/b*x^2/((1/b*a+x^2)*b*x)^(1/2)+3/2/b^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))
*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b
/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b
*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*Elliptic
F(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^3 + a*x)^(3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^3+axx^2}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**4/(x*(a + b*x**2))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(bx^3 + ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^3 + a*x)^(3/2), x)
```

## 3.69 $\int \frac{x^3}{\left(ax+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=115

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{ax+bx^3}}-\frac{x}{b\sqrt{ax+bx^3}}$$

```
[Out] -(x/(b*Sqrt[a*x + b*x^3])) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2
)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1
/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.0998051, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2022, 2011, 329, 220}

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{ax+bx^3}}-\frac{x}{b\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x/(b*Sqrt[a*x + b*x^3])) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2
)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1
/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(ax+bx^3\right)^{3/2}}\,dx &= -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{2b}\\
&= -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{2b\sqrt{ax+bx^3}}\\
&= -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{b\sqrt{ax+bx^3}}\\
&= -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{ax+bx^3}}
\end{aligned}$$

**Mathematica [C]** time = 0.0196673, size = 54, normalized size = 0.47

$$\frac{x\left(\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-1\right)}{b\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (x*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]
))/(b*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.015, size = 130, normalized size = 1.1

$$-\frac{x}{b}\frac{1}{\sqrt{\left(\frac{a}{b}+x^2\right)bx}}+\frac{1}{2\,b^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x)^(3/2),x)
```

```
[Out] -1/b*x/((1/b*a+x^2)*b*x)^(1/2)+1/2/b^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b
/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(
-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*
b)^(1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*x^3 + a*x)^(3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^3+ax}\,x}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(x*(a + b*x**2))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*x^3 + a*x)^(3/2), x)
```

## 3.70 $\int \frac{x^2}{\left(ax+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=254

$$-\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+bx^3}}+\frac{}{a\sqrt{a}}$$

```
[Out] x^2/(a*Sqrt[a*x + b*x^3]) - (x*(a + b*x^2))/(a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x
)*Sqrt[a*x + b*x^3]) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(
a^(3/4)*b^(3/4)*Sqrt[a*x + b*x^3]) - (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(
1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.209029, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2023, 2032, 329, 305, 220, 1196}

$$-\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+bx^3}}+\frac{x^2}{a\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3)^(3/2),x]
```

```
[Out] x^2/(a*Sqrt[a*x + b*x^3]) - (x*(a + b*x^2))/(a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x
)*Sqrt[a*x + b*x^3]) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(
a^(3/4)*b^(3/4)*Sqrt[a*x + b*x^3]) - (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(
1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(ax+bx^3\right)^{3/2}}\,dx &= \frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{2a}\\
&= \frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{2a\sqrt{ax+bx^3}}\\
&= \frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax+bx^3}}\\
&= \frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{a}\sqrt{b}\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\Bigg(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,dx}{\sqrt{a}\sqrt{b}\sqrt{ax+bx^3}}\\
&= \frac{x^2}{a\sqrt{ax+bx^3}}-\frac{x\left(a+bx^2\right)}{a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\Bigg(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}}{a^{3/4}b^{3/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0152682, size = 56, normalized size = 0.22

$$\frac{2x^2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)}{3a\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^2)/a)])/
(3*a*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.015, size = 184, normalized size = 0.7

$$\frac{x^2}{a}\frac{1}{\sqrt{\left(\frac{a}{b}+x^2\right)bx}}-\frac{1}{2\,ab}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\left(-2\frac{\sqrt{-ab}}{b}\mathrm{EllipticE}\left(\sqrt{\frac{b}{\sqrt{-a}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x)^(3/2),x)
```

```
[Out] x^2/a/((1/b*a+x^2)*b*x)^(1/2)-1/2/a/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/
(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-
a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-
a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF((
(x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^3 + a*x)^(3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^3+ax}}{b^2x^4+2\,abx^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x)**(3/2),x)

[Out] Integral(x**2/(x*(a + b*x**2))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="giac")

[Out] integrate(x^2/(b*x^3 + a*x)^(3/2), x)
```

## 3.71 $\int \frac{x}{\left(ax+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=114

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{ax+bx^3}}+\frac{x}{a\sqrt{ax+bx^3}}$$

```
[Out] x/(a*Sqrt[a*x + b*x^3]) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2]
)/(2*a^(5/4)*b^(1/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.083586, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2023, 2011, 329, 220}

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{ax+bx^3}}+\frac{x}{a\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3)^(3/2),x]
```

```
[Out] x/(a*Sqrt[a*x + b*x^3]) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2]
)/(2*a^(5/4)*b^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
```

```
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(ax+bx^3\right)^{3/2}}\,dx &= \frac{x}{a\sqrt{ax+bx^3}}+\frac{\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{2a}\\
&= \frac{x}{a\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{2a\sqrt{ax+bx^3}}\\
&= \frac{x}{a\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax+bx^3}}\\
&= \frac{x}{a\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0192975, size = 54, normalized size = 0.47

$$
\frac{x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+x}{a\sqrt{x\left(a+bx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (x + x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/
(a*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.016, size = 132, normalized size = 1.2

$$
\frac{x}{a}\frac{1}{\sqrt{\left(\frac{a}{b}+x^2\right)bx}}+\frac{1}{2\,ab}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{b\left(x+\frac{1}{b}\sqrt{-a}\right.}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x)^(3/2),x)
```

```
[Out] x/a/((1/b*a+x^2)*b*x)^(1/2)+1/2/a/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-
a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*
b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^
(1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{x}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^3 + a*x)^(3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}}{b^2x^5 + 2\,abx^3 + a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b^2*x^5 + 2*a*b*x^3 + a^2*x), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x\left(a + bx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x/(x*(a + b*x**2))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3 + ax\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*x^3 + a*x)^(3/2), x)
```

## 3.72 $\int \frac{1}{\left(ax+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=273

$$\frac{3\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{7/4}\sqrt{ax+bx^3}}+\frac{3\sqrt{b}x\left(a+bx^2\right)}{a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{3\sqrt{ax+bx^3}}{a^2x}-3\sqrt[4]{b}\sqrt{x}$$

```
[Out] 1/(a*Sqrt[a*x + b*x^3]) + (3*Sqrt[b]*x*(a + b*x^2))/(a^2*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[a*x + b*x^3]) - (3*Sqrt[a*x + b*x^3])/(a^2*x) - (3*b^(1/4)*Sqrt[x]
*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[
2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(7/4)*Sqrt[a*x + b*x^3]) + (3
*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*
x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[
a*x + b*x^3])
```

---

**Rubi [A]** time = 0.223823, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2006, 2025, 2032, 329, 305, 220, 1196}

$$\frac{3\sqrt{b}x\left(a+bx^2\right)}{a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{3\sqrt{ax+bx^3}}{a^2x}+\frac{3\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{ax+bx^3}}-3\sqrt[4]{b}\sqrt{x}(\sqrt{a}+$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(-3/2),x]
```

```
[Out] 1/(a*Sqrt[a*x + b*x^3]) + (3*Sqrt[b]*x*(a + b*x^2))/(a^2*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[a*x + b*x^3]) - (3*Sqrt[a*x + b*x^3])/(a^2*x) - (3*b^(1/4)*Sqrt[x]
*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[
2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(7/4)*Sqrt[a*x + b*x^3]) + (3
*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*
x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[
a*x + b*x^3])
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(ax+bx^3\right)^{3/2}}\,dx &= \frac{1}{a\sqrt{ax+bx^3}}+\frac{3\int \frac{1}{x\sqrt{ax+bx^3}}\,dx}{2a}\\
&= \frac{1}{a\sqrt{ax+bx^3}}-\frac{3\sqrt{ax+bx^3}}{a^2x}+\frac{(3b)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{2a^2}\\
&= \frac{1}{a\sqrt{ax+bx^3}}-\frac{3\sqrt{ax+bx^3}}{a^2x}+\frac{\left(3b\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{2a^2\sqrt{ax+bx^3}}\\
&= \frac{1}{a\sqrt{ax+bx^3}}-\frac{3\sqrt{ax+bx^3}}{a^2x}+\frac{\left(3b\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{a^2\sqrt{ax+bx^3}}\\
&= \frac{1}{a\sqrt{ax+bx^3}}-\frac{3\sqrt{ax+bx^3}}{a^2x}+\frac{\left(3\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{a^{3/2}\sqrt{ax+bx^3}}-\frac{\left(3\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)}{a^{3/}}\\
&= \frac{1}{a\sqrt{ax+bx^3}}+\frac{3\sqrt{b}x\left(a+bx^2\right)}{a^2\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{3\sqrt{ax+bx^3}}{a^2x}-\frac{3\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\,\mathrm{ta}\right.}{a^{7/4}\sqrt{ax+bx^3}}
\end{aligned}$$

**Mathematica [C]** time = 0.0127408, size = 51, normalized size = 0.19

$$-\frac{2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{bx^2}{a}\right)}{a\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^2)/a)])/(a
*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.018, size = 206, normalized size = 0.8

$$-2\,\frac{bx^2+a}{a^2\sqrt{x\left(bx^2+a\right)}}-\frac{bx^2}{a^2}\frac{1}{\sqrt{\left(\frac{a}{b}+x^2\right)bx}}+\frac{3}{2\,a^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\Bigl(-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x)^(3/2),x)
```

```
[Out] -2*(b*x^2+a)/a^2/(x*(b*x^2+a))^(1/2)-b*x^2/a^2/((1/b*a+x^2)*b*x)^(1/2)+3/2/
a^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a
*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2
)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),
1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2
))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(-3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{b^2x^6+2\,abx^4+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**3)**(-3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3 + ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(-3/2), x)
```

## 3.73 $\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=139

$$-\frac{5b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{9/4}\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}+\frac{1}{ax\sqrt{ax+bx^3}}$$

```
[Out] 1/(a*x*Sqrt[a*x + b*x^3]) - (5*Sqrt[a*x + b*x^3])/(3*a^2*x^2) - (5*b^(3/4)*
Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(6*a^(9/4)*Sqrt[a*x + b*x
^3])
```

---

**Rubi [A]** time = 0.14217, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2023, 2025, 2011, 329, 220}

$$-\frac{5b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{9/4}\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}+\frac{1}{ax\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3)^(3/2)),x]
```

```
[Out] 1/(a*x*Sqrt[a*x + b*x^3]) - (5*Sqrt[a*x + b*x^3])/(3*a^2*x^2) - (5*b^(3/4)*
Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(6*a^(9/4)*Sqrt[a*x + b*x
^3])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx &= \frac{1}{ax\sqrt{ax+bx^3}}+\frac{5\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx}{2a}\\
&= \frac{1}{ax\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}-\frac{(5b)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{6a^2}\\
&= \frac{1}{ax\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}-\frac{\left(5b\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{6a^2\sqrt{ax+bx^3}}\\
&= \frac{1}{ax\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}-\frac{\left(5b\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3a^2\sqrt{ax+bx^3}}\\
&= \frac{1}{ax\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}-\frac{5b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{9/4}\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0168496, size = 56, normalized size = 0.4

$$
-\frac{2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3ax\sqrt{x\left(a+bx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^2)/a)])/(3
*a*x*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.018, size = 150, normalized size = 1.1

$$
-\frac{2}{3\,a^2x^2}\sqrt{bx^3+ax}-\frac{bx}{a^2}\frac{1}{\sqrt{\left(\frac{a}{b}+x^2\right)bx}}-\frac{5}{6\,a^2}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-ab}}\left(x-\frac{\sqrt{-ab}}{b}\right)}\sqrt{-bx\frac{1}{\sqrt{-ab}}}\text{Elliptic}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x)^(3/2),x)
```

```
[Out] -2/3*(b*x^3+a*x)^(1/2)/a^2/x^2-b*x/a^2/((1/b*a+x^2)*b*x)^(1/2)-5/6/a^2*(-a*
b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2
))*b/(-a*b)^(1/2))^(1/2)*(-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*Ellipt
icF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^3+ax}}{b^2x^7+2\,abx^5+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b^2*x^7 + 2*a*b*x^5 + a^2*x^3), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(x*(a + b*x**2))**(3/2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x), x)
```

## 3.74 $\int \frac{1}{x^2\left(ax+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=306

$$-\frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10a^{11/4}\sqrt{ax+bx^3}}-\frac{21b^{3/2}x\left(a+bx^2\right)}{5a^3\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)}{5}$$

```
[Out] 1/(a*x^2*Sqrt[a*x + b*x^3]) - (21*b^(3/2)*x*(a + b*x^2))/(5*a^3*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (7*Sqrt[a*x + b*x^3])/(5*a^2*x^3) + (21*b*S
qrt[a*x + b*x^3])/(5*a^3*x) + (21*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])
/a^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[a*x + b*x^3]) - (21*b^(5/4)*Sqrt[x]*(Sqrt
[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcT
an[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(10*a^(11/4)*Sqrt[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.322572, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2023, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{21b^{3/2}x\left(a+bx^2\right)}{5a^3\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}-\frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10a^{11/4}\sqrt{ax+bx^3}}+\frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)}{5a^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3)^(3/2)),x]
```

```
[Out] 1/(a*x^2*Sqrt[a*x + b*x^3]) - (21*b^(3/2)*x*(a + b*x^2))/(5*a^3*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (7*Sqrt[a*x + b*x^3])/(5*a^2*x^3) + (21*b*S
qrt[a*x + b*x^3])/(5*a^3*x) + (21*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])
/a^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[a*x + b*x^3]) - (21*b^(5/4)*Sqrt[x]*(Sqrt
[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcT
an[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(10*a^(11/4)*Sqrt[a*x + b*x^3])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

### Rubi steps

$$\int \frac{1}{x^2\left(ax+bx^3\right)^{3/2}}\,dx = \frac{1}{ax^2\sqrt{ax+bx^3}} + \frac{7\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx}{2a}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} - \frac{(21b)\int \frac{1}{x\sqrt{ax+bx^3}}\,dx}{10a^2}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{\left(21b^2\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{10a^3}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{\left(21b^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{10a^3\sqrt{ax+bx^3}}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{\left(21b^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right.}{5a^3\sqrt{ax+bx^3}}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{\left(21b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x,\right.}{5a^{5/2}\sqrt{ax+bx^3}}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{21b^{3/2}x\left(a+bx^2\right)}{5a^3\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} + \frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\right.}{}$$

**Mathematica [C]** time = 0.0162168, size = 56, normalized size = 0.18

$$-\frac{2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5ax^2\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((b*x^2)/a)])/(
5*a*x^2*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.02, size = 228, normalized size = 0.8

$$-\frac{2}{5\,x^3a^2}\sqrt{bx^3+ax}+\frac{\left(16\,bx^2+16\,a\right)b}{5\,a^3}\frac{1}{\sqrt{x\left(bx^2+a\right)}}+\frac{b^2x^2}{a^3}\frac{1}{\sqrt{\left(\frac{a}{b}+x^2\right)bx}}-\frac{21\,b}{10\,a^3}\sqrt{-ab}\sqrt{b\left(x+\frac{1}{b}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{b}{\sqrt{-}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x)^(3/2),x)
```

```
[Out] -2/5*(b*x^3+a*x)^(1/2)/x^3/a^2+16/5*(b*x^2+a)*b/a^3/(x*(b*x^2+a))^(1/2)+b^2
*x^2/a^3/((1/b*a+x^2)*b*x)^(1/2)-21/10*b/a^3*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1
/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(
-x*b/(-a*b)^(1/2))^(1/2)/(b*x^3+a*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x
+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*Elli
```

```
pticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^3+ax}}{b^2x^8+2\,abx^6+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x*(a + b*x**2))**(3/2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x^2), x)
```

## 3.75 $\int \frac{x^{29/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=159

$$-\frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}}-\frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}}+\frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5}-\frac{9a\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{2b^{11/2}}-\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[Out] -x^(25/2)/(7*b*(a*x + b*x^3)^(7/2)) - (9*x^(19/2))/(35*b^2*(a*x + b*x^3)^(5
/2)) - (3*x^(13/2))/(5*b^3*(a*x + b*x^3)^(3/2)) - (3*x^(7/2))/(b^4*Sqrt[a*x
 + b*x^3]) + (9*Sqrt[x]*Sqrt[a*x + b*x^3])/(2*b^5) - (9*a*ArcTanh[(Sqrt[b]*
x^(3/2))/Sqrt[a*x + b*x^3]])/(2*b^(11/2))
```

---

**Rubi [A]** time = 0.248376, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2022, 2024, 2029, 206}

$$-\frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}}-\frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}}+\frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5}-\frac{9a\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{2b^{11/2}}-\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(29/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(25/2)/(7*b*(a*x + b*x^3)^(7/2)) - (9*x^(19/2))/(35*b^2*(a*x + b*x^3)^(5
/2)) - (3*x^(13/2))/(5*b^3*(a*x + b*x^3)^(3/2)) - (3*x^(7/2))/(b^4*Sqrt[a*x
 + b*x^3]) + (9*Sqrt[x]*Sqrt[a*x + b*x^3])/(2*b^5) - (9*a*ArcTanh[(Sqrt[b]*
x^(3/2))/Sqrt[a*x + b*x^3]])/(2*b^(11/2))
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{29/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} + \frac{9\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}$$

$$= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} + \frac{9\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{5b^2}$$

$$= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}} + \frac{3\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{b^3}$$

$$= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\int \frac{x^{5/2}}{\sqrt{ax+bx^3}}\,dx}{b^4}$$

$$= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5} -$$

$$= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5} -$$

$$= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax+bx^3}}{2b^5} -$$

**Mathematica [A]** time = 0.18079, size = 130, normalized size = 0.82

$$\frac{\sqrt{x}\left(\sqrt{b}x\left(1218a^2b^2x^4+1050a^3bx^2+315a^4+528ab^3x^6+35b^4x^8\right)-\frac{315\sqrt{a}\left(a+bx^2\right)^4\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{70b^{11/2}\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(29/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*(Sqrt[b]*x*(315*a^4 + 1050*a^3*b*x^2 + 1218*a^2*b^2*x^4 + 528*a*b^
3*x^6 + 35*b^4*x^8) - (315*Sqrt[a]*(a + b*x^2)^4*ArcSinh[(Sqrt[b]*x)/Sqrt[a
]])/Sqrt[1 + (b*x^2)/a]))/(70*b^(11/2)*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.04, size = 212, normalized size = 1.3

$$-\frac{1}{70\left(bx^2+a\right)^4}\sqrt{x\left(bx^2+a\right)}\left(-35\,x^9b^{9/2}+315\,\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)x^6ab^3\sqrt{bx^2+a}-528\,b^{7/2}x^7a+945\,\ln\left(x\sqrt{b}+\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(29/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/70*(x*(b*x^2+a))^(1/2)/b^(11/2)*(-35*x^9*b^(9/2)+315*ln(x*b^(1/2)+(b*x^2
+a)^(1/2))*x^6*a*b^3*(b*x^2+a)^(1/2)-528*b^(7/2)*x^7*a+945*ln(x*b^(1/2)+(b*
x^2+a)^(1/2))*x^4*a^2*b^2*(b*x^2+a)^(1/2)-1218*b^(5/2)*x^5*a^2+945*ln(x*b^(
1/2)+(b*x^2+a)^(1/2))*x^2*a^3*b*(b*x^2+a)^(1/2)-1050*b^(3/2)*x^3*a^3+315*ln
(x*b^(1/2)+(b*x^2+a)^(1/2))*a^4*(b*x^2+a)^(1/2)-315*b^(1/2)*x*a^4)/x^(1/2)/
(b*x^2+a)^4
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{29}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(29/2)/(b*x^3 + a*x)^(9/2), x)
```

---

**Fricas [A]** time = 1.58916, size = 845, normalized size = 5.31

$$\left[\frac{315\left(ab^4x^8+4a^2b^3x^6+6a^3b^2x^4+4a^4bx^2+a^5\right)\sqrt{b}\log\left(2bx^2-2\sqrt{bx^3+ax}\sqrt{b}\sqrt{x}+a\right)+2\bigl(35b^5x^8+528ab^4x^6+121}{140\left(b^{10}x^8+4ab^9x^6+6a^2b^8x^4+4a^3b^7x^2+a^4b^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)
*sqrt(b)*log(2*b*x^2 - 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) + 2*(35*b^5
*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2*x^2 + 315*a^4*b)*sqr
t(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7
*x^2 + a^4*b^6), 1/70*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a
^4*b*x^2 + a^5)*sqrt(-b)*arctan(sqrt(b*x^3 + a*x)*sqrt(-b)/(b*x^(3/2))) + (
35*b^5*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2*x^2 + 315*a^4*
b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a
^3*b^7*x^2 + a^4*b^6)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(29/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.32596, size = 123, normalized size = 0.77

$$\frac{\left(\left(\left(x^2\left(\frac{35\,x^2}{b}+\frac{528\,a}{b^2}\right)+\frac{1218\,a^2}{b^3}\right)x^2+\frac{1050\,a^3}{b^4}\right)x^2+\frac{315\,a^4}{b^5}\right)x}{70\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{9\,a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/70*(((x^2*(35*x^2/b + 528*a/b^2) + 1218*a^2/b^3)*x^2 + 1050*a^3/b^4)*x^2
+ 315*a^4/b^5)*x/(b*x^2 + a)^(7/2) + 9/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2
+ a)))/b^(11/2)
```

## 3.76 $\int \frac{x^{27/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=126

$$-\frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}}+\frac{128\sqrt{ax+bx^3}}{35b^5\sqrt{x}}-\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[Out] -x^(23/2)/(7*b*(a*x + b*x^3)^(7/2)) - (8*x^(17/2))/(35*b^2*(a*x + b*x^3)^(5
/2)) - (16*x^(11/2))/(35*b^3*(a*x + b*x^3)^(3/2)) - (64*x^(5/2))/(35*b^4*Sq
rt[a*x + b*x^3]) + (128*Sqrt[a*x + b*x^3])/(35*b^5*Sqrt[x])
```

---

**Rubi [A]** time = 0.200279, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$-\frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}}+\frac{128\sqrt{ax+bx^3}}{35b^5\sqrt{x}}-\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(27/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(23/2)/(7*b*(a*x + b*x^3)^(7/2)) - (8*x^(17/2))/(35*b^2*(a*x + b*x^3)^(5
/2)) - (16*x^(11/2))/(35*b^3*(a*x + b*x^3)^(3/2)) - (64*x^(5/2))/(35*b^4*Sq
rt[a*x + b*x^3]) + (128*Sqrt[a*x + b*x^3])/(35*b^5*Sqrt[x])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{27/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}+\frac{8\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}\\
&= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}}+\frac{48\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35b^2}\\
&= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}}+\frac{64\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{35b^3}\\
&= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}}+\frac{128\int \frac{x^{3/2}}{\sqrt{ax+bx^3}}}{35b^4}\\
&= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}}+\frac{128\sqrt{ax+bx}}{35b^5\sqrt{x}}
\end{aligned}$$

**Mathematica [A]** time = 0.0366427, size = 77, normalized size = 0.61

$$\frac{\sqrt{x}\left(560a^2b^2x^4+448a^3bx^2+128a^4+280ab^3x^6+35b^4x^8\right)}{35b^5\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(27/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*(128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3*x^6 + 35*b^
4*x^8))/(35*b^5*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.007, size = 70, normalized size = 0.6

$$\frac{\left(bx^2+a\right)\left(35\,x^8b^4+280\,ax^6b^3+560\,a^2x^4b^2+448\,a^3x^2b+128\,a^4\right)}{35\,b^5}x^{\frac{9}{2}}\left(bx^3+ax\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(27/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/35*(b*x^2+a)*(35*b^4*x^8+280*a*b^3*x^6+560*a^2*b^2*x^4+448*a^3*b*x^2+128*
a^4)*x^(9/2)/b^5/(b*x^3+a*x)^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{27}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(27/2)/(b*x^3 + a*x)^(9/2), x)
```

---

**Fricas [A]** time = 1.46616, size = 234, normalized size = 1.86

$$\frac{\left(35\,b^{4}x^{8}+280\,ab^{3}x^{6}+560\,a^{2}b^{2}x^{4}+448\,a^{3}bx^{2}+128\,a^{4}\right)\sqrt{bx^{3}+ax}\sqrt{x}}{35\left(b^{9}x^{9}+4\,ab^{8}x^{7}+6\,a^{2}b^{7}x^{5}+4\,a^{3}b^{6}x^{3}+a^{4}b^{5}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^4*x^8 + 280*a*b^3*x^6 + 560*a^2*b^2*x^4 + 448*a^3*b*x^2 + 128*a^
4)*sqrt(b*x^3 + a*x)*sqrt(x)/(b^9*x^9 + 4*a*b^8*x^7 + 6*a^2*b^7*x^5 + 4*a^3
*b^6*x^3 + a^4*b^5*x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(27/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.28052, size = 108, normalized size = 0.86

$$\frac{35\,\sqrt{bx^{2}+a}+\frac{140\left(bx^{2}+a\right)^{3}a-70\left(bx^{2}+a\right)^{2}a^{2}+28\left(bx^{2}+a\right)a^{3}-5\,a^{4}}{\left(bx^{2}+a\right)^{\frac{7}{2}}}}{35\,b^{5}}-\frac{128\,\sqrt{a}}{35\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*(35*sqrt(b*x^2 + a) + (140*(b*x^2 + a)^3*a - 70*(b*x^2 + a)^2*a^2 + 28
*(b*x^2 + a)*a^3 - 5*a^4)/(b*x^2 + a)^(7/2))/b^5 - 128/35*sqrt(a)/b^5
```

## 3.77 $\int \frac{x^{25/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=130

$$-\frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}}-\frac{x^{3/2}}{b^4\sqrt{ax+bx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}}-\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[Out] -x^(21/2)/(7*b*(a*x + b*x^3)^(7/2)) - x^(15/2)/(5*b^2*(a*x + b*x^3)^(5/2))
- x^(9/2)/(3*b^3*(a*x + b*x^3)^(3/2)) - x^(3/2)/(b^4*Sqrt[a*x + b*x^3]) + A
rcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x + b*x^3]]/b^(9/2)
```

---

**Rubi [A]** time = 0.205549, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2022, 2029, 206}

$$-\frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}}-\frac{x^{3/2}}{b^4\sqrt{ax+bx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}}-\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(25/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(21/2)/(7*b*(a*x + b*x^3)^(7/2)) - x^(15/2)/(5*b^2*(a*x + b*x^3)^(5/2))
- x^(9/2)/(3*b^3*(a*x + b*x^3)^(3/2)) - x^(3/2)/(b^4*Sqrt[a*x + b*x^3]) + A
rcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x + b*x^3]]/b^(9/2)
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{25/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} + \frac{\int \frac{x^{19/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{b} \\
&= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}} + \frac{\int \frac{x^{13/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{b^2} \\
&= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}} - \frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}} + \frac{\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{b^3} \\
&= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}} - \frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\int \frac{\sqrt{x}}{\sqrt{ax+bx^3}}\,dx}{b^4} \\
&= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}} - \frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x,\right.}{b^4} \\
&= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}} - \frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.252861, size = 120, normalized size = 0.92

$$\frac{\sqrt{x}\left(105\sqrt{a}\left(a+bx^2\right)^3\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \sqrt{b}x\left(350a^2bx^2+105a^3+406ab^2x^4+176b^3x^6\right)\right)}{105b^{9/2}\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(25/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*(-(Sqrt[b]*x*(105*a^3 + 350*a^2*b*x^2 + 406*a*b^2*x^4 + 176*b^3*x^
6)) + 105*Sqrt[a]*(a + b*x^2)^3*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqr
t[a]]))/(105*b^(9/2)*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.022, size = 198, normalized size = 1.5

$$\frac{1}{105\left(bx^2+a\right)^4}\sqrt{x\left(bx^2+a\right)}\left(105\,\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)x^6b^3\sqrt{bx^2+a} - 176\,x^7b^{7/2} + 315\,\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)x^4ab^2\sqrt{bx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(25/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/105*(x*(b*x^2+a))^(1/2)/b^(9/2)*(105*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*x^6*b^
3*(b*x^2+a)^(1/2)-176*x^7*b^(7/2)+315*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*x^4*a*b
^2*(b*x^2+a)^(1/2)-406*b^(5/2)*x^5*a+315*ln(x*b^(1/2)+(b*x^2+a)^(1/2))*x^2*
a^2*b*(b*x^2+a)^(1/2)-350*b^(3/2)*x^3*a^2+105*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
*a^3*(b*x^2+a)^(1/2)-105*b^(1/2)*x*a^3)/x^(1/2)/(b*x^2+a)^4
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{25}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(25/2)/(b*x^3 + a*x)^(9/2), x)
```

---

**Fricas [A]** time = 1.47245, size = 783, normalized size = 6.02

$$\left[\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{b}\log\left(2\,bx^2+2\,\sqrt{bx^3+ax}\sqrt{b}\sqrt{x}+a\right)-2\,(176\,b^4x^6+406\,ab^3x^4+3}{210\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqr
t(b)*log(2*b*x^2 + 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) - 2*(176*b^4*x^
6 + 406*a*b^3*x^4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(x))
/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*
(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-b)*a
rctan(sqrt(b*x^3 + a*x)*sqrt(-b)/(b*x^(3/2))) + (176*b^4*x^6 + 406*a*b^3*x^
4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^9*x^8 + 4*a*
b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(25/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.40829, size = 105, normalized size = 0.81

$$-\frac{\left(2\left(x^2\left(\frac{88\,x^2}{b}+\frac{203\,a}{b^2}\right)+\frac{175\,a^2}{b^3}\right)x^2+\frac{105\,a^3}{b^4}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(2*(x^2*(88*x^2/b + 203*a/b^2) + 175*a^2/b^3)*x^2 + 105*a^3/b^4)*x/(
b*x^2 + a)^(7/2) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

## 3.78 $\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=101

$$-\frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}-\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[Out] -x^(19/2)/(7*b*(a*x + b*x^3)^(7/2)) - (6*x^(13/2))/(35*b^2*(a*x + b*x^3)^(5
/2)) - (8*x^(7/2))/(35*b^3*(a*x + b*x^3)^(3/2)) - (16*Sqrt[x])/(35*b^4*Sqrt
[a*x + b*x^3])
```

---

**Rubi [A]** time = 0.161451, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$-\frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}-\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(23/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(19/2)/(7*b*(a*x + b*x^3)^(7/2)) - (6*x^(13/2))/(35*b^2*(a*x + b*x^3)^(5
/2)) - (8*x^(7/2))/(35*b^3*(a*x + b*x^3)^(3/2)) - (16*Sqrt[x])/(35*b^4*Sqrt
[a*x + b*x^3])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}} + \frac{6\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b} \\ &= -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}} + \frac{24\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35b^2} \\ &= -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}} + \frac{16\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{35b^3} \\ &= -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}} - \frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}\end{aligned}$$

**Mathematica [A]** time = 0.0302392, size = 66, normalized size = 0.65

$$-\frac{\sqrt{x}\left(56a^2bx^2+16a^3+70ab^2x^4+35b^3x^6\right)}{35b^4\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(23/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -(Sqrt[x]*(16*a^3 + 56*a^2*b*x^2 + 70*a*b^2*x^4 + 35*b^3*x^6))/(35*b^4*(a +
 b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.005, size = 59, normalized size = 0.6

$$-\frac{\left(bx^2+a\right)\left(35\,x^6b^3+70\,ax^4b^2+56\,a^2x^2b+16\,a^3\right)}{35\,b^4}x^{\frac{9}{2}}\left(bx^3+ax\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/35*(b*x^2+a)*(35*b^3*x^6+70*a*b^2*x^4+56*a^2*b*x^2+16*a^3)*x^(9/2)/b^4/(
b*x^3+a*x)^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{23}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(23/2)/(b*x^3 + a*x)^(9/2), x)
```

---

**Fricas [A]** time = 1.4131, size = 207, normalized size = 2.05

$$-\frac{\left(35\,b^{3}x^{6}+70\,ab^{2}x^{4}+56\,a^{2}bx^{2}+16\,a^{3}\right)\sqrt{bx^{3}+ax}\sqrt{x}}{35\left(b^{8}x^{9}+4\,ab^{7}x^{7}+6\,a^{2}b^{6}x^{5}+4\,a^{3}b^{5}x^{3}+a^{4}b^{4}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 70*a*b^2*x^4 + 56*a^2*b*x^2 + 16*a^3)*sqrt(b*x^3 + a*x)
*sqrt(x)/(b^8*x^9 + 4*a*b^7*x^7 + 6*a^2*b^6*x^5 + 4*a^3*b^5*x^3 + a^4*b^4*x
)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(23/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.27062, size = 86, normalized size = 0.85

$$\frac{16}{35\sqrt{a}b^{4}}-\frac{35\left(bx^{2}+a\right)^{3}-35\left(bx^{2}+a\right)^{2}a+21\left(bx^{2}+a\right)a^{2}-5\,a^{3}}{35\left(bx^{2}+a\right)^{\frac{7}{2}}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 16/35/(sqrt(a)*b^4) - 1/35*(35*(b*x^2 + a)^3 - 35*(b*x^2 + a)^2*a + 21*(b*x
^2 + a)*a^2 - 5*a^3)/((b*x^2 + a)^(7/2)*b^4)
```

## 3.79 $\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=25

$$\frac{x^{21/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] x^(21/2)/(7*a*(a*x + b*x^3)^(7/2))
```

---

**Rubi [A]** time = 0.0369776, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{x^{21/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(21/2)/(a*x + b*x^3)^(9/2),x]

[Out] x^(21/2)/(7*a*(a*x + b*x^3)^(7/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=\frac{x^{21/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

**Mathematica [A]** time = 0.0168311, size = 25, normalized size = 1.

$$\frac{x^{21/2}}{7a\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(21/2)/(a*x + b*x^3)^(9/2),x]

[Out] x^(21/2)/(7*a*(x*(a + b*x^2))^(7/2))
```

---

**Maple [A]** time = 0.004, size = 27, normalized size = 1.1

$$\frac{bx^2+a}{7\,a}x^{\frac{23}{2}}\left(bx^3+ax\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/7*(b*x^2+a)/a*x^(23/2)/(b*x^3+a*x)^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{21}{2}}}{\left(bx^3 + ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(21/2)/(b*x^3 + a*x)^(9/2), x)
```

---

**Fricas [B]** time = 1.42907, size = 130, normalized size = 5.2

$$\frac{\sqrt{bx^3 + ax}\,x^{\frac{13}{2}}}{7\left(ab^4x^8 + 4\,a^2b^3x^6 + 6\,a^3b^2x^4 + 4\,a^4bx^2 + a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(b*x^3 + a*x)*x^(13/2)/(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 +
 4*a^4*b*x^2 + a^5)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(21/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.2361, size = 23, normalized size = 0.92

$$\frac{x^7}{7\left(bx^2 + a\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/7*x^7/((b*x^2 + a)^(7/2)*a)
```

## 3.80 $\int \frac{x^{19/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=76

$$-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{3/2}}{105b^3\left(ax+bx^3\right)^{3/2}}-\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[Out] -x^(15/2)/(7*b*(a*x + b*x^3)^(7/2)) - (4*x^(9/2))/(35*b^2*(a*x + b*x^3)^(5/
2)) - (8*x^(3/2))/(105*b^3*(a*x + b*x^3)^(3/2))
```

---

**Rubi [A]** time = 0.118776, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{3/2}}{105b^3\left(ax+bx^3\right)^{3/2}}-\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(19/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(15/2)/(7*b*(a*x + b*x^3)^(7/2)) - (4*x^(9/2))/(35*b^2*(a*x + b*x^3)^(5/
2)) - (8*x^(3/2))/(105*b^3*(a*x + b*x^3)^(3/2))
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{19/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= -\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}+\frac{4\int \frac{x^{13/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}\\
&= -\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}+\frac{8\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35b^2}\\
&= -\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{3/2}}{105b^3\left(ax+bx^3\right)^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0256023, size = 55, normalized size = 0.72

$$-\frac{\sqrt{x}\left(8a^2+28abx^2+35b^2x^4\right)}{105b^3\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(19/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -(Sqrt[x]*(8*a^2 + 28*a*b*x^2 + 35*b^2*x^4))/(105*b^3*(a + b*x^2)^3*Sqrt[x*
(a + b*x^2)])
```

---

**Maple [A]** time = 0.005, size = 48, normalized size = 0.6

$$-\frac{\left(bx^2+a\right)\left(35\,x^4b^2+28\,ax^2b+8\,a^2\right)}{105\,b^3}x^{\frac{9}{2}}\left(bx^3+ax\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/105*(b*x^2+a)*(35*b^2*x^4+28*a*b*x^2+8*a^2)*x^(9/2)/b^3/(b*x^3+a*x)^(9/2
)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{19}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(19/2)/(b*x^3 + a*x)^(9/2), x)
```

---

**Fricas [A]** time = 1.42644, size = 184, normalized size = 2.42

$$-\frac{\left(35\,b^2x^4+28\,abx^2+8\,a^2\right)\sqrt{bx^3+ax}\sqrt{x}}{105\left(b^7x^9+4\,ab^6x^7+6\,a^2b^5x^5+4\,a^3b^4x^3+a^4b^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^2*x^4 + 28*a*b*x^2 + 8*a^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(b^7*x^9
 + 4*a*b^6*x^7 + 6*a^2*b^5*x^5 + 4*a^3*b^4*x^3 + a^4*b^3*x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.24878, size = 68, normalized size = 0.89

$$\frac{8}{105\,a^{\frac{3}{2}}b^3} - \frac{35\left(bx^2+a\right)^2 - 42\left(bx^2+a\right)a + 15\,a^2}{105\left(bx^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 8/105/(a^(3/2)*b^3) - 1/105*(35*(b*x^2 + a)^2 - 42*(b*x^2 + a)*a + 15*a^2)/
((b*x^2 + a)^(7/2)*b^3)
```

## 3.81 $\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=51

$$\frac{2x^{15/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] x^(17/2)/(7*a*(a*x + b*x^3)^(7/2)) + (2*x^(15/2))/(35*a^2*(a*x + b*x^3)^(5/
2))
```

---

**Rubi [A]** time = 0.0747788, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$\frac{2x^{15/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(17/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(17/2)/(7*a*(a*x + b*x^3)^(7/2)) + (2*x^(15/2))/(35*a^2*(a*x + b*x^3)^(5/
2))
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= \frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{2\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}\\ &= \frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{2x^{15/2}}{35a^2\left(ax+bx^3\right)^{5/2}}\end{aligned}$$

**Mathematica [A]** time = 0.0210368, size = 44, normalized size = 0.86

$$\frac{x^{9/2}\sqrt{x\left(a+bx^2\right)}\left(7a+2bx^2\right)}{35a^2\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(17/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(9/2)*Sqrt[x*(a + b*x^2)]*(7*a + 2*b*x^2))/(35*a^2*(a + b*x^2)^4)
```

---

**Maple [A]** time = 0.004, size = 37, normalized size = 0.7

$$\frac{\left(bx^2+a\right)\left(2\,bx^2+7\,a\right)}{35\,a^2}x^{\frac{19}{2}}\left(bx^3+ax\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/35*(b*x^2+a)*x^(19/2)*(2*b*x^2+7*a)/a^2/(b*x^3+a*x)^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{17}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(17/2)/(b*x^3 + a*x)^(9/2), x)
```

---

**Fricas [A]** time = 1.43253, size = 159, normalized size = 3.12

$$\frac{\left(2\,bx^6+7\,ax^4\right)\sqrt{bx^3+ax}\sqrt{x}}{35\left(a^2b^4x^8+4\,a^3b^3x^6+6\,a^4b^2x^4+4\,a^5bx^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(2*b*x^6 + 7*a*x^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^2*b^4*x^8 + 4*a^3*b^3
*x^6 + 6*a^4*b^2*x^4 + 4*a^5*b*x^2 + a^6)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.33704, size = 39, normalized size = 0.76

$$\frac{x^5\left(\frac{2\,bx^2}{a^2}+\frac{7}{a}\right)}{35\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*x^5*(2*b*x^2/a^2 + 7/a)/(b*x^2 + a)^(7/2)
```

## 3.82 $\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=51

$$-\frac{2x^{5/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[Out] -x^(11/2)/(7*b*(a*x + b*x^3)^(7/2)) - (2*x^(5/2))/(35*b^2*(a*x + b*x^3)^(5/
2))
```

---

**Rubi [A]** time = 0.0736555, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$-\frac{2x^{5/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(11/2)/(7*b*(a*x + b*x^3)^(7/2)) - (2*x^(5/2))/(35*b^2*(a*x + b*x^3)^(5/
2))
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= -\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}+\frac{2\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}\\ &= -\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{2x^{5/2}}{35b^2\left(ax+bx^3\right)^{5/2}}\end{aligned}$$

**Mathematica [A]** time = 0.0195845, size = 44, normalized size = 0.86

$$-\frac{\sqrt{x}\left(2a+7bx^2\right)}{35b^2\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -(Sqrt[x]*(2*a + 7*b*x^2))/(35*b^2*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

---

**Maple [A]** time = 0.006, size = 37, normalized size = 0.7

$$-\frac{\left(bx^{2}+a\right)\left(7\,bx^{2}+2\,a\right)}{35\,b^{2}}x^{\frac{9}{2}}\left(bx^{3}+ax\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/35*(b*x^2+a)*(7*b*x^2+2*a)*x^(9/2)/b^2/(b*x^3+a*x)^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{15}{2}}}{\left(bx^{3}+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(15/2)/(b*x^3 + a*x)^(9/2), x)
```

---

**Fricas [A]** time = 1.59191, size = 158, normalized size = 3.1

$$-\frac{\sqrt{bx^{3}+ax}\left(7\,bx^{2}+2\,a\right)\sqrt{x}}{35\left(b^{6}x^{9}+4\,ab^{5}x^{7}+6\,a^{2}b^{4}x^{5}+4\,a^{3}b^{3}x^{3}+a^{4}b^{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*sqrt(b*x^3 + a*x)*(7*b*x^2 + 2*a)*sqrt(x)/(b^6*x^9 + 4*a*b^5*x^7 + 6*
a^2*b^4*x^5 + 4*a^3*b^3*x^3 + a^4*b^2*x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.26349, size = 45, normalized size = 0.88

$$-\frac{7\,bx^{2}+2\,a}{35\,\left(bx^{2}+a\right)^{\frac{7}{2}}b^{2}}+\frac{2}{35\,a^{\frac{5}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(7*b*x^2 + 2*a)/((b*x^2 + a)^(7/2)*b^2) + 2/35/(a^(5/2)*b^2)
```

## 3.83 $\int \frac{x^{13/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=76

$$\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{9/2}}{105a^3\left(ax+bx^3\right)^{3/2}}+\frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] x^(13/2)/(7*a*(a*x + b*x^3)^(7/2)) + (4*x^(11/2))/(35*a^2*(a*x + b*x^3)^(5/
2)) + (8*x^(9/2))/(105*a^3*(a*x + b*x^3)^(3/2))
```

---

**Rubi [A]** time = 0.112928, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{9/2}}{105a^3\left(ax+bx^3\right)^{3/2}}+\frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(13/2)/(7*a*(a*x + b*x^3)^(7/2)) + (4*x^(11/2))/(35*a^2*(a*x + b*x^3)^(5/
2)) + (8*x^(9/2))/(105*a^3*(a*x + b*x^3)^(3/2))
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{13/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= \frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{4\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}\\ &= \frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35a^2}\\ &= \frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{9/2}}{105a^3\left(ax+bx^3\right)^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.020886, size = 55, normalized size = 0.72

$$\frac{x^{5/2}\sqrt{x\left(a+bx^2\right)}\left(35a^2+28abx^2+8b^2x^4\right)}{105a^3\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(5/2)*Sqrt[x*(a + b*x^2)]*(35*a^2 + 28*a*b*x^2 + 8*b^2*x^4))/(105*a^3*(a
 + b*x^2)^4)
```

---

**Maple [A]** time = 0.007, size = 48, normalized size = 0.6

$$\frac{\left(bx^2+a\right)\left(8\,b^2x^4+28\,abx^2+35\,a^2\right)}{105\,a^3}x^{\frac{15}{2}}\left(bx^3+ax\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/105*(b*x^2+a)*x^(15/2)*(8*b^2*x^4+28*a*b*x^2+35*a^2)/a^3/(b*x^3+a*x)^(9/2
)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(13/2)/(b*x^3 + a*x)^(9/2), x)
```

---

**Fricas [A]** time = 1.65983, size = 185, normalized size = 2.43

$$\frac{\left(8\,b^2x^6+28\,abx^4+35\,a^2x^2\right)\sqrt{bx^3+ax}\sqrt{x}}{105\left(a^3b^4x^8+4\,a^4b^3x^6+6\,a^5b^2x^4+4\,a^6bx^2+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*b^2*x^6 + 28*a*b*x^4 + 35*a^2*x^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^3*
b^4*x^8 + 4*a^4*b^3*x^6 + 6*a^5*b^2*x^4 + 4*a^6*b*x^2 + a^7)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.27394, size = 58, normalized size = 0.76

$$\frac{\left(4\,x^2\left(\frac{2\,b^2x^2}{a^3}+\frac{7\,b}{a^2}\right)+\frac{35}{a}\right)x^3}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(4*x^2*(2*b^2*x^2/a^3 + 7*b/a^2) + 35/a)*x^3/(b*x^2 + a)^(7/2)
```

## 3.84 $\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=25

$$-\frac{x^{7/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[Out] -x^(7/2)/(7*b*(a*x + b*x^3)^(7/2))
```

---

**Rubi [A]** time = 0.0370535, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$-\frac{x^{7/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(7/2)/(7*b*(a*x + b*x^3)^(7/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{9/2}}\,dx=-\frac{x^{7/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

**Mathematica [A]** time = 0.0135167, size = 25, normalized size = 1.

$$-\frac{x^{7/2}}{7b\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(7/2)/(7*b*(x*(a + b*x^2))^(7/2))
```

---

**Maple [A]** time = 0.003, size = 27, normalized size = 1.1

$$-\frac{bx^2+a}{7b}x^{\frac{9}{2}}\left(bx^3+ax\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/7*(b*x^2+a)/b*x^(9/2)/(b*x^3+a*x)^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{11}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(11/2)/(b*x^3 + a*x)^(9/2), x)
```

---

**Fricas [B]** time = 1.70886, size = 132, normalized size = 5.28

$$-\frac{\sqrt{bx^3+ax}\sqrt{x}}{7\left(b^5x^9+4\,ab^4x^7+6\,a^2b^3x^5+4\,a^3b^2x^3+a^4bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/7*sqrt(b*x^3 + a*x)*sqrt(x)/(b^5*x^9 + 4*a*b^4*x^7 + 6*a^2*b^3*x^5 + 4*a
^3*b^2*x^3 + a^4*b*x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.27413, size = 31, normalized size = 1.24

$$-\frac{1}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{1}{7\,a^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/7/((b*x^2 + a)^(7/2)*b) + 1/7/(a^(7/2)*b)
```

## 3.85 $\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=101

$$\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}+\frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] x^(9/2)/(7*a*(a*x + b*x^3)^(7/2)) + (6*x^(7/2))/(35*a^2*(a*x + b*x^3)^(5/2)
) + (8*x^(5/2))/(35*a^3*(a*x + b*x^3)^(3/2)) + (16*x^(3/2))/(35*a^4*Sqrt[a*
x + b*x^3])
```

---

**Rubi [A]** time = 0.157272, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}+\frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(9/2)/(7*a*(a*x + b*x^3)^(7/2)) + (6*x^(7/2))/(35*a^2*(a*x + b*x^3)^(5/2)
) + (8*x^(5/2))/(35*a^3*(a*x + b*x^3)^(3/2)) + (16*x^(3/2))/(35*a^4*Sqrt[a*
x + b*x^3])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{6\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a} \\
&= \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{24\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35a^2} \\
&= \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}} + \frac{16\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{35a^3} \\
&= \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}} + \frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}
\end{aligned}
$$

**Mathematica [A]** time = 0.0231574, size = 66, normalized size = 0.65

$$\frac{\sqrt{x}\sqrt{x\left(a+bx^2\right)}\left(70a^2bx^2+35a^3+56ab^2x^4+16b^3x^6\right)}{35a^4\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[x*(a + b*x^2)]*(35*a^3 + 70*a^2*b*x^2 + 56*a*b^2*x^4 + 16*b^3
*x^6))/(35*a^4*(a + b*x^2)^4)
```

---

**Maple [A]** time = 0.005, size = 59, normalized size = 0.6

$$\frac{\left(bx^2+a\right)\left(16\,b^3x^6+56\,b^2x^4a+70\,bx^2a^2+35\,a^3\right)}{35\,a^4}x^{\frac{11}{2}}\left(bx^3+ax\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/35*(b*x^2+a)*x^(11/2)*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/a^4/(
b*x^3+a*x)^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{9}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(9/2)/(b*x^3 + a*x)^(9/2), x)
```

---

**Fricas [A]** time = 1.50133, size = 203, normalized size = 2.01

$$\frac{\left(16\,b^3x^6+56\,ab^2x^4+70\,a^2bx^2+35\,a^3\right)\sqrt{bx^3+ax}\sqrt{x}}{35\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*b^3*x^6 + 56*a*b^2*x^4 + 70*a^2*b*x^2 + 35*a^3)*sqrt(b*x^3 + a*x)*
sqrt(x)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.31053, size = 74, normalized size = 0.73

$$\frac{\left(2\left(4\,x^2\left(\frac{2\,b^3x^2}{a^4}+\frac{7\,b^2}{a^3}\right)+\frac{35\,b}{a^2}\right)x^2+\frac{35}{a}\right)x}{35\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*(2*(4*x^2*(2*b^3*x^2/a^4 + 7*b^2/a^3) + 35*b/a^2)*x^2 + 35/a)*x/(b*x^2
 + a)^(7/2)
```

## 3.86 $\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{9/2}} \, dx$

**Optimal**. Leaf size=130

$$\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}}+\frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] x^(7/2)/(7*a*(a*x + b*x^3)^(7/2)) + x^(5/2)/(5*a^2*(a*x + b*x^3)^(5/2)) + x
^(3/2)/(3*a^3*(a*x + b*x^3)^(3/2)) + Sqrt[x]/(a^4*Sqrt[a*x + b*x^3]) - ArcT
anh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]]/a^(9/2)
```

---

**Rubi [A]** time = 0.202778, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2023, 2029, 206}

$$\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}}+\frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(7/2)/(7*a*(a*x + b*x^3)^(7/2)) + x^(5/2)/(5*a^2*(a*x + b*x^3)^(5/2)) + x
^(3/2)/(3*a^3*(a*x + b*x^3)^(3/2)) + Sqrt[x]/(a^4*Sqrt[a*x + b*x^3]) - ArcT
anh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]]/a^(9/2)
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{a}$$

$$= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}} + \frac{\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{a^2}$$

$$= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}} + \frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}} + \frac{\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{3/2}}\,dx}{a^3}$$

$$= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}} + \frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3}}\,dx}{a^4}$$

$$= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}} + \frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} - \frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, -\right.}{a^4}$$

$$= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}} + \frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}}$$

**Mathematica [C]** time = 0.0117814, size = 43, normalized size = 0.33

$$\frac{x^{7/2}\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};\frac{bx^2}{a}+1\right)}{7a\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(7/2)*Hypergeometric2F1[-7/2, 1, -5/2, 1 + (b*x^2)/a])/(7*a*(x*(a + b*x^
2))^(7/2))
```

---

**Maple [B]** time = 0.012, size = 217, normalized size = 1.7

$$-\frac{1}{105\left(bx^2+a\right)^4}\sqrt{x\left(bx^2+a\right)}\left(105\ln\left(2\frac{\sqrt{a}\sqrt{bx^2+a}+a}{x}\right)x^6b^3\sqrt{bx^2+a} - 105\sqrt{a}x^6b^3 + 315\ln\left(2\frac{\sqrt{a}\sqrt{bx^2+a}+a}{x}\right)x^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/105*(x*(b*x^2+a))^(1/2)/a^(9/2)*(105*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)
*x^6*b^3*(b*x^2+a)^(1/2)-105*a^(1/2)*x^6*b^3+315*ln(2*(a^(1/2)*(b*x^2+a)^(1
/2)+a)/x)*x^4*a*b^2*(b*x^2+a)^(1/2)-350*a^(3/2)*x^4*b^2+315*ln(2*(a^(1/2)*(
b*x^2+a)^(1/2)+a)/x)*x^2*a^2*b*(b*x^2+a)^(1/2)-406*a^(5/2)*x^2*b+105*ln(2*(
a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a^3*(b*x^2+a)^(1/2)-176*a^(7/2))/x^(1/2)/(b*x
^2+a)^4
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/(b*x^3 + a*x)^(9/2), x)
```

---

**Fricas [A]** time = 1.29511, size = 803, normalized size = 6.18

$$\left[\frac{105\left(b^4x^9+4\,ab^3x^7+6\,a^2b^2x^5+4\,a^3bx^3+a^4x\right)\sqrt{a}\log\left(\frac{bx^3+2\,ax-2\,\sqrt{bx^3+ax}\sqrt{a}\sqrt{x}}{x^3}\right)+2\left(105\,ab^3x^6+350\,a^2b^2x^4+406\,a^3\right.}{210\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x)*s
qrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))/x^3) + 2*(
105*a*b^3*x^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 + a*x
)*sqrt(x))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9
*x), 1/105*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*
x)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x)*sqrt(-a)/(a*sqrt(x))) + (105*a*b^3*x^6
 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 + a*x)*sqrt(x))/(a
^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.28796, size = 154, normalized size = 1.18

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}-\frac{105\,\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+176\,\sqrt{-a}}{105\,\sqrt{-a}a^{\frac{9}{2}}}+\frac{105\left(bx^2+a\right)^3+35\left(bx^2+a\right)^2a+21\left(bx^2+a\right)a^2+15\,a^3}{105\left(bx^2+a\right)^{\frac{7}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) - 1/105*(105*sqrt(a)*arctan
(sqrt(a)/sqrt(-a)) + 176*sqrt(-a))/(sqrt(-a)*a^(9/2)) + 1/105*(105*(b*x^2 +
 a)^3 + 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 + 15*a^3)/((b*x^2 + a)^(7/2
)*a^4)
```

## 3.87 $\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=126

$$\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}}-\frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}}+\frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] x^(5/2)/(7*a*(a*x + b*x^3)^(7/2)) + (8*x^(3/2))/(35*a^2*(a*x + b*x^3)^(5/2)
) + (16*Sqrt[x])/(35*a^3*(a*x + b*x^3)^(3/2)) + 64/(35*a^4*Sqrt[x]*Sqrt[a*x
 + b*x^3]) - (128*Sqrt[a*x + b*x^3])/(35*a^5*x^(3/2))
```

---

**Rubi [A]** time = 0.192445, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}}-\frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}}+\frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(5/2)/(7*a*(a*x + b*x^3)^(7/2)) + (8*x^(3/2))/(35*a^2*(a*x + b*x^3)^(5/2)
) + (16*Sqrt[x])/(35*a^3*(a*x + b*x^3)^(3/2)) + 64/(35*a^4*Sqrt[x]*Sqrt[a*x
 + b*x^3]) - (128*Sqrt[a*x + b*x^3])/(35*a^5*x^(3/2))
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{8\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a} \\
&= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{48\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35a^2} \\
&= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{64\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{3/2}}\,dx}{35a^3} \\
&= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}}+\frac{128\int \frac{1}{x^{3/2}\sqrt{ax+b}}}{35a^4} \\
&= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}}-\frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.0297992, size = 77, normalized size = 0.61

$$-\frac{\sqrt{x\left(a+bx^2\right)}\left(560a^2b^2x^4+280a^3bx^2+35a^4+448ab^3x^6+128b^4x^8\right)}{35a^5x^{3/2}\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -(Sqrt[x*(a + b*x^2)]*(35*a^4 + 280*a^3*b*x^2 + 560*a^2*b^2*x^4 + 448*a*b^3
*x^6 + 128*b^4*x^8))/(35*a^5*x^(3/2)*(a + b*x^2)^4)
```

---

**Maple [A]** time = 0.004, size = 70, normalized size = 0.6

$$-\frac{\left(bx^2+a\right)\left(128\,b^4x^8+448\,b^3x^6a+560\,b^2x^4a^2+280\,bx^2a^3+35\,a^4\right)}{35\,a^5}x^{\frac{7}{2}}\left(bx^3+ax\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/35*x^(7/2)*(b*x^2+a)*(128*b^4*x^8+448*a*b^3*x^6+560*a^2*b^2*x^4+280*a^3*
b*x^2+35*a^4)/a^5/(b*x^3+a*x)^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(b*x^3 + a*x)^(9/2), x)
```

---

**Fricas [A]** time = 1.67136, size = 239, normalized size = 1.9

$$-\frac{\left(128\,b^4x^8+448\,ab^3x^6+560\,a^2b^2x^4+280\,a^3bx^2+35\,a^4\right)\sqrt{bx^3+ax}\sqrt{x}}{35\left(a^5b^4x^{10}+4\,a^6b^3x^8+6\,a^7b^2x^6+4\,a^8bx^4+a^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(128*b^4*x^8 + 448*a*b^3*x^6 + 560*a^2*b^2*x^4 + 280*a^3*b*x^2 + 35*a
^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^5*b^4*x^10 + 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6
 + 4*a^8*b*x^4 + a^9*x^2)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.44085, size = 96, normalized size = 0.76

$$-\frac{\left(\left(x^2\left(\frac{93\,b^4x^2}{a^5}+\frac{308\,b^3}{a^4}\right)+\frac{350\,b^2}{a^3}\right)x^2+\frac{140\,b}{a^2}\right)x}{35\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{\sqrt{b+\frac{a}{x^2}}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*((x^2*(93*b^4*x^2/a^5 + 308*b^3/a^4) + 350*b^2/a^3)*x^2 + 140*b/a^2)*
x/(b*x^2 + a)^(7/2) - sqrt(b + a/x^2)/a^5
```

## 3.88 $\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=159

$$\frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}}+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}-\frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}+\frac{9b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{11/2}}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] x^(3/2)/(7*a*(a*x + b*x^3)^(7/2)) + (9*Sqrt[x])/(35*a^2*(a*x + b*x^3)^(5/2)
) + 3/(5*a^3*Sqrt[x]*(a*x + b*x^3)^(3/2)) + 3/(a^4*x^(3/2)*Sqrt[a*x + b*x^3
]) - (9*Sqrt[a*x + b*x^3])/(2*a^5*x^(5/2)) + (9*b*ArcTanh[(Sqrt[a]*Sqrt[x])
/Sqrt[a*x + b*x^3]])/(2*a^(11/2))
```

---

**Rubi [A]** time = 0.240548, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2023, 2025, 2029, 206}

$$\frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}}+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}-\frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}+\frac{9b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{11/2}}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(3/2)/(7*a*(a*x + b*x^3)^(7/2)) + (9*Sqrt[x])/(35*a^2*(a*x + b*x^3)^(5/2)
) + 3/(5*a^3*Sqrt[x]*(a*x + b*x^3)^(3/2)) + 3/(a^4*x^(3/2)*Sqrt[a*x + b*x^3
]) - (9*Sqrt[a*x + b*x^3])/(2*a^5*x^(5/2)) + (9*b*ArcTanh[(Sqrt[a]*Sqrt[x])
/Sqrt[a*x + b*x^3]])/(2*a^(11/2))
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{9\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{5/2}}\,dx}{5a^2}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}} + \frac{3\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{3/2}}\,dx}{a^3}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} + \frac{9\int \frac{1}{x^{5/2}\sqrt{ax+}}}{a^4}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} - \frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} - \frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} - \frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}$$

**Mathematica [C]** time = 0.0196505, size = 44, normalized size = 0.28

$$-\frac{bx^{7/2}\,{}_2F_1\left(-\frac{7}{2},2;-\frac{5}{2};\frac{bx^2}{a}+1\right)}{7a^2\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -(b*x^(7/2)*Hypergeometric2F1[-7/2, 2, -5/2, 1 + (b*x^2)/a])/(7*a^2*(x*(a +
 b*x^2))^(7/2))
```

---

**Maple [A]** time = 0.013, size = 234, normalized size = 1.5

$$\frac{1}{70\left(bx^2+a\right)^4}\sqrt{x\left(bx^2+a\right)}\left(315\ln\left(2\,\frac{\sqrt{a}\sqrt{bx^2+a}+a}{x}\right)x^8b^4\sqrt{bx^2+a}-315\sqrt{a}x^8b^4+945\ln\left(2\,\frac{\sqrt{a}\sqrt{bx^2+a}+a}{x}\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/70*(x*(b*x^2+a))^(1/2)/a^(11/2)*(315*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*
x^8*b^4*(b*x^2+a)^(1/2)-315*a^(1/2)*x^8*b^4+945*ln(2*(a^(1/2)*(b*x^2+a)^(1/
2)+a)/x)*x^6*a*b^3*(b*x^2+a)^(1/2)-1050*a^(3/2)*x^6*b^3+945*ln(2*(a^(1/2)*(
b*x^2+a)^(1/2)+a)/x)*x^4*a^2*b^2*(b*x^2+a)^(1/2)-1218*a^(5/2)*x^4*b^2+315*l
n(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*x^2*a^3*b*(b*x^2+a)^(1/2)-528*a^(7/2)*x^
2*b-35*a^(9/2))/x^(5/2)/(b*x^2+a)^4
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\left(bx^3 + ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(b*x^3 + a*x)^(9/2), x)
```

---

**Fricas [A]** time = 1.3772, size = 884, normalized size = 5.56

$$\left[\frac{315\left(b^5x^{11} + 4\,ab^4x^9 + 6\,a^2b^3x^7 + 4\,a^3b^2x^5 + a^4bx^3\right)\sqrt{a}\log\left(\frac{bx^3+2\,ax+2\,\sqrt{bx^3+ax}\sqrt{a}\sqrt{x}}{x^3}\right) - 2\left(315\,ab^4x^8 + 1050\,a^2b^3x^6 + 1218}{140\left(a^6b^4x^{11} + 4\,a^7b^3x^9 + 6\,a^8b^2x^7 + 4\,a^9bx^5 + a^{10}x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(b^5*x^11 + 4*a*b^4*x^9 + 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b
*x^3)*sqrt(a)*log((b*x^3 + 2*a*x + 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))/x^3
) - 2*(315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2
+ 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*
b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3), -1/70*(315*(b^5*x^11 + 4*a*b^4*x^9 + 6*a
^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x)*s
qrt(-a)/(a*sqrt(x))) + (315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4
 + 528*a^4*b*x^2 + 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11 + 4*a^7
*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.32919, size = 142, normalized size = 0.89

$$-\frac{1}{70}\,b\left(\frac{315\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^5}+\frac{2\left(140\left(bx^2+a\right)^3+35\left(bx^2+a\right)^2a+14\left(bx^2+a\right)a^2+5\,a^3\right)}{\left(bx^2+a\right)^{\frac{7}{2}}a^5}+\frac{35\,\sqrt{bx^2+a}}{a^5bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/70*b*(315*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^5) + 2*(140*(b*x^
2 + a)^3 + 35*(b*x^2 + a)^2*a + 14*(b*x^2 + a)*a^2 + 5*a^3)/((b*x^2 + a)^(7
/2)*a^5) + 35*sqrt(b*x^2 + a)/(a^5*b*x^2))
```

## 3.89 $\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=152

$$\frac{256b\sqrt{ax+bx^3}}{21a^6x^{3/2}}-\frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}}+\frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}}+\frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}}+\frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}}+\frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] Sqrt[x]/(7*a*(a*x + b*x^3)^(7/2)) + 2/(7*a^2*Sqrt[x]*(a*x + b*x^3)^(5/2)) +
 16/(21*a^3*x^(3/2)*(a*x + b*x^3)^(3/2)) + 32/(7*a^4*x^(5/2)*Sqrt[a*x + b*x
^3]) - (128*Sqrt[a*x + b*x^3])/(21*a^5*x^(7/2)) + (256*b*Sqrt[a*x + b*x^3])
/(21*a^6*x^(3/2))
```

---

**Rubi [A]** time = 0.233374, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{256b\sqrt{ax+bx^3}}{21a^6x^{3/2}}-\frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}}+\frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}}+\frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}}+\frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}}+\frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a*x + b*x^3)^(9/2),x]
```

```
[Out] Sqrt[x]/(7*a*(a*x + b*x^3)^(7/2)) + 2/(7*a^2*Sqrt[x]*(a*x + b*x^3)^(5/2)) +
 16/(21*a^3*x^(3/2)*(a*x + b*x^3)^(3/2)) + 32/(7*a^4*x^(5/2)*Sqrt[a*x + b*x
^3]) - (128*Sqrt[a*x + b*x^3])/(21*a^5*x^(7/2)) + (256*b*Sqrt[a*x + b*x^3])
/(21*a^6*x^(3/2))
```

### Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

### Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

### Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{9/2}}\,dx &= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{10\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{7/2}}\,dx}{7a} \\
&= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{5/2}}\,dx}{7a^2} \\
&= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}} + \frac{32\int \frac{1}{x^{5/2}\left(ax+bx^3\right)^{3/2}}\,dx}{7a^3} \\
&= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}} + 128\int \\
&= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}} - \frac{128\sqrt{a}}{21a} \\
&= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}} - \frac{128\sqrt{a}}{21a}
\end{aligned}$$

**Mathematica [A]** time = 0.024839, size = 88, normalized size = 0.58

$$\frac{\sqrt{x\left(a+bx^2\right)}\left(1120a^2b^3x^6+560a^3b^2x^4+70a^4bx^2-7a^5+896ab^4x^8+256b^5x^{10}\right)}{21a^6x^{7/2}\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x*(a + b*x^2)]*(-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^
3*x^6 + 896*a*b^4*x^8 + 256*b^5*x^10))/(21*a^6*x^(7/2)*(a + b*x^2)^4)
```

---

**Maple [A]** time = 0.006, size = 81, normalized size = 0.5

$$-\frac{\left(bx^2+a\right)\left(-256\,b^5x^{10}-896\,b^4x^8a-1120\,b^3x^6a^2-560\,b^2x^4a^3-70\,bx^2a^4+7\,a^5\right)}{21\,a^6}x^{\frac{3}{2}}\left(bx^3+ax\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/21*x^(3/2)*(b*x^2+a)*(-256*b^5*x^10-896*a*b^4*x^8-1120*a^2*b^3*x^6-560*a
^3*b^2*x^4-70*a^4*b*x^2+7*a^5)/a^6/(b*x^3+a*x)^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(b*x^3 + a*x)^(9/2), x)
```

---

**Fricas [A]** time = 1.85057, size = 265, normalized size = 1.74

$$\frac{\left(256\,b^{5}x^{10}+896\,ab^{4}x^{8}+1120\,a^{2}b^{3}x^{6}+560\,a^{3}b^{2}x^{4}+70\,a^{4}bx^{2}-7\,a^{5}\right)\sqrt{bx^{3}+ax}\sqrt{x}}{21\left(a^{6}b^{4}x^{12}+4\,a^{7}b^{3}x^{10}+6\,a^{8}b^{2}x^{8}+4\,a^{9}bx^{6}+a^{10}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(256*b^5*x^10 + 896*a*b^4*x^8 + 1120*a^2*b^3*x^6 + 560*a^3*b^2*x^4 + 7
0*a^4*b*x^2 - 7*a^5)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^6*b^4*x^12 + 4*a^7*b^3*x^
10 + 6*a^8*b^2*x^8 + 4*a^9*b*x^6 + a^10*x^4)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.38026, size = 116, normalized size = 0.76

$$\frac{\left(\left(x^{2}\left(\frac{158\,b^{5}x^{2}}{a^{6}}+\frac{511\,b^{4}}{a^{5}}\right)+\frac{560\,b^{3}}{a^{4}}\right)x^{2}+\frac{210\,b^{2}}{a^{3}}\right)x}{21\left(bx^{2}+a\right)^{\frac{7}{2}}}-\frac{\left(b+\frac{a}{x^{2}}\right)^{\frac{3}{2}}-15\sqrt{b+\frac{a}{x^{2}}}b}{3\,a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/21*((x^2*(158*b^5*x^2/a^6 + 511*b^4/a^5) + 560*b^3/a^4)*x^2 + 210*b^2/a^3
)*x/(b*x^2 + a)^(7/2) - 1/3*((b + a/x^2)^(3/2) - 15*sqrt(b + a/x^2)*b)/a^6
```

## 3.90 $\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=189

$$-\frac{99b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^{13/2}}+\frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}}-\frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}}+\frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}}+\frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}}+\frac{1}{35a^2x^{3/2}\big(}$$

```
[Out] 1/(7*a*Sqrt[x]*(a*x + b*x^3)^(7/2)) + 11/(35*a^2*x^(3/2)*(a*x + b*x^3)^(5/2
)) + 33/(35*a^3*x^(5/2)*(a*x + b*x^3)^(3/2)) + 33/(5*a^4*x^(7/2)*Sqrt[a*x +
 b*x^3]) - (33*Sqrt[a*x + b*x^3])/(4*a^5*x^(9/2)) + (99*b*Sqrt[a*x + b*x^3]
)/(8*a^6*x^(5/2)) - (99*b^2*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]])/(
8*a^(13/2))
```

---

**Rubi [A]** time = 0.292728, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2023, 2025, 2029, 206}

$$-\frac{99b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^{13/2}}+\frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}}-\frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}}+\frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}}+\frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}}+\frac{1}{35a^2x^{3/2}\big(}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] 1/(7*a*Sqrt[x]*(a*x + b*x^3)^(7/2)) + 11/(35*a^2*x^(3/2)*(a*x + b*x^3)^(5/2
)) + 33/(35*a^3*x^(5/2)*(a*x + b*x^3)^(3/2)) + 33/(5*a^4*x^(7/2)*Sqrt[a*x +
 b*x^3]) - (33*Sqrt[a*x + b*x^3])/(4*a^5*x^(9/2)) + (99*b*Sqrt[a*x + b*x^3]
)/(8*a^6*x^(5/2)) - (99*b^2*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]])/(
8*a^(13/2))
```

### Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

### Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

### Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{9/2}}\,dx = \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{7/2}}\,dx}{7a}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{99\int \frac{1}{x^{5/2}\left(ax+bx^3\right)^{5/2}}\,dx}{35a^2}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33\int \frac{1}{x^{7/2}\left(ax+bx^3\right)^{3/2}}\,dx}{5a^3}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} +$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} -$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} -$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} -$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}} -$$

**Mathematica [C]** time = 0.0193527, size = 46, normalized size = 0.24

$$\frac{b^2x^{7/2}\,{}_2F_1\left(-\frac{7}{2},3;-\frac{5}{2};\frac{bx^2}{a}+1\right)}{7a^3\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] (b^2*x^(7/2)*Hypergeometric2F1[-7/2, 3, -5/2, 1 + (b*x^2)/a])/(7*a^3*(x*(a
+ b*x^2))^(7/2))
```

---

**Maple [A]** time = 0.013, size = 247, normalized size = 1.3

$$-\frac{1}{280\left(bx^2+a\right)^4}\sqrt{x\left(bx^2+a\right)}\Bigg(3465\ln\left(2\frac{\sqrt{a}\sqrt{bx^2+a}+a}{x}\right)x^{10}b^5\sqrt{bx^2+a}-3465\sqrt{a}x^{10}b^5+10395\ln\Bigg(2\frac{\sqrt{a}\sqrt{bx^2+a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/280*(x*(b*x^2+a))^(1/2)/a^(13/2)*(3465*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/
x)*x^10*b^5*(b*x^2+a)^(1/2)-3465*a^(1/2)*x^10*b^5+10395*ln(2*(a^(1/2)*(b*x^
2+a)^(1/2)+a)/x)*x^8*a*b^4*(b*x^2+a)^(1/2)-11550*a^(3/2)*x^8*b^4+10395*ln(2
*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*x^6*a^2*b^3*(b*x^2+a)^(1/2)-13398*a^(5/2)*x
^6*b^3+3465*ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*x^4*a^3*b^2*(b*x^2+a)^(1/2)
-5808*a^(7/2)*x^4*b^2-385*a^(9/2)*x^2*b+70*a^(11/2))/x^(9/2)/(b*x^2+a)^4
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3 + ax\right)^{\frac{9}{2}}\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(9/2)*sqrt(x)), x)
```

---

**Fricas [A]** time = 1.42645, size = 963, normalized size = 5.1

$$\left[\frac{3465 \left(b^6x^{13} + 4\,ab^5x^{11} + 6\,a^2b^4x^9 + 4\,a^3b^3x^7 + a^4b^2x^5\right)\sqrt{a}\log\left(\frac{bx^3+2\,ax-2\,\sqrt{bx^3+ax}\sqrt{a}\sqrt{x}}{x^3}\right) + 2\left(3465\,ab^5x^{10} + 11550\,a^2b^4}{560\left(a^7b^4x^{13} + 4\,a^8b^3x^{11} + 6\,a^9b^2x^9 + 4\,a^{10}bx^7 + a^{11}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/560*(3465*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^4
*b^2*x^5)*sqrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))
/x^3) + 2*(3465*a*b^5*x^10 + 11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 5808*a
^4*b^2*x^4 + 385*a^5*b*x^2 - 70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^4*x^
13 + 4*a^8*b^3*x^11 + 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5), 1/280*(3465
*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^4*b^2*x^5)*sq
rt(-a)*arctan(sqrt(b*x^3 + a*x)*sqrt(-a)/(a*sqrt(x))) + (3465*a*b^5*x^10 +
11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 5808*a^4*b^2*x^4 + 385*a^5*b*x^2 -
70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^4*x^13 + 4*a^8*b^3*x^11 + 6*a^9*b
^2*x^9 + 4*a^10*b*x^7 + a^11*x^5)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.35876, size = 165, normalized size = 0.87

$$\frac{1}{280}\,b^{2}\left(\frac{3465\,\arctan\left(\frac{\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{6}}+\frac{8\left(350\left(bx^{2}+a\right)^{3}+70\left(bx^{2}+a\right)^{2}a+21\left(bx^{2}+a\right)a^{2}+5\,a^{3}\right)}{\left(bx^{2}+a\right)^{\frac{7}{2}}a^{6}}+\frac{35\left(19\left(bx^{2}+a\right)^{\frac{3}{2}}-21\right.}{a^{6}b^{2}x^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/280*b^2*(3465*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^6) + 8*(350*(b
*x^2 + a)^3 + 70*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 + 5*a^3)/((b*x^2 + a)
^(7/2)*a^6) + 35*(19*(b*x^2 + a)^(3/2) - 21*sqrt(b*x^2 + a)*a)/(a^6*b^2*x^4
))
```

## 3.91 $\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx$

**Optimal**. Leaf size=180

$$-\frac{1024b^2\sqrt{ax+bx^3}}{35a^7x^{3/2}}+\frac{512b\sqrt{ax+bx^3}}{35a^6x^{7/2}}-\frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}}+\frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}}+\frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}}+\frac{12}{35a^2x^{5/2}\left(ax+\right.}$$

```
[Out] 1/(7*a*x^(3/2)*(a*x + b*x^3)^(7/2)) + 12/(35*a^2*x^(5/2)*(a*x + b*x^3)^(5/2
)) + 8/(7*a^3*x^(7/2)*(a*x + b*x^3)^(3/2)) + 64/(7*a^4*x^(9/2)*Sqrt[a*x + b
*x^3]) - (384*Sqrt[a*x + b*x^3])/(35*a^5*x^(11/2)) + (512*b*Sqrt[a*x + b*x^
3])/(35*a^6*x^(7/2)) - (1024*b^2*Sqrt[a*x + b*x^3])/(35*a^7*x^(3/2))
```

---

**Rubi [A]** time = 0.284987, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$-\frac{1024b^2\sqrt{ax+bx^3}}{35a^7x^{3/2}}+\frac{512b\sqrt{ax+bx^3}}{35a^6x^{7/2}}-\frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}}+\frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}}+\frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}}+\frac{12}{35a^2x^{5/2}\left(ax+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] 1/(7*a*x^(3/2)*(a*x + b*x^3)^(7/2)) + 12/(35*a^2*x^(5/2)*(a*x + b*x^3)^(5/2
)) + 8/(7*a^3*x^(7/2)*(a*x + b*x^3)^(3/2)) + 64/(7*a^4*x^(9/2)*Sqrt[a*x + b
*x^3]) - (384*Sqrt[a*x + b*x^3])/(35*a^5*x^(11/2)) + (512*b*Sqrt[a*x + b*x^
3])/(35*a^6*x^(7/2)) - (1024*b^2*Sqrt[a*x + b*x^3])/(35*a^7*x^(3/2))
```

### Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

### Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

### Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

### Rubi steps

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12\int \frac{1}{x^{5/2}\left(ax+bx^3\right)^{7/2}}\,dx}{7a}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{24\int \frac{1}{x^{7/2}\left(ax+bx^3\right)^{5/2}}\,dx}{7a^2}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64\int \frac{1}{x^{9/2}\left(ax+bx^3\right)^{3/2}}\,dx}{7a^3}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}} + \frac{3}{}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}} - \frac{3}{}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}} - \frac{3}{}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}} - \frac{3}{}$$

**Mathematica [A]** time = 0.0310446, size = 99, normalized size = 0.55

$$-\frac{\sqrt{x\left(a+bx^2\right)}\left(4480a^2b^4x^8+2240a^3b^3x^6+280a^4b^2x^4-28a^5bx^2+7a^6+3584ab^5x^{10}+1024b^6x^{12}\right)}{35a^7x^{11/2}\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] -(Sqrt[x*(a + b*x^2)]*(7*a^6 - 28*a^5*b*x^2 + 280*a^4*b^2*x^4 + 2240*a^3*b^
3*x^6 + 4480*a^2*b^4*x^8 + 3584*a*b^5*x^10 + 1024*b^6*x^12))/(35*a^7*x^(11/
2)*(a + b*x^2)^4)
```

---

**Maple [A]** time = 0.007, size = 92, normalized size = 0.5

$$-\frac{\left(bx^2+a\right)\left(1024\,b^6x^{12}+3584\,b^5x^{10}a+4480\,b^4x^8a^2+2240\,b^3x^6a^3+280\,b^2x^4a^4-28\,bx^2a^5+7\,a^6\right)}{35\,a^7}\frac{1}{\sqrt{x}}\left(bx^3+ax\right)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/35*(b*x^2+a)*(1024*b^6*x^12+3584*a*b^5*x^10+4480*a^2*b^4*x^8+2240*a^3*b^
3*x^6+280*a^4*b^2*x^4-28*a^5*b*x^2+7*a^6)/x^(1/2)/a^7/(b*x^3+a*x)^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{3}+ax\right)^{\frac{9}{2}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(9/2)*x^(3/2)), x)
```

---

**Fricas [A]** time = 2.28488, size = 298, normalized size = 1.66

$$-\frac{\left(1024\,b^{6}x^{12}+3584\,ab^{5}x^{10}+4480\,a^{2}b^{4}x^{8}+2240\,a^{3}b^{3}x^{6}+280\,a^{4}b^{2}x^{4}-28\,a^{5}bx^{2}+7\,a^{6}\right)\sqrt{bx^{3}+ax}\sqrt{x}}{35\left(a^{7}b^{4}x^{14}+4\,a^{8}b^{3}x^{12}+6\,a^{9}b^{2}x^{10}+4\,a^{10}bx^{8}+a^{11}x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(1024*b^6*x^12 + 3584*a*b^5*x^10 + 4480*a^2*b^4*x^8 + 2240*a^3*b^3*x^
6 + 280*a^4*b^2*x^4 - 28*a^5*b*x^2 + 7*a^6)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^7*
b^4*x^14 + 4*a^8*b^3*x^12 + 6*a^9*b^2*x^10 + 4*a^10*b*x^8 + a^11*x^6)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.40183, size = 136, normalized size = 0.76

$$-\frac{\left(\left(2\,x^{2}\left(\frac{281\,b^{6}x^{2}}{a^{7}}+\frac{896\,b^{5}}{a^{6}}\right)+\frac{1925\,b^{4}}{a^{5}}\right)x^{2}+\frac{700\,b^{3}}{a^{4}}\right)x}{35\left(bx^{2}+a\right)^{\frac{7}{2}}}-\frac{\left(b+\frac{a}{x^{2}}\right)^{\frac{5}{2}}-10\left(b+\frac{a}{x^{2}}\right)^{\frac{3}{2}}b+75\sqrt{b+\frac{a}{x^{2}}}b^{2}}{5\,a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*((2*x^2*(281*b^6*x^2/a^7 + 896*b^5/a^6) + 1925*b^4/a^5)*x^2 + 700*b^3
/a^4)*x/(b*x^2 + a)^(7/2) - 1/5*((b + a/x^2)^(5/2) - 10*(b + a/x^2)^(3/2)*b
 + 75*sqrt(b + a/x^2)*b^2)/a^7
```

## 3.92 $\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx$

**Optimal**. Leaf size=55

$$\frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

```
[Out] (x*Sqrt[a*x + b*x^4])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/
(3*b^(3/2))
```

---

**Rubi [A]** time = 0.0698222, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2024, 2029, 206}

$$\frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x*Sqrt[a*x + b*x^4])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/
(3*b^(3/2))
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx &= \frac{x\sqrt{ax+bx^4}}{3b} - \frac{a\int \frac{x}{\sqrt{ax+bx^4}}\,dx}{2b} \\
&= \frac{x\sqrt{ax+bx^4}}{3b} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax+bx^4}}\right)}{3b} \\
&= \frac{x\sqrt{ax+bx^4}}{3b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.0327577, size = 81, normalized size = 1.47

$$
\frac{\sqrt{b}x^2\left(a+bx^3\right)-a\sqrt{x}\sqrt{a+bx^3}\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)}{3b^{3/2}\sqrt{x\left(a+bx^3\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x + b*x^4],x]
```

```
[Out] (Sqrt[b]*x^2*(a + b*x^3) - a*Sqrt[x]*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/
2))/Sqrt[a + b*x^3]])/(3*b^(3/2)*Sqrt[x*(a + b*x^3)])
```

---

**Maple [C]** time = 0.032, size = 997, normalized size = 18.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a*x)^(1/2),x)
```

```
[Out] 1/3*x*(b*x^4+a*x)^(1/2)/b-a*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3))*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*(-
b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)*
(x-1/b*(-b^2*a)^(1/3))^2*(1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))/(-1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)*(1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(
1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b
*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))/(-b^2*a)^(1/3)/(b*x*(x-1/b*(-b^2*a)^(1/3))*(x+1/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(1/3)-
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(1/b*(-b^2*a)^(1/3)*EllipticF(((-3/2
/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2),((3/2/b*(-b^2
*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/
2)/b*(-b^2*a)^(1/3))/(1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/
(3/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-1/b*(-b^2*a)^
(1/3)*EllipticPi(((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/
(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3
)))^(1/2),(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-
b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)),((3/2/b*(-b^2*a)^(1/3)+1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))/(1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(3/2/b*(-b^2*a)^(
```

```
1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^4+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(b*x^4 + a*x), x)
```

---

**Fricas [A]** time = 1.97972, size = 315, normalized size = 5.73

$$\left[\frac{4\,\sqrt{bx^4+ax}bx + a\sqrt{b}\log\left(-8\,b^2x^6 - 8\,abx^3 - a^2 + 4\left(2\,bx^4+ax\right)\sqrt{bx^4+ax}\sqrt{b}\right)}{12\,b^2}, \frac{2\,\sqrt{bx^4+ax}bx + a\sqrt{-b}\arctan\left(\frac{2\,\sqrt{bx^4+}}{2\,bx}\right.}{6\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*sqrt(b*x^4 + a*x)*b*x + a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2
 + 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x)*sqrt(b)))/b^2, 1/6*(2*sqrt(b*x^4 + a
*x)*b*x + a*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2*b*x^3 + a)))/
b^2]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{x\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x*(a + b*x**3)), x)
```

---

**Giac [A]** time = 1.32322, size = 61, normalized size = 1.11

$$\frac{\sqrt{bx^4+ax}x}{3\,b} + \frac{a\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*x^4 + a*x)*x/b + 1/3*a*arctan(sqrt(b + a/x^3)/sqrt(-b))/(sqrt(-b
)*b)
```

## 3.93 $\int \frac{x}{\sqrt{ax+bx^4}}\,dx$

**Optimal**. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/(3*Sqrt[b])
```

---

**Rubi [A]** time = 0.0322918, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2029, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/(3*Sqrt[b])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{ax+bx^4}}\,dx &= \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax+bx^4}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}\end{aligned}$$

**Mathematica [A]** time = 0.0131221, size = 61, normalized size = 1.91

$$\frac{2\sqrt{x}\sqrt{a+bx^3}\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)}{3\sqrt{b}\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/(3*S
qrt[b]*Sqrt[x*(a + b*x^3)])
```

---

**Maple [C]** time = 0.015, size = 979, normalized size = 30.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a*x)^(1/2),x)
```

```
[Out] 2*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*((-3/2/b*(-b^2*a)^(
1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)*(x-1/b*(-b^2*a)^(1/3))^2*(
1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/
(-1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3
)))^(1/2)*(1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-
b^2*a)^(1/3)))^(1/2)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
*b/(-b^2*a)^(1/3)/(b*x*(x-1/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3)))^(1/2)*(1/b*(-b^2*a)^(1/3)*EllipticF(((-3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2),((3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(1/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(3/2/b*(-b^2*a)^(1/3)-1/
2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))-1/b*(-b^2*a)^(1/3)*EllipticPi(((-3/2/
b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2),(-1/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/
2)/b*(-b^2*a)^(1/3)),((3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(1/2/b*(-b^2*a)^(1/3
)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(3/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b
^2*a)^(1/3)))^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^4 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*x^4 + a*x), x)
```

---

**Fricas [A]** time = 1.89581, size = 225, normalized size = 7.03

$$\left[\frac{\log\left(-8\,b^{2}x^{6}-8\,abx^{3}-a^{2}-4\left(2\,bx^{4}+ax\right)\sqrt{bx^{4}+ax}\sqrt{b}\right)}{6\,\sqrt{b}},-\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{bx^{4}+ax}\sqrt{-bx}}{2\,bx^{3}+a}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x)
*sqrt(b))/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2*b
*x^3 + a))/b]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(a + b*x**3)), x)
```

---

**Giac [A]** time = 1.30927, size = 31, normalized size = 0.97

$$-\frac{2\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b)
```

## 3.94 $\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx$

**Optimal**. Leaf size=23

$$-\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

```
[Out] (-2*Sqrt[a*x + b*x^4])/(3*a*x^2)
```

---

**Rubi [A]** time = 0.0337861, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2014}

$$-\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x + b*x^4]),x]

[Out] (-2*Sqrt[a*x + b*x^4])/(3*a*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx=-\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

**Mathematica [A]** time = 0.0106894, size = 23, normalized size = 1.

$$-\frac{2\sqrt{x\left(a+bx^3\right)}}{3ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x + b*x^4]),x]

[Out] (-2*Sqrt[x*(a + b*x^3)])/(3*a*x^2)
```

---

**Maple [A]** time = 0.003, size = 27, normalized size = 1.2

$$-\frac{2\,b{x}^{3}+2\,a}{3\,ax}\frac{1}{\sqrt{b{x}^{4}+ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a*x)^(1/2),x)
```

```
[Out] -2/3*(b*x^3+a)/x/a/(b*x^4+a*x)^(1/2)
```

---

**Maxima [A]** time = 1.06738, size = 35, normalized size = 1.52

$$-\frac{2\left(bx^4+ax\right)}{3\sqrt{bx^3+a}ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))
```

---

**Fricas [A]** time = 1.26757, size = 43, normalized size = 1.87

$$-\frac{2\sqrt{bx^4+ax}}{3\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x^4 + a*x)/(a*x^2)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^2\sqrt{x\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x*(a + b*x**3))), x)
```

---

**Giac [A]** time = 1.19666, size = 19, normalized size = 0.83

$$-\frac{2\sqrt{b+\frac{a}{x^3}}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b + a/x^3)/a
```

## 3.95 $\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx$

**Optimal**. Leaf size=48

$$\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}-\frac{2\sqrt{ax+bx^4}}{9ax^5}$$

```
[Out] (-2*Sqrt[a*x + b*x^4])/(9*a*x^5) + (4*b*Sqrt[a*x + b*x^4])/(9*a^2*x^2)
```

---

**Rubi [A]** time = 0.0664935, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2016, 2014}

$$\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}-\frac{2\sqrt{ax+bx^4}}{9ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^4])/(9*a*x^5) + (4*b*Sqrt[a*x + b*x^4])/(9*a^2*x^2)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx &= -\frac{2\sqrt{ax+bx^4}}{9ax^5}-\frac{(2b)\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax+bx^4}}{9ax^5}+\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}\end{aligned}$$

**Mathematica [A]** time = 0.0137281, size = 31, normalized size = 0.65

$$-\frac{2\left(a-2bx^3\right)\sqrt{x\left(a+bx^3\right)}}{9a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*(a - 2*b*x^3)*Sqrt[x*(a + b*x^3)])/(9*a^2*x^5)
```

---

**Maple [A]** time = 0.004, size = 35, normalized size = 0.7

$$-\frac{\left(2\,b{x}^{3}+2\,a\right)\left(-2\,b{x}^{3}+a\right)}{9\,{a}^{2}{x}^{4}}\frac{1}{\sqrt{b{x}^{4}+ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a*x)^(1/2),x)
```

```
[Out] -2/9*(b*x^3+a)*(-2*b*x^3+a)/x^4/a^2/(b*x^4+a*x)^(1/2)
```

---

**Maxima [A]** time = 1.01426, size = 51, normalized size = 1.06

$$\frac{2\left(2\,b^{2}x^{7}+abx^{4}-a^{2}x\right)}{9\,\sqrt{bx^{3}+a}a^{2}x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(sqrt(b*x^3 + a)*a^2*x^(11/2))
```

---

**Fricas [A]** time = 1.19889, size = 63, normalized size = 1.31

$$\frac{2\,\sqrt{bx^{4}+ax}\left(2\,bx^{3}-a\right)}{9\,a^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*sqrt(b*x^4 + a*x)*(2*b*x^3 - a)/(a^2*x^5)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{5}\sqrt{x\left(a+bx^{3}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(x*(a + b*x**3))), x)
```

---

**Giac [A]** time = 1.19973, size = 36, normalized size = 0.75

$$-\frac{2\left(\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}-3\sqrt{b+\frac{a}{x^3}}b\right)}{9\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*((b + a/x^3)^(3/2) - 3*sqrt(b + a/x^3)*b)/a^2
```

## 3.96 $\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx$

**Optimal**. Leaf size=74

$$-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}-\frac{2\sqrt{ax+bx^4}}{15ax^8}$$

```
[Out] (-2*Sqrt[a*x + b*x^4])/(15*a*x^8) + (8*b*Sqrt[a*x + b*x^4])/(45*a^2*x^5) -
(16*b^2*Sqrt[a*x + b*x^4])/(45*a^3*x^2)
```

---

**Rubi [A]** time = 0.101669, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2016, 2014}

$$-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}-\frac{2\sqrt{ax+bx^4}}{15ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^4])/(15*a*x^8) + (8*b*Sqrt[a*x + b*x^4])/(45*a^2*x^5) -
(16*b^2*Sqrt[a*x + b*x^4])/(45*a^3*x^2)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx &= -\frac{2\sqrt{ax+bx^4}}{15ax^8}-\frac{(4b)\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx}{5a}\\ &= -\frac{2\sqrt{ax+bx^4}}{15ax^8}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}+\frac{\left(8b^2\right)\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx}{15a^2}\\ &= -\frac{2\sqrt{ax+bx^4}}{15ax^8}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}\end{aligned}$$

**Mathematica [A]** time = 0.0146873, size = 44, normalized size = 0.59

$$-\frac{2\sqrt{x\left(a+bx^3\right)}\left(3a^2-4abx^3+8b^2x^6\right)}{45a^3x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[x*(a + b*x^3)]*(3*a^2 - 4*a*b*x^3 + 8*b^2*x^6))/(45*a^3*x^8)
```

---

**Maple [A]** time = 0.005, size = 48, normalized size = 0.7

$$-\frac{\left(2\,b{x}^{3}+2\,a\right)\left(8\,{b}^{2}{x}^{6}-4\,ab{x}^{3}+3\,{a}^{2}\right)}{45\,{x}^{7}{a}^{3}}\frac{1}{\sqrt{b{x}^{4}+ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^4+a*x)^(1/2),x)
```

```
[Out] -2/45*(b*x^3+a)*(8*b^2*x^6-4*a*b*x^3+3*a^2)/x^7/a^3/(b*x^4+a*x)^(1/2)
```

---

**Maxima [A]** time = 1.08143, size = 68, normalized size = 0.92

$$-\frac{2\left(8\,b^{3}x^{10}+4\,ab^{2}x^{7}-a^{2}bx^{4}+3\,a^{3}x\right)}{45\,\sqrt{bx^{3}+a}a^{3}x^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/45*(8*b^3*x^10 + 4*a*b^2*x^7 - a^2*b*x^4 + 3*a^3*x)/(sqrt(b*x^3 + a)*a^3
*x^(17/2))
```

---

**Fricas [A]** time = 1.18574, size = 90, normalized size = 1.22

$$-\frac{2\left(8\,b^{2}x^{6}-4\,abx^{3}+3\,a^{2}\right)\sqrt{bx^{4}+ax}}{45\,a^{3}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(8*b^2*x^6 - 4*a*b*x^3 + 3*a^2)*sqrt(b*x^4 + a*x)/(a^3*x^8)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{8}\sqrt{x\left(a+bx^{3}\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**8*sqrt(x*(a + b*x**3))), x)
```

---

**Giac [A]** time = 1.23487, size = 58, normalized size = 0.78

$$-\frac{2\left(3\left(b+\frac{a}{x^3}\right)^{\frac{5}{2}}-10\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}b+15\sqrt{b+\frac{a}{x^3}}b^2\right)}{45\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/45*(3*(b + a/x^3)^(5/2) - 10*(b + a/x^3)^(3/2)*b + 15*sqrt(b + a/x^3)*b^
2)/a^3
```

## 3.97 $\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=224

$$\frac{\sqrt{ax+bx^4}}{2b}-\frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

```
[Out] Sqrt[a*x + b*x^4]/(2*b) - (a^(2/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ell
ipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1
/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

---

**Rubi [A]** time = 0.212868, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2024, 2011, 329, 225}

$$\frac{\sqrt{ax+bx^4}}{2b}-\frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x + b*x^4],x]
```

```
[Out] Sqrt[a*x + b*x^4]/(2*b) - (a^(2/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ell
ipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1
/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx &= \frac{\sqrt{ax+bx^4}}{2b} - \frac{a\int \frac{1}{\sqrt{ax+bx^4}}\,dx}{4b}\\
&= \frac{\sqrt{ax+bx^4}}{2b} - \frac{\left(a\sqrt{x}\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{4b\sqrt{ax+bx^4}}\\
&= \frac{\sqrt{ax+bx^4}}{2b} - \frac{\left(a\sqrt{x}\sqrt{a+bx^3}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{2b\sqrt{ax+bx^4}}\\
&= \frac{\sqrt{ax+bx^4}}{2b} - \frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}
\end{aligned}$$

**Mathematica** **[C]** time = 0.0230682, size = 64, normalized size = 0.29

$$\frac{x\left(-a\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)+a+bx^3\right)}{2b\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b
*x^3)/a)]))/(2*b*Sqrt[x*(a + b*x^3)])
```

---

**Maple** **[C]** time = 0.014, size = 688, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a*x)^(1/2),x)
```

```
[Out] 1/2*(b*x^4+a*x)^(1/2)/b-1/2*a*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a
)^(1/3))*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*
(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2
)*(x-1/b*(-b^2*a)^(1/3))^2*(1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))/(-1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)*(1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)
^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)/(-3/2/b*(-b^2*a)^(1/3)+1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3))/(-b^2*a)^(1/3)/(b*x*(x-1/b*(-b^2*a)^(1/3))*(x+
1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(1/3
)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2),((3/2/b*(-b^2*a)^(1/3)+1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
))/(1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(3/2/b*(-b^2*a)^(1
/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^4 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(b*x^4 + a*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + ax}x^2}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a*x)*x^2/(b*x^3 + a), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x*(a + b*x**3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^4 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(b*x^4 + a*x), x)
```

## 3.98 $\int \frac{1}{\sqrt{ax+bx^4}}\,dx$

**Optimal**. Leaf size=197

$$\frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

```
[Out] (x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt
[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^
(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

---

**Rubi [A]** time = 0.130435, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2011, 329, 225}

$$\frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt
[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^
(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
```

```
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{ax+bx^4}}\,dx &= \frac{\left(\sqrt{x}\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{\sqrt{ax+bx^4}}\\
&= \frac{\left(2\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^4}}\\
&= \frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0096315, size = 49, normalized size = 0.25

$$
\frac{2x\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)}{\sqrt{x\left(a+bx^3\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)])/Sq
rt[x*(a + b*x^3)]
```

---

**Maple [C]** time = 0.016, size = 671, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a*x)^(1/2),x)
```

```
[Out] 2*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*((-3/2/b*(-b^2*a)^(
1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)*(x-1/b*(-b^2*a)^(1/3))^2*(
1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/
(-1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3
)))^(1/2)*(1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-
b^2*a)^(1/3)))^(1/2)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
*b/(-b^2*a)^(1/3)/(b*x*(x-1/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2
*a)^(1/3)))^(1/2),((3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*(1
```

```
/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(1/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(3/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3)))^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^4 + a*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{bx^4 + ax}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*x^4 + a*x), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**4), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*x^4 + a*x), x)
```

## 3.99 $\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx$

**Optimal**. Leaf size=225

$$-\frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{5ax^3}$$

```
[Out] (-2*Sqrt[a*x + b*x^4])/(5*a*x^3) - (2*b*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2
]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt
[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt[(b^(1/3)*x*(a^(
1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]
)
```

---

**Rubi [A]** time = 0.186875, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2025, 2011, 329, 225}

$$-\frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{5ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^4])/(5*a*x^3) - (2*b*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2
]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt
[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt[(b^(1/3)*x*(a^(
1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]
)
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx &= -\frac{2\sqrt{ax+bx^4}}{5ax^3}-\frac{(2b)\int \frac{1}{\sqrt{ax+bx^4}}\,dx}{5a}\\
&= -\frac{2\sqrt{ax+bx^4}}{5ax^3}-\frac{\left(2b\sqrt{x}\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{5a\sqrt{ax+bx^4}}\\
&= -\frac{2\sqrt{ax+bx^4}}{5ax^3}-\frac{\left(4b\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{5a\sqrt{ax+bx^4}}\\
&= -\frac{2\sqrt{ax+bx^4}}{5ax^3}-\frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}
\end{aligned}
$$

**Mathematica [C]** time = 0.013207, size = 53, normalized size = 0.24

$$
-\frac{2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};-\frac{bx^3}{a}\right)}{5x^2\sqrt{x\left(a+bx^3\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((b*x^3)/a)])/(5
*x^2*Sqrt[x*(a + b*x^3)])
```

---

**Maple [C]** time = 0.018, size = 696, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a*x)^(1/2),x)
```

```
[Out] -2/5*(b*x^4+a*x)^(1/2)/a/x^3-4/5*b^2/a*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x
/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/
3)))^(1/2)*(x-1/b*(-b^2*a)^(1/3))^2*(1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(
1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(-1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b
*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)*(1/b*(-b^2*a)^(1/3)*(x+1/2/b
*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(-1/2/b*(-b^2*a)^(1/3)+1/2*
I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)/(-3/2/b*(-b^2*a)^
(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(-b^2*a)^(1/3)/(b*x*(x-1/b*(-b^2*a)^(
1/3))*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^
2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2),((3/2/b*(-b^2*a)^(1/3)
+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^
2*a)^(1/3))/(1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(3/2/b*(-
b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + axx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x^3), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + ax}}{bx^7 + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a*x)/(b*x^7 + a*x^4), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{x\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x*(a + b*x**3))), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + axx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x^3), x)
```

# 3.100 $\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx$

**Optimal**. Leaf size=503

$$\frac{5\left(1-\sqrt{3}\right)a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{bx}\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{bx}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{bx}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{bx}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{bx}\right)^2}}\sqrt{ax+bx^4}}+\frac{5\sqrt[4]{3}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{}$$

```
[Out] (-5*(1 + Sqrt[3])*a*x*(a + b*x^3))/(8*b^(5/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)*Sqrt[a*x + b*x^4]) + (x^2*Sqrt[a*x + b*x^4])/(4*b) + (5*3^(1/4)*a^(4
/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - S
qrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/
(8*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]) + (5*(1 - Sqrt[3])*a^(4/3)*x*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(16*3^(1/4)*b^(5/3)
*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*Sqrt[a*x + b*x^4])
```

---

**Rubi [A]** time = 0.514097, antiderivative size = 503, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2024, 2032, 329, 308, 225, 1881}

$$\frac{5\left(1-\sqrt{3}\right)a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{bx}\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{bx}+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{bx}+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{bx}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{bx}\right)^2}}\sqrt{ax+bx^4}}+\frac{5\sqrt[4]{3}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{8b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a*x + b*x^4],x]
```

```
[Out] (-5*(1 + Sqrt[3])*a*x*(a + b*x^3))/(8*b^(5/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)*Sqrt[a*x + b*x^4]) + (x^2*Sqrt[a*x + b*x^4])/(4*b) + (5*3^(1/4)*a^(4
/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - S
qrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/
(8*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]) + (5*(1 - Sqrt[3])*a^(4/3)*x*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(16*3^(1/4)*b^(5/3)
*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*Sqrt[a*x + b*x^4])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
```

```
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx &= \frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{(5a)\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx}{8b} \\
&= \frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{\left(5a\sqrt{x}\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{8b\sqrt{ax+bx^4}} \\
&= \frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{\left(5a\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{4b\sqrt{ax+bx^4}} \\
&= \frac{x^2\sqrt{ax+bx^4}}{4b} + \frac{\left(5a\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{8b^{5/3}\sqrt{ax+bx^4}} + \frac{\left(5\left(1-\sqrt{3}\right)a^{5/3}\sqrt{x}\sqrt{a+b}\right.}{8b^{5/3}} \\
&= -\frac{5\left(1+\sqrt{3}\right)ax\left(a+bx^3\right)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}} + \frac{x^2\sqrt{ax+bx^4}}{4b} + \frac{5\sqrt[4]{3}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0239624, size = 66, normalized size = 0.13

$$
\frac{x^3\left(-a\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)+a+bx^3\right)}{4b\sqrt{x\left(a+bx^3\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x^3*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -
((b*x^3)/a)]))/(4*b*Sqrt[x*(a + b*x^3)])
```

---

**Maple [C]** time = 0.016, size = 1079, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a*x)^(1/2),x)
```

```
[Out] 1/4*x^2*(b*x^4+a*x)^(1/2)/b-5/8/b*a*(x*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))
+(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*((-3/2/b*(-b^2*a)^(1
/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)*(x-1/b*(-b^2*a)^(1/3))^2*(1
/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(
-1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)
))^(1/2)*(1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*
a)^(1/3))/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b
^2*a)^(1/3)))^(1/2)*(((-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
)/b*(-b^2*a)^(1/3)+1/b^2*(-b^2*a)^(2/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/
2)/b*(-b^2*a)^(1/3))*b/(-b^2*a)^(1/3)*EllipticF(((-3/2/b*(-b^2*a)^(1/3)+1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2
```

```
*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2),((3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/
2)/b*(-b^2*a)^(1/3))*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/
(1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(3/2/b*(-b^2*a)^(1/3)
-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+(1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3))*EllipticE(((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2
*a)^(1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*
(-b^2*a)^(1/3)))^(1/2),((3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3
))*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(1/2/b*(-b^2*a)^(1
/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(3/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(
-b^2*a)^(1/3)))^(1/2))*b/(-b^2*a)^(1/3)))/(b*x*(x-1/b*(-b^2*a)^(1/3))*(x+1/
2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(1/3)-
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{bx^4 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/sqrt(b*x^4 + a*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + ax}\,x^4}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a*x)*x^4/(b*x^3 + a), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{x\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(x*(a + b*x**3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{bx^4 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/sqrt(b*x^4 + a*x), x)
```

## 3.101 $\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx$

**Optimal**. Leaf size=474

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}-\frac{\sqrt[4]{3}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{}$$

```
[Out] ((1 + Sqrt[3])*x*(a + b*x^3))/(b^(2/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)*
Sqrt[a*x + b*x^4]) - (3^(1/4)*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*
EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/
3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]) - ((1 - Sq
rt[3])*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/
3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sq
rt[3])/4])/(2*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

---

**Rubi [A]** time = 0.390082, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2032, 329, 308, 225, 1881}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}-\frac{\sqrt[4]{3}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x + b*x^4],x]
```

```
[Out] ((1 + Sqrt[3])*x*(a + b*x^3))/(b^(2/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)*
Sqrt[a*x + b*x^4]) - (3^(1/4)*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*
EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/
3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]) - ((1 - Sq
rt[3])*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/
3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sq
rt[3])/4])/(2*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \frac{\left(\sqrt{x}\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax+bx^4}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^4}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{b^{2/3}\sqrt{ax+bx^4}} - \frac{\left(\left(1-\sqrt{3}\right)a^{2/3}\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\Big(\int \frac{1}{\sqrt{a+bx}}}{b^{2/3}\sqrt{ax+bx^4}}$$

$$= \frac{\left(1+\sqrt{3}\right)x\left(a+bx^3\right)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}} - \frac{\sqrt[4]{3}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\Big(\cos^{-1}\Big(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{}}}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

**Mathematica [C]** time = 0.0117792, size = 53, normalized size = 0.11

$$\frac{2x^3\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)}{5\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)])
/(5*Sqrt[x*(a + b*x^3)])
```

---

**Maple [C]** time = 0.015, size = 1054, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a*x)^(1/2),x)
```

```
[Out] (x*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a
)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))+(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)
/b*(-b^2*a)^(1/3))*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*
x/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1
/3)))^(1/2)*(x-1/b*(-b^2*a)^(1/3))^2*(1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^
(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(-1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)*(1/b*(-b^2*a)^(1/3)*(x+1/2/
b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(-1/2/b*(-b^2*a)^(1/3)+1/2
*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)*(((-1/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/b*(-b^2*a)^(1/3)+1/b^2*(-b^2*a)^(2
/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*b/(-b^2*a)^(1/3
)*EllipticF(((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2
/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(
1/2),((3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*(1/2/b*(-b^2*a)
^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/
b*(-b^2*a)^(1/3))/(3/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1
/2))+(1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(((-3/2
/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2),((3/2/b*(-b^2
*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/
2)/b*(-b^2*a)^(1/3))/(1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/
(3/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))*b/(-b^2*a)^(1
/3)))/(b*x*(x-1/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(
-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/
2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^4+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*x^4 + a*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4+ax}\,x}{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a*x)*x/(b*x^3 + a), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x*(a + b*x**3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^4+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*x^4 + a*x), x)
```

## 3.102 $\int \frac{1}{x\sqrt{ax+bx^4}}\,dx$

**Optimal**. Leaf size=497

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}-\frac{2\sqrt[4]{3}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{}$$

```
[Out] (2*(1 + Sqrt[3])*b^(1/3)*x*(a + b*x^3))/(a*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)*Sqrt[a*x + b*x^4]) - (2*Sqrt[a*x + b*x^4])/(a*x) - (2*3^(1/4)*b^(1/3)*x
*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^
(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3
])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(a^(2
/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*Sqrt[a*x + b*x^4]) - ((1 - Sqrt[3])*b^(1/3)*x*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
+ (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(1/4)*a^(2/3)*Sqrt[(b^(1/
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x
 + b*x^4])
```

---

**Rubi [A]** time = 0.472092, antiderivative size = 497, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2025, 2032, 329, 308, 225, 1881}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}-\frac{2\sqrt[4]{3}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (2*(1 + Sqrt[3])*b^(1/3)*x*(a + b*x^3))/(a*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)*Sqrt[a*x + b*x^4]) - (2*Sqrt[a*x + b*x^4])/(a*x) - (2*3^(1/4)*b^(1/3)*x
*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^
(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3
])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(a^(2
/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*Sqrt[a*x + b*x^4]) - ((1 - Sqrt[3])*b^(1/3)*x*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
+ (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(1/4)*a^(2/3)*Sqrt[(b^(1/
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x
 + b*x^4])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

### Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

### Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{ax+bx^4}}\,dx &= -\frac{2\sqrt{ax+bx^4}}{ax}+\frac{(2b)\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx}{a}\\
&= -\frac{2\sqrt{ax+bx^4}}{ax}+\frac{\left(2b\sqrt{x}\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{a\sqrt{ax+bx^4}}\\
&= -\frac{2\sqrt{ax+bx^4}}{ax}+\frac{\left(4b\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax+bx^4}}\\
&= -\frac{2\sqrt{ax+bx^4}}{ax}-\frac{\left(2\sqrt[3]{b}\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax+bx^4}}-\left(2\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt{x}\right.\\
&= \frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}x\left(a+bx^3\right)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{ax}-\frac{2\sqrt[4]{3}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\right.}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}}}
\end{aligned}$$

**Mathematica [C]** time = 0.0114298, size = 48, normalized size = 0.1

$$-\frac{2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\frac{bx^3}{a}\right)}{\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/6, 1/2, 5/6, -((b*x^3)/a)])/Sq
rt[x*(a + b*x^3)]
```

---

**Maple [C]** time = 0.015, size = 1083, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a*x)^(1/2),x)
```

```
[Out] -2*(b*x^3+a)/a/((b*x^3+a)*x)^(1/2)+2*b/a*(x*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))+(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*((-3/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2)*(x-1/b*(-b^2*a)^(1/3))
^2*(1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/
3))/(-1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^
(1/3)))^(1/2)*(1/b*(-b^2*a)^(1/3)*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(
-b^2*a)^(1/3))/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x-1/
b*(-b^2*a)^(1/3)))^(1/2)*(((-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^
(1/3))/b*(-b^2*a)^(1/3)+1/b^2*(-b^2*a)^(2/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*b/(-b^2*a)^(1/3)*EllipticF(((-3/2/b*(-b^2*a)^(1/3
)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
```

```
(-b^2*a)^(1/3))/(x-1/b*(-b^2*a)^(1/3)))^(1/2),((3/2/b*(-b^2*a)^(1/3)+1/2*I*
3^(1/2)/b*(-b^2*a)^(1/3))*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1
/3))/(1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(3/2/b*(-b^2*a)^
(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+(1/2/b*(-b^2*a)^(1/3)+1/2*I*3
^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*
(-b^2*a)^(1/3))*x/(-1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(x
-1/b*(-b^2*a)^(1/3)))^(1/2),((3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)
^(1/3))*(1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(1/2/b*(-b^2*
a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))/(3/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2
)/b*(-b^2*a)^(1/3)))^(1/2))*b/(-b^2*a)^(1/3)))/(b*x*(x-1/b*(-b^2*a)^(1/3))*
(x+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*(x+1/2/b*(-b^2*a)^(
1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + axx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^4 + ax}}{bx^5 + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a*x)/(b*x^5 + a*x^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x*(a + b*x**3))), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + axx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x), x)
```

# 3.103 $\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx$

**Optimal**. Leaf size=174

$$\frac{63b^4\sqrt{ax+b\sqrt{x}}}{64a^5}-\frac{21b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^4}+\frac{21b^2x\sqrt{ax+b\sqrt{x}}}{40a^3}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{11/2}}-\frac{9bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^2}+\frac{2x^2\sqrt{ax+}}{5a}$$

```
[Out] (63*b^4*Sqrt[b*Sqrt[x] + a*x])/(64*a^5) - (21*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] +
a*x])/(32*a^4) + (21*b^2*x*Sqrt[b*Sqrt[x] + a*x])/(40*a^3) - (9*b*x^(3/2)*S
qrt[b*Sqrt[x] + a*x])/(20*a^2) + (2*x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a) - (63*
b^5*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(64*a^(11/2))
```

---

**Rubi [A]** time = 0.151043, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 670, 640, 620, 206}

$$\frac{63b^4\sqrt{ax+b\sqrt{x}}}{64a^5}-\frac{21b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^4}+\frac{21b^2x\sqrt{ax+b\sqrt{x}}}{40a^3}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{11/2}}-\frac{9bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^2}+\frac{2x^2\sqrt{ax+}}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (63*b^4*Sqrt[b*Sqrt[x] + a*x])/(64*a^5) - (21*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] +
a*x])/(32*a^4) + (21*b^2*x*Sqrt[b*Sqrt[x] + a*x])/(40*a^3) - (9*b*x^(3/2)*S
qrt[b*Sqrt[x] + a*x])/(20*a^2) + (2*x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a) - (63*
b^5*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(64*a^(11/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{(9b)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{5a}\\
&= -\frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} + \frac{\left(63b^2\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{40a^2}\\
&= \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{\left(21b^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{16a^3}\\
&= -\frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} + \frac{\left(63b^4\right)\text{Subs}}{}\\
&= \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+}}{5a}\\
&= \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+}}{5a}\\
&= \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+}}{5a}
\end{aligned}
$$

**Mathematica [A]** time = 0.170274, size = 151, normalized size = 0.87

$$
\frac{\left(a\sqrt{x}+b\right)\left(\sqrt{a}\sqrt{x}\sqrt{\frac{a\sqrt{x}}{b}+1}\left(168a^2b^2x-144a^3bx^{3/2}+128a^4x^2-210ab^3\sqrt{x}+315b^4\right)-315b^{9/2}\sqrt[4]{x}\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)\right)}{320a^{11/2}\sqrt{\frac{a\sqrt{x}}{b}+1}\sqrt{ax+b\sqrt{x}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] ((b + a*Sqrt[x])*(Sqrt[a]*Sqrt[1 + (a*Sqrt[x])/b]*Sqrt[x]*(315*b^4 - 210*a*
b^3*Sqrt[x] + 168*a^2*b^2*x - 144*a^3*b*x^(3/2) + 128*a^4*x^2) - 315*b^(9/2
)*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]]))/(320*a^(11/2)*Sqrt[1 + (a*Sq
rt[x])/b]*Sqrt[b*Sqrt[x] + a*x])
```

---

**Maple [A]** time = 0.06, size = 223, normalized size = 1.3

$$\frac{1}{640}\sqrt{b\sqrt{x}+ax}\left(-544\sqrt{x}a^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}b+256\,x\left(b\sqrt{x}+ax\right)^{3/2}a^{9/2}-1300\sqrt{x}a^{5/2}\sqrt{b\sqrt{x}+axb^{3}}+880\,a^{5/2}\left(b\sqrt{x}+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] 1/640*(b*x^(1/2)+a*x)^(1/2)*(-544*x^(1/2)*a^(7/2)*(b*x^(1/2)+a*x)^(3/2)*b+2
56*x*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)-1300*x^(1/2)*a^(5/2)*(b*x^(1/2)+a*x)^(1/
2)*b^3+880*a^(5/2)*(b*x^(1/2)+a*x)^(3/2)*b^2+1280*(x^(1/2)*(b+a*x^(1/2)))^(
1/2)*a^(3/2)*b^4-640*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^
(1/2)+b)/a^(1/2))*a*b^5-650*a^(3/2)*(b*x^(1/2)+a*x)^(1/2)*b^4+325*ln(1/2*(2
*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^5)/(x^(1/2)*(b+a
*x^(1/2)))^(1/2)/a^(13/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a*x + b*sqrt(x)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*x + b*sqrt(x)), x)
```

---

**Giac [A]** time = 1.39757, size = 150, normalized size = 0.86

$$\frac{1}{320}\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\sqrt{x}\left(\frac{8\sqrt{x}}{a}-\frac{9b}{a^2}\right)+\frac{21b^2}{a^3}\right)\sqrt{x}-\frac{105b^3}{a^4}\right)\sqrt{x}+\frac{315b^4}{a^5}\right)+\frac{63b^5\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b}\right.\right.\right.}{128a^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/320*sqrt(a*x + b*sqrt(x))*(2*(4*(2*sqrt(x)*(8*sqrt(x)/a - 9*b/a^2) + 21*b
^2/a^3)*sqrt(x) - 105*b^3/a^4)*sqrt(x) + 315*b^4/a^5) + 63/128*b^5*log(abs(
-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(11/2)
```

## 3.104 $\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx$

**Optimal**. Leaf size=116

$$\frac{5b^2\sqrt{ax+b\sqrt{x}}}{4a^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{7/2}}-\frac{5b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^2}+\frac{2x\sqrt{ax+b\sqrt{x}}}{3a}$$

```
[Out] (5*b^2*Sqrt[b*Sqrt[x] + a*x])/(4*a^3) - (5*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])
/(6*a^2) + (2*x*Sqrt[b*Sqrt[x] + a*x])/(3*a) - (5*b^3*ArcTanh[(Sqrt[a]*Sqrt
[x])/Sqrt[b*Sqrt[x] + a*x]])/(4*a^(7/2))
```

---

**Rubi [A]** time = 0.0868686, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2018, 670, 640, 620, 206}

$$\frac{5b^2\sqrt{ax+b\sqrt{x}}}{4a^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{7/2}}-\frac{5b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^2}+\frac{2x\sqrt{ax+b\sqrt{x}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (5*b^2*Sqrt[b*Sqrt[x] + a*x])/(4*a^3) - (5*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])
/(6*a^2) + (2*x*Sqrt[b*Sqrt[x] + a*x])/(3*a) - (5*b^3*ArcTanh[(Sqrt[a]*Sqrt
[x])/Sqrt[b*Sqrt[x] + a*x]])/(4*a^(7/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(5b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{3a}\\
&= -\frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} + \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^2}\\
&= \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{8a^3}\\
&= \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^3}\\
&= \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^{7/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.106052, size = 129, normalized size = 1.11

$$
\frac{5b^4\left(\frac{a\sqrt{x}}{b}+1\right)\left(\frac{16a^3x^{3/2}}{15b^3} - \frac{4a^2x}{3b^2} + \frac{2a\sqrt{x}}{b} - \frac{2\sqrt{a}\,\sqrt[4]{x}\sinh^{-1}\left(\frac{\sqrt{a}\,\sqrt[4]{x}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{\frac{a\sqrt{x}}{b}+1}}\right)}{8a^4\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (5*b^4*(1 + (a*Sqrt[x])/b)*((2*a*Sqrt[x])/b - (4*a^2*x)/(3*b^2) + (16*a^3*x
^(3/2))/(15*b^3) - (2*Sqrt[a]*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]])/(
Sqrt[b]*Sqrt[1 + (a*Sqrt[x])/b])))/(8*a^4*Sqrt[(b + a*Sqrt[x])*Sqrt[x]])
```

---

**Maple [B]** time = 0.007, size = 181, normalized size = 1.6

$$
\frac{1}{24}\sqrt{b\sqrt{x}+ax}\left(16\,\left(b\sqrt{x}+ax\right)^{3/2}a^{5/2} - 36\,\sqrt{b\sqrt{x}+ax}a^{5/2}\sqrt{x}b - 18\,\sqrt{b\sqrt{x}+ax}a^{3/2}b^2 + 48\,a^{3/2}\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}b^2 - 24\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] 1/24*(b*x^(1/2)+a*x)^(1/2)/a^(9/2)*(16*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)-36*(b*
x^(1/2)+a*x)^(1/2)*a^(5/2)*x^(1/2)*b-18*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b^2+4
8*a^(3/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^2-24*a*ln(1/2*(2*(x^(1/2)*(b+a*x^
(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^3+9*ln(1/2*(2*a*x^(1/2)+2*(
b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^3)/(x^(1/2)*(b+a*x^(1/2)))^(1/
2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(a*x + b*sqrt(x)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*x + b*sqrt(x)), x)
```

---

**Giac [A]** time = 1.36586, size = 112, normalized size = 0.97

$$\frac{1}{12}\sqrt{ax+b\sqrt{x}}\left(2\sqrt{x}\left(\frac{4\sqrt{x}}{a}-\frac{5b}{a^{2}}\right)+\frac{15b^{2}}{a^{3}}\right)+\frac{5b^{3}\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)*(4*sqrt(x)/a - 5*b/a^2) + 15*b^2/a^3)
 + 5/8*b^3*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b
))/a^(7/2)
```

## 3.105 $\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx$

**Optimal**. Leaf size=56

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}$$

```
[Out] (2*Sqrt[b*Sqrt[x] + a*x])/a - (2*b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x]
 + a*x]])/a^(3/2)
```

---

**Rubi [A]** time = 0.059945, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2010, 2013, 620, 206}

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (2*Sqrt[b*Sqrt[x] + a*x])/a - (2*b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x]
 + a*x]])/a^(3/2)
```

Rule 2010

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[(-2*Sqrt
[a*x^j + b*x^n])/(b*(n - 2)*x^(n - 1)), x] - Dist[(a*(2*n - j - 2))/(b*(n -
 2)), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt{x}+ax}}{a} - \frac{b\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx}{2a}$$

$$= \frac{2\sqrt{b\sqrt{x}+ax}}{a} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{2\sqrt{b\sqrt{x}+ax}}{a} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a}$$

$$= \frac{2\sqrt{b\sqrt{x}+ax}}{a} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}$$

**Mathematica [A]** time = 0.0479315, size = 88, normalized size = 1.57

$$\frac{2\sqrt{a}\sqrt{x}\left(a\sqrt{x}+b\right)-2b^{3/2}\sqrt[4]{x}\sqrt{\frac{a\sqrt{x}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{a^{3/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (2*Sqrt[a]*(b + a*Sqrt[x])*Sqrt[x] - 2*b^(3/2)*Sqrt[1 + (a*Sqrt[x])/b]*x^(1
/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]])/(a^(3/2)*Sqrt[b*Sqrt[x] + a*x])
```

---

**Maple [A]** time = 0.003, size = 83, normalized size = 1.5

$$-\sqrt{b\sqrt{x}+ax}\left(b\ln\left(\frac{1}{2}\left(2\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}\sqrt{a}+2a\sqrt{x}+b\right)\frac{1}{\sqrt{a}}\right)-2\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}\sqrt{a}\right)\frac{1}{\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}}a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] -(b*x^(1/2)+a*x)^(1/2)*(b*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2
*a*x^(1/2)+b)/a^(1/2))-2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2))/(x^(1/2)*(b
+a*x^(1/2)))^(1/2)/a^(3/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x + b*sqrt(x)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*sqrt(x)), x)
```

---

**Giac [A]** time = 1.3343, size = 73, normalized size = 1.3

$$\frac{b\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}\right) - b\right|\right)}{a^{\frac{3}{2}}} + \frac{2\sqrt{ax + b\sqrt{x}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] b*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(3/2
) + 2*sqrt(a*x + b*sqrt(x))/a
```

## 3.106 $\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx$

**Optimal**. Leaf size=25

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])
```

---

**Rubi [A]** time = 0.0379828, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$$

**Mathematica [A]** time = 0.0080369, size = 25, normalized size = 1.

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])
```

---

**Maple [C]** time = 0.011, size = 160, normalized size = 6.4

$$-\frac{1}{b^2x}\sqrt{b\sqrt{x}+ax}\,\Big(4\,\left(b\sqrt{x}+ax\right)^{3/2}\sqrt{a}-2\,\sqrt{b\sqrt{x}+ax}a^{3/2}x-\ln\Big(\frac{1}{2}\Big(2\,a\sqrt{x}+2\,\sqrt{b\sqrt{x}+ax}\sqrt{a}+b\Big)\frac{1}{\sqrt{a}}\Big)xab-2\,a^{3/2}\sqrt{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] -(b*x^(1/2)+a*x)^(1/2)*(4*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)-2*(b*x^(1/2)+a*x)^(
1/2)*a^(3/2)*x-ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/
2))*x*a*b-2*a^(3/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*x+ln(1/2*(2*(x^(1/2)*(b+a
*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x*a*b)/(x^(1/2)*(b+a*x^(1/
2)))^(1/2)/b^2/x/a^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x), x)
```

---

**Fricas [A]** time = 2.43133, size = 51, normalized size = 2.04

$$-\frac{4\,\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4*sqrt(a*x + b*sqrt(x))/(b*sqrt(x))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*x + b*sqrt(x))), x)
```

---

**Giac [A]** time = 1.22164, size = 34, normalized size = 1.36

$$\frac{4}{\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))
```

# 3.107 $\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx$

**Optimal**. Leaf size=84

$$-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{15b^3\sqrt{x}}+\frac{16a\sqrt{ax+b\sqrt{x}}}{15b^2x}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) + (16*a*Sqrt[b*Sqrt[x] + a*x])/(15
*b^2*x) - (32*a^2*Sqrt[b*Sqrt[x] + a*x])/(15*b^3*Sqrt[x])
```

---

**Rubi [A]** time = 0.11658, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{15b^3\sqrt{x}}+\frac{16a\sqrt{ax+b\sqrt{x}}}{15b^2x}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) + (16*a*Sqrt[b*Sqrt[x] + a*x])/(15
*b^2*x) - (32*a^2*Sqrt[b*Sqrt[x] + a*x])/(15*b^3*Sqrt[x])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx &= -\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}-\frac{(4a)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{5b}\\ &= -\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}+\frac{16a\sqrt{b\sqrt{x}+ax}}{15b^2x}+\frac{\left(8a^2\right)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{15b^2}\\ &= -\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}+\frac{16a\sqrt{b\sqrt{x}+ax}}{15b^2x}-\frac{32a^2\sqrt{b\sqrt{x}+ax}}{15b^3\sqrt{x}}\end{aligned}$$

**Mathematica [A]** time = 0.0481187, size = 48, normalized size = 0.57

$$-\frac{4\sqrt{ax+b\sqrt{x}}\left(8a^2x-4ab\sqrt{x}+3b^2\right)}{15b^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(3*b^2 - 4*a*b*Sqrt[x] + 8*a^2*x))/(15*b^3*x^(3/2
))
```

---

**Maple [C]** time = 0.01, size = 218, normalized size = 2.6

$$-\frac{1}{15\,b^4}\sqrt{b\sqrt{x}+ax}\left(60\,\left(b\sqrt{x}+ax\right)^{3/2}a^{5/2}x^{5/2}-30\,\sqrt{b\sqrt{x}+ax}a^{7/2}x^{7/2}-15\,\ln\left(1/2\,\frac{2\,a\sqrt{x}+2\,\sqrt{b\sqrt{x}+ax}\sqrt{a}+b}{\sqrt{a}}\right)x^{7/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] -1/15*(b*x^(1/2)+a*x)^(1/2)*(60*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*x^(5/2)-30*(b
*x^(1/2)+a*x)^(1/2)*a^(7/2)*x^(7/2)-15*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x
)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(7/2)*a^3*b-30*a^(7/2)*x^(7/2)*(x^(1/2)*(b+a*
x^(1/2)))^(1/2)+15*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1
/2)+b)/a^(1/2))*x^(7/2)*a^3*b+12*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*x^(3/2)*b^2-
28*a^(3/2)*(b*x^(1/2)+a*x)^(3/2)*b*x^2)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b^4/a
^(1/2)/x^(7/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^2), x)
```

---

**Fricas [A]** time = 2.31516, size = 103, normalized size = 1.23

$$\frac{4\left(4\,abx-\left(8\,a^2x+3\,b^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{15\,b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/15*(4*a*b*x - (8*a^2*x + 3*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^3*x^2)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*x + b*sqrt(x))), x)
```

---

**Giac [A]** time = 1.25171, size = 113, normalized size = 1.35

$$\frac{4\left(20\,a\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^2+15\,\sqrt{a}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+3\,b^2\right)}{15\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(20*a*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 15*sqrt(a)*b*(sqrt
(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 3*b^2)/(sqrt(a)*sqrt(x) - sqrt(a*x +
 b*sqrt(x)))^5
```

## 3.108 $\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx$

**Optimal**. Leaf size=142

$$-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{105b^3x^{3/2}}-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{315b^5\sqrt{x}}+\frac{256a^3\sqrt{ax+b\sqrt{x}}}{315b^4x}+\frac{32a\sqrt{ax+b\sqrt{x}}}{63b^2x^2}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(9*b*x^(5/2)) + (32*a*Sqrt[b*Sqrt[x] + a*x])/(63
*b^2*x^2) - (64*a^2*Sqrt[b*Sqrt[x] + a*x])/(105*b^3*x^(3/2)) + (256*a^3*Sqr
t[b*Sqrt[x] + a*x])/(315*b^4*x) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(315*b^5*
Sqrt[x])
```

---

**Rubi [A]** time = 0.203379, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{105b^3x^{3/2}}-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{315b^5\sqrt{x}}+\frac{256a^3\sqrt{ax+b\sqrt{x}}}{315b^4x}+\frac{32a\sqrt{ax+b\sqrt{x}}}{63b^2x^2}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(9*b*x^(5/2)) + (32*a*Sqrt[b*Sqrt[x] + a*x])/(63
*b^2*x^2) - (64*a^2*Sqrt[b*Sqrt[x] + a*x])/(105*b^3*x^(3/2)) + (256*a^3*Sqr
t[b*Sqrt[x] + a*x])/(315*b^4*x) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(315*b^5*
Sqrt[x])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} - \frac{(8a)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{9b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} + \frac{\left(16a^2\right)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{21b^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}} - \frac{\left(64a^3\right)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{105b^3}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}} + \frac{256a^3\sqrt{b\sqrt{x}+ax}}{315b^4x} + \frac{\left(128a^4\right)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}}{315b^4}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}} + \frac{256a^3\sqrt{b\sqrt{x}+ax}}{315b^4x} - \frac{512a^4\sqrt{b\sqrt{x}+ax}}{315b^5\sqrt{x}}$$

**Mathematica [A]** time = 0.0528787, size = 72, normalized size = 0.51

$$-\frac{4\sqrt{ax+b\sqrt{x}}\left(48a^2b^2x-64a^3bx^{3/2}+128a^4x^2-40ab^3\sqrt{x}+35b^4\right)}{315b^5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(35*b^4 - 40*a*b^3*Sqrt[x] + 48*a^2*b^2*x - 64*a^
3*b*x^(3/2) + 128*a^4*x^2))/(315*b^5*x^(5/2))
```

---

**Maple [C]** time = 0.013, size = 262, normalized size = 1.9

$$-\frac{1}{315\,b^6}\sqrt{b\sqrt{x}+ax}\left(1260\,\left(b\sqrt{x}+ax\right)^{3/2}a^{9/2}x^{9/2}-630\,\sqrt{b\sqrt{x}+ax}\,a^{11/2}x^{11/2}-315\,\ln\left(1/2\,\frac{2\,a\sqrt{x}+2\,\sqrt{b\sqrt{x}+ax}\sqrt{a}+b}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] -1/315*(b*x^(1/2)+a*x)^(1/2)*(1260*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)*x^(9/2)-63
0*(b*x^(1/2)+a*x)^(1/2)*a^(11/2)*x^(11/2)-315*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1
/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(11/2)*a^5*b-630*a^(11/2)*x^(11/2)*(x^
(1/2)*(b+a*x^(1/2)))^(1/2)+315*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1
/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(11/2)*a^5*b+492*(b*x^(1/2)+a*x)^(3/2)*a^(5/2
)*x^(7/2)*b^2+140*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*x^(5/2)*b^4-748*a^(7/2)*(b*
x^(1/2)+a*x)^(3/2)*b*x^4-300*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*x^3*b^3)/(x^(1/2
)*(b+a*x^(1/2)))^(1/2)/b^6/a^(1/2)/x^(11/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^3), x)
```

---

**Fricas [A]** time = 2.33588, size = 155, normalized size = 1.09

$$\frac{4\left(64\,a^{3}bx^{2}+40\,ab^{3}x-\left(128\,a^{4}x^{2}+48\,a^{2}b^{2}x+35\,b^{4}\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{315\,b^{5}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/315*(64*a^3*b*x^2 + 40*a*b^3*x - (128*a^4*x^2 + 48*a^2*b^2*x + 35*b^4)*sq
rt(x))*sqrt(a*x + b*sqrt(x))/(b^5*x^3)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^{3}\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a*x + b*sqrt(x))), x)
```

---

**Giac [A]** time = 1.16857, size = 197, normalized size = 1.39

$$\frac{4\Bigl(1008\,a^{2}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{4}+1680\,a^{\frac{3}{2}}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{3}+1080\,ab^{2}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{2}+315\,\sqrt{a}b^{3}\Bigl(}{315\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/315*(1008*a^2*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 1680*a^(3/2)*
b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 1080*a*b^2*(sqrt(a)*sqrt(x)
 - sqrt(a*x + b*sqrt(x)))^2 + 315*sqrt(a)*b^3*(sqrt(a)*sqrt(x) - sqrt(a*x +
 b*sqrt(x))) + 35*b^4)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^9
```

## 3.109 $\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx$

**Optimal**. Leaf size=200

$$-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{1001b^5x^{3/2}}+\frac{1280a^3\sqrt{ax+b\sqrt{x}}}{3003b^4x^2}-\frac{160a^2\sqrt{ax+b\sqrt{x}}}{429b^3x^{5/2}}-\frac{4096a^6\sqrt{ax+b\sqrt{x}}}{3003b^7\sqrt{x}}+\frac{2048a^5\sqrt{ax+b\sqrt{x}}}{3003b^6x}+\frac{48a\sqrt{ax+b}}{143b^2x^3}$$

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(13*b*x^(7/2)) + (48*a*Sqrt[b*Sqrt[x] + a*x])/(1
43*b^2*x^3) - (160*a^2*Sqrt[b*Sqrt[x] + a*x])/(429*b^3*x^(5/2)) + (1280*a^3
*Sqrt[b*Sqrt[x] + a*x])/(3003*b^4*x^2) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(1
001*b^5*x^(3/2)) + (2048*a^5*Sqrt[b*Sqrt[x] + a*x])/(3003*b^6*x) - (4096*a^
6*Sqrt[b*Sqrt[x] + a*x])/(3003*b^7*Sqrt[x])
```

---

**Rubi [A]** time = 0.296884, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{1001b^5x^{3/2}}+\frac{1280a^3\sqrt{ax+b\sqrt{x}}}{3003b^4x^2}-\frac{160a^2\sqrt{ax+b\sqrt{x}}}{429b^3x^{5/2}}-\frac{4096a^6\sqrt{ax+b\sqrt{x}}}{3003b^7\sqrt{x}}+\frac{2048a^5\sqrt{ax+b\sqrt{x}}}{3003b^6x}+\frac{48a\sqrt{ax+b}}{143b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(13*b*x^(7/2)) + (48*a*Sqrt[b*Sqrt[x] + a*x])/(1
43*b^2*x^3) - (160*a^2*Sqrt[b*Sqrt[x] + a*x])/(429*b^3*x^(5/2)) + (1280*a^3
*Sqrt[b*Sqrt[x] + a*x])/(3003*b^4*x^2) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(1
001*b^5*x^(3/2)) + (2048*a^5*Sqrt[b*Sqrt[x] + a*x])/(3003*b^6*x) - (4096*a^
6*Sqrt[b*Sqrt[x] + a*x])/(3003*b^7*Sqrt[x])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} - \frac{(12a)\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx}{13b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} + \frac{\left(120a^2\right)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{143b^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} - \frac{\left(320a^3\right)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{429b^3}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2} + \frac{\left(640a^4\right)\int \frac{}{x^2\sqrt{b}}}{1001b^4}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2} - \frac{512a^4\sqrt{b\sqrt{x}+}}{1001b^5x^{3/2}}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2} - \frac{512a^4\sqrt{b\sqrt{x}+}}{1001b^5x^{3/2}}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2} - \frac{512a^4\sqrt{b\sqrt{x}+}}{1001b^5x^{3/2}}$$

**Mathematica [A]** time = 0.059094, size = 96, normalized size = 0.48

$$-\frac{4\sqrt{ax+b\sqrt{x}}\left(384a^4b^2x^2-320a^3b^3x^{3/2}+280a^2b^4x-512a^5bx^{5/2}+1024a^6x^3-252ab^5\sqrt{x}+231b^6\right)}{3003b^7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(231*b^6 - 252*a*b^5*Sqrt[x] + 280*a^2*b^4*x - 32
0*a^3*b^3*x^(3/2) + 384*a^4*b^2*x^2 - 512*a^5*b*x^(5/2) + 1024*a^6*x^3))/(3
003*b^7*x^(7/2))
```

---

**Maple [C]** time = 0.013, size = 306, normalized size = 1.5

$$-\frac{1}{3003\,b^8}\sqrt{b\sqrt{x}+ax}\left(12012\,\left(b\sqrt{x}+ax\right)^{3/2}a^{13/2}x^{13/2}-6006\,\sqrt{b\sqrt{x}+ax}a^{15/2}x^{15/2}-3003\,\ln\left(1/2\,\frac{2\,a\sqrt{x}+2\,\sqrt{b\sqrt{x}+}}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] -1/3003*(b*x^(1/2)+a*x)^(1/2)*(12012*(b*x^(1/2)+a*x)^(3/2)*a^(13/2)*x^(13/2
)-6006*(b*x^(1/2)+a*x)^(1/2)*a^(15/2)*x^(15/2)-3003*ln(1/2*(2*a*x^(1/2)+2*(
b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(15/2)*a^7*b-6006*a^(15/2)*x^(15
/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)+3003*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1
/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(15/2)*a^7*b+5868*(b*x^(1/2)+a*x)^(3/
2)*a^(9/2)*x^(11/2)*b^2+3052*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*x^(9/2)*b^4-7916
*a^(11/2)*(b*x^(1/2)+a*x)^(3/2)*b*x^6+924*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*x^(
```

```
7/2)*b^6-4332*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*x^5*b^3-1932*(b*x^(1/2)+a*x)^(3
/2)*a^(3/2)*x^4*b^5)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b^8/a^(1/2)/x^(15/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^4), x)
```

---

**Fricas [A]** time = 2.41402, size = 212, normalized size = 1.06

$$\frac{4\left(512\,a^{5}bx^{3}+320\,a^{3}b^{3}x^{2}+252\,ab^{5}x-\left(1024\,a^{6}x^{3}+384\,a^{4}b^{2}x^{2}+280\,a^{2}b^{4}x+231\,b^{6}\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{3003\,b^{7}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3003*(512*a^5*b*x^3 + 320*a^3*b^3*x^2 + 252*a*b^5*x - (1024*a^6*x^3 + 384
*a^4*b^2*x^2 + 280*a^2*b^4*x + 231*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^7
*x^4)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{4}\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a*x + b*sqrt(x))), x)
```

---

**Giac [A]** time = 1.27784, size = 281, normalized size = 1.4

$$\frac{4\Bigl(27456\,a^{3}\bigl(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\bigr)^{6}+72072\,a^{\frac{5}{2}}b\bigl(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\bigr)^{5}+80080\,a^{2}b^{2}\bigl(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\bigr)^{4}+48048\,a}{3003\Bigl(\sqrt{a}\sqrt{x}-\sqrt{ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3003*(27456*a^3*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^6 + 72072*a^(5/
2)*b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5 + 80080*a^2*b^2*(sqrt(a)*s
qrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 48048*a^(3/2)*b^3*(sqrt(a)*sqrt(x) - sq
rt(a*x + b*sqrt(x)))^3 + 16380*a*b^4*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x
)))^2 + 3003*sqrt(a)*b^5*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 231*b^
6)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^13
```

## 3.110 $\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=197

$$\frac{693b^4\sqrt{ax+b\sqrt{x}}}{64a^6}-\frac{231b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^5}+\frac{231b^2x\sqrt{ax+b\sqrt{x}}}{40a^4}-\frac{693b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{13/2}}-\frac{99bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^3}+22x^2$$

```
[Out] (-4*x^3)/(a*Sqrt[b*Sqrt[x] + a*x]) + (693*b^4*Sqrt[b*Sqrt[x] + a*x])/(64*a^
6) - (231*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(32*a^5) + (231*b^2*x*Sqrt[b*S
qrt[x] + a*x])/(40*a^4) - (99*b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(20*a^3) + (
22*x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a^2) - (693*b^5*ArcTanh[(Sqrt[a]*Sqrt[x])/
Sqrt[b*Sqrt[x] + a*x]])/(64*a^(13/2))
```

---

**Rubi [A]** time = 0.175458, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 668, 670, 640, 620, 206}

$$\frac{693b^4\sqrt{ax+b\sqrt{x}}}{64a^6}-\frac{231b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^5}+\frac{231b^2x\sqrt{ax+b\sqrt{x}}}{40a^4}-\frac{693b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{13/2}}-\frac{99bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^3}+22x^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^3)/(a*Sqrt[b*Sqrt[x] + a*x]) + (693*b^4*Sqrt[b*Sqrt[x] + a*x])/(64*a^
6) - (231*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(32*a^5) + (231*b^2*x*Sqrt[b*S
qrt[x] + a*x])/(40*a^4) - (99*b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(20*a^3) + (
22*x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a^2) - (693*b^5*ArcTanh[(Sqrt[a]*Sqrt[x])/
Sqrt[b*Sqrt[x] + a*x]])/(64*a^(13/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
```

```
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^7}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}}+\frac{22\,\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}}+\frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2}-\frac{(99b)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{5a^2}\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}}-\frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3}+\frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2}+\frac{\left(693b^2\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x,\right.}{40a^3}\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}}+\frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4}-\frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3}+\frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2}-\left(231b^3\right)\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}}-\frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5}+\frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4}-\frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3}+\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}}+\frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6}-\frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5}+\frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4}-\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}}+\frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6}-\frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5}+\frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4}-\\
&= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}}+\frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6}-\frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5}+\frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4}-
\end{aligned}
$$

**Mathematica [C]** time = 0.059961, size = 64, normalized size = 0.32

$$\frac{4x^{7/2}\sqrt{\frac{a\sqrt{x}}{b}+1}\,{}_2F_1\left(\frac{3}{2},\frac{13}{2};\frac{15}{2};-\frac{a\sqrt{x}}{b}\right)}{13b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^(7/2)*Hypergeometric2F1[3/2, 13/2, 15/2, -((a*
Sqrt[x])/b)])/(13*b*Sqrt[b*Sqrt[x] + a*x])
```

---

**Maple [B]** time = 0.012, size = 549, normalized size = 2.8

$$\frac{1}{640}\sqrt{b\sqrt{x}+ax}\Bigg(256\left(b\sqrt{x}+ax\right)^{3/2}a^{13/2}x^{2}-352\left(b\sqrt{x}+ax\right)^{3/2}a^{11/2}x^{3/2}b-4060\sqrt{b\sqrt{x}+ax}a^{9/2}x^{3/2}b^{3}+528\,\big(b\sqrt{x}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(1/2)+a*x)^(3/2),x)
```

```
[Out] 1/640*(b*x^(1/2)+a*x)^(1/2)*(256*(b*x^(1/2)+a*x)^(3/2)*a^(13/2)*x^2-352*(b*
x^(1/2)+a*x)^(3/2)*a^(11/2)*x^(3/2)*b-4060*(b*x^(1/2)+a*x)^(1/2)*a^(9/2)*x^
(3/2)*b^3+528*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)*x*b^2+3136*(b*x^(1/2)+a*x)^(3/2
)*a^(7/2)*x^(1/2)*b^3-10150*(b*x^(1/2)+a*x)^(1/2)*a^(7/2)*x*b^4+8960*a^(7/2
)*x*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^4+2000*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*b^
4-8120*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*x^(1/2)*b^5+17920*a^(5/2)*x^(1/2)*(x^(
1/2)*(b+a*x^(1/2)))^(1/2)*b^5-2560*a^(5/2)*(x^(1/2)*(b+a*x^(1/2)))^(3/2)*b^
4-2030*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b^6+8960*a^(3/2)*(x^(1/2)*(b+a*x^(1/2)
))^(1/2)*b^6+2030*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^
(1/2))*x^(1/2)*a^2*b^6+1015*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(
1/2)+b)/a^(1/2))*x*a^3*b^5-8960*a^2*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)
*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(1/2)*b^6-4480*a^3*ln(1/2*(2*(x^(1/2)*(b
+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x*b^5+1015*ln(1/2*(2*a*x
^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^7-4480*a*ln(1/2*(2*(
x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^7)/a^(15/2)/
(x^(1/2)*(b+a*x^(1/2)))^(1/2)/(b+a*x^(1/2))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{3}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(a*x + b*sqrt(x))^(3/2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(a*x + b*sqrt(x))**(3/2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.111 $\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}} dx$

**Optimal**. Leaf size=139

$$\frac{35b^2\sqrt{ax+b\sqrt{x}}}{4a^4}-\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{9/2}}-\frac{35b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^3}+\frac{14x\sqrt{ax+b\sqrt{x}}}{3a^2}-\frac{4x^2}{a\sqrt{ax+b\sqrt{x}}}$$

```
[Out] (-4*x^2)/(a*Sqrt[b*Sqrt[x] + a*x]) + (35*b^2*Sqrt[b*Sqrt[x] + a*x])/(4*a^4)
 - (35*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(6*a^3) + (14*x*Sqrt[b*Sqrt[x] + a*
x])/(3*a^2) - (35*b^3*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(4*
a^(9/2))
```

---

**Rubi [A]** time = 0.126971, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 668, 670, 640, 620, 206}

$$\frac{35b^2\sqrt{ax+b\sqrt{x}}}{4a^4}-\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{9/2}}-\frac{35b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^3}+\frac{14x\sqrt{ax+b\sqrt{x}}}{3a^2}-\frac{4x^2}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^2)/(a*Sqrt[b*Sqrt[x] + a*x]) + (35*b^2*Sqrt[b*Sqrt[x] + a*x])/(4*a^4)
 - (35*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(6*a^3) + (14*x*Sqrt[b*Sqrt[x] + a*
x])/(3*a^2) - (35*b^3*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(4*
a^(9/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
```

```
+ 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^5}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{14\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a} \\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{(35b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{3a^2} \\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} + \frac{\left(35b^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^3} \\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{\left(35b^3\right)\text{Subst}\left(\int \right.}{8} \\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{\left(35b^3\right)\text{Subst}\left(\int \right.}{} \\
&= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{35b^3\tanh^{-1}\left(\right.}{4a^{9/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0430175, size = 64, normalized size = 0.46

$$\frac{4x^{5/2}\sqrt{\frac{a\sqrt{x}}{b}+1}\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};-\frac{a\sqrt{x}}{b}\right)}{9b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^(5/2)*Hypergeometric2F1[3/2, 9/2, 11/2, -((a*S
qrt[x])/b)])/(9*b*Sqrt[b*Sqrt[x] + a*x])
```

---

**Maple [B]** time = 0.01, size = 503, normalized size = 3.6

$$\frac{1}{24}\sqrt{b\sqrt{x}+ax}\left(16\,x\left(b\sqrt{x}+ax\right)^{3/2}a^{9/2}-60\,\sqrt{b\sqrt{x}+ax}a^{9/2}x^{3/2}b+32\,\sqrt{x}a^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}b-150\,\sqrt{b\sqrt{x}+ax}a^{7/2}xb^{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/2)+a*x)^(3/2),x)
```

```
[Out] 1/24*(b*x^(1/2)+a*x)^(1/2)/a^(11/2)*(16*x*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)-60*
(b*x^(1/2)+a*x)^(1/2)*a^(9/2)*x^(3/2)*b+32*x^(1/2)*a^(7/2)*(b*x^(1/2)+a*x)^
(3/2)*b-150*(b*x^(1/2)+a*x)^(1/2)*a^(7/2)*x*b^2+240*a^(7/2)*x*(x^(1/2)*(b+a
*x^(1/2)))^(1/2)*b^2-120*a^3*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2
)+2*a*x^(1/2)+b)/a^(1/2))*x*b^3+16*a^(5/2)*(b*x^(1/2)+a*x)^(3/2)*b^2-120*x^
(1/2)*a^(5/2)*(b*x^(1/2)+a*x)^(1/2)*b^3+480*a^(5/2)*x^(1/2)*(x^(1/2)*(b+a*x
^(1/2)))^(1/2)*b^3-96*a^(5/2)*(x^(1/2)*(b+a*x^(1/2)))^(3/2)*b^2-240*a^2*ln(
1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(1/2
)*b^4+15*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x*
a^3*b^3-30*a^(3/2)*(b*x^(1/2)+a*x)^(1/2)*b^4+240*(x^(1/2)*(b+a*x^(1/2)))^(1
/2)*a^(3/2)*b^4-120*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(
1/2)+b)/a^(1/2))*a*b^5+30*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/
2)+b)/a^(1/2))*x^(1/2)*a^2*b^4+15*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/
2)*a^(1/2)+b)/a^(1/2))*a*b^5)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/(b+a*x^(1/2))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x + b*sqrt(x))^(3/2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(a*x + b*sqrt(x))**(3/2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.112 $\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=77

$$\frac{6\sqrt{ax+b\sqrt{x}}}{a^2}-\frac{6b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{5/2}}-\frac{4x}{a\sqrt{ax+b\sqrt{x}}}$$

```
[Out] (-4*x)/(a*Sqrt[b*Sqrt[x] + a*x]) + (6*Sqrt[b*Sqrt[x] + a*x])/a^2 - (6*b*Arc
Tanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/a^(5/2)
```

---

**Rubi [A]** time = 0.0742663, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2018, 668, 640, 620, 206}

$$\frac{6\sqrt{ax+b\sqrt{x}}}{a^2}-\frac{6b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{5/2}}-\frac{4x}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x)/(a*Sqrt[b*Sqrt[x] + a*x]) + (6*Sqrt[b*Sqrt[x] + a*x])/a^2 - (6*b*Arc
Tanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/a^(5/2)
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^3}{\left(bx+ax^2\right)^{3/2}}\,dx,x,\sqrt{x}\right)\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{a}\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{(3b)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{a^2}\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{(6b)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^2}\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{6b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{5/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0378811, size = 64, normalized size = 0.83

$$
\frac{4x^{3/2}\sqrt{\frac{a\sqrt{x}}{b}+1}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};-\frac{a\sqrt{x}}{b}\right)}{5b\sqrt{ax+b\sqrt{x}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^(3/2)*Hypergeometric2F1[3/2, 5/2, 7/2, -((a*Sq
rt[x])/b)])/(5*b*Sqrt[b*Sqrt[x] + a*x])
```

---

**Maple [B]** time = 0.006, size = 237, normalized size = 3.1

$$
-\sqrt{b\sqrt{x}+ax}\left(3\ln\left(1/2\,\frac{2\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}\sqrt{a}+2\,a\sqrt{x}+b}{\sqrt{a}}\right)xa^2b-6\,a^{5/2}x\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}+6\,\ln\left(1/2\,\frac{2\sqrt{\sqrt{x}\,(b+a\sqrt{x}}}{}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/2)+a*x)^(3/2),x)
```

```
[Out] -(b*x^(1/2)+a*x)^(1/2)/a^(5/2)*(3*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a
^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x*a^2*b-6*a^(5/2)*x*(x^(1/2)*(b+a*x^(1/2)))^
```

```
(1/2)+6*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1
/2))*x^(1/2)*a*b^2-12*a^(3/2)*x^(1/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b+4*a^(
3/2)*(x^(1/2)*(b+a*x^(1/2)))^(3/2)+3*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2
)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^3-6*a^(1/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/
2)*b^2)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/(b+a*x^(1/2))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(a*x + b*sqrt(x))^(3/2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x/(a*x + b*sqrt(x))**(3/2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.113 $\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=25

$$\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

```
[Out] (4*Sqrt[x])/(b*Sqrt[b*Sqrt[x] + a*x])
```

---

**Rubi [A]** time = 0.0051318, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2000}

$$\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sqrt[x] + a*x)^(-3/2),x]
```

```
[Out] (4*Sqrt[x])/(b*Sqrt[b*Sqrt[x] + a*x])
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4\sqrt{x}}{b\sqrt{b\sqrt{x}+ax}}$$

**Mathematica [A]** time = 0.0164931, size = 25, normalized size = 1.

$$\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sqrt[x] + a*x)^(-3/2),x]
```

```
[Out] (4*Sqrt[x])/(b*Sqrt[b*Sqrt[x] + a*x])
```

---

**Maple [C]** time = 0.008, size = 404, normalized size = 16.2

$$\frac{1}{b^2}\sqrt{b\sqrt{x}+ax}\left(2\sqrt{b\sqrt{x}+ax}a^{5/2}x+\ln\left(\frac{1}{2}\left(2\,a\sqrt{x}+2\sqrt{b\sqrt{x}+ax}\sqrt{a}+b\right)\frac{1}{\sqrt{a}}\right)xa^2b+2\,a^{5/2}x\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}-\ln\left(\frac{1}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/2)+a*x)^(3/2),x)
```

```
[Out] (b*x^(1/2)+a*x)^(1/2)*(2*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*x+ln(1/2*(2*a*x^(1/2
)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x*a^2*b+2*a^(5/2)*x*(x^(1/2)*
(b+a*x^(1/2)))^(1/2)-ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^
(1/2)+b)/a^(1/2))*x*a^2*b+4*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*x^(1/2)*b+2*ln(1/
2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(1/2)*a*b^2+4*
a^(3/2)*x^(1/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b-4*a^(3/2)*(x^(1/2)*(b+a*x^(
1/2)))^(3/2)-2*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+
b)/a^(1/2))*x^(1/2)*a*b^2+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)*b^2+ln(1/2*(2*a*x
^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*b^3+2*a^(1/2)*(x^(1/2)*(
b+a*x^(1/2)))^(1/2)*b^2-ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a
*x^(1/2)+b)/a^(1/2))*b^3)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b^2/(b+a*x^(1/2))^2
/a^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*sqrt(x))^(-3/2), x)
```

---

**Fricas [A]** time = 2.28203, size = 77, normalized size = 3.08

$$\frac{4\sqrt{ax+b\sqrt{x}}\left(a\sqrt{x}-b\right)}{a^{2}bx-b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4*sqrt(a*x + b*sqrt(x))*(a*sqrt(x) - b)/(a^2*b*x - b^3)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*sqrt(x))**(-3/2), x)
```

**Giac [A]** time = 1.14704, size = 46, normalized size = 1.84

$$\frac{4}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 4/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*sqrt(a))
```

## 3.114 $\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=79

$$\frac{32a\sqrt{ax+b\sqrt{x}}}{3b^3\sqrt{x}}-\frac{16\sqrt{ax+b\sqrt{x}}}{3b^2x}+\frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

```
[Out] 4/(b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]) - (16*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x)
+ (32*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^3*Sqrt[x])
```

---

**Rubi [A]** time = 0.122479, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{32a\sqrt{ax+b\sqrt{x}}}{3b^3\sqrt{x}}-\frac{16\sqrt{ax+b\sqrt{x}}}{3b^2x}+\frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]) - (16*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x)
+ (32*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^3*Sqrt[x])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}} + \frac{4\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}} - \frac{16\sqrt{b\sqrt{x}+ax}}{3b^2x} - \frac{(8a)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{3b^2}$$

$$= \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}} - \frac{16\sqrt{b\sqrt{x}+ax}}{3b^2x} + \frac{32a\sqrt{b\sqrt{x}+ax}}{3b^3\sqrt{x}}$$

**Mathematica [A]** time = 0.0524482, size = 48, normalized size = 0.61

$$\frac{4\left(8a^2x+4ab\sqrt{x}-b^2\right)}{3b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (4*(-b^2 + 4*a*b*Sqrt[x] + 8*a^2*x))/(3*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])
```

---

**Maple [C]** time = 0.011, size = 524, normalized size = 6.6

$$\frac{1}{3\,b^4}\sqrt{b\sqrt{x}+ax}\left(24\,\left(b\sqrt{x}+ax\right)^{3/2}a^{7/2}x^{5/2}-6\,\sqrt{b\sqrt{x}+ax}a^{9/2}x^{7/2}-3\,\ln\left(1/2\,\frac{2\,a\sqrt{x}+2\,\sqrt{b\sqrt{x}+ax}\sqrt{a}+b}{\sqrt{a}}\right)x^{7/2}a^4b-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(1/2)+a*x)^(3/2),x)
```

```
[Out] 1/3*(b*x^(1/2)+a*x)^(1/2)*(24*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*x^(5/2)-6*(b*x^
(1/2)+a*x)^(1/2)*a^(9/2)*x^(7/2)-3*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1
/2)*a^(1/2)+b)/a^(1/2))*x^(7/2)*a^4*b-6*a^(9/2)*x^(7/2)*(x^(1/2)*(b+a*x^(1/
2)))^(1/2)+3*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)
/a^(1/2))*x^(7/2)*a^4*b+44*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*x^2*b-12*(b*x^(1/2
)+a*x)^(1/2)*a^(7/2)*x^3*b-6*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^
(1/2)+b)/a^(1/2))*x^3*a^3*b^2-12*a^(7/2)*x^3*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*
b-12*a^(7/2)*x^(5/2)*(x^(1/2)*(b+a*x^(1/2)))^(3/2)+6*ln(1/2*(2*(x^(1/2)*(b+
a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^3*a^3*b^2+16*(b*x^(1/2)
+a*x)^(3/2)*a^(3/2)*x^(3/2)*b^2-6*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*x^(5/2)*b^2
-3*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(5/2)*
a^2*b^3-6*a^(5/2)*x^(5/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^2+3*ln(1/2*(2*(x^
(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(5/2)*a^2*b^3-
4*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*x*b^3)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b^4/a^
(1/2)/(b+a*x^(1/2))^2/x^(5/2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)
```

---

**Fricas [A]** time = 1.87056, size = 132, normalized size = 1.67

$$-\frac{4\left(4\,a^{2}bx-b^{3}-\left(8\,a^{3}x-5\,ab^{2}\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{3\left(a^{2}b^{3}x^{2}-b^{5}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -4/3*(4*a^2*b*x - b^3 - (8*a^3*x - 5*a*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/
(a^2*b^3*x^2 - b^5*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(a*x + b*sqrt(x))**(3/2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)
```

## 3.115 $\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{512a^3\sqrt{ax+b\sqrt{x}}}{35b^5\sqrt{x}}-\frac{256a^2\sqrt{ax+b\sqrt{x}}}{35b^4x}+\frac{192a\sqrt{ax+b\sqrt{x}}}{35b^3x^{3/2}}-\frac{32\sqrt{ax+b\sqrt{x}}}{7b^2x^2}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

```
[Out] 4/(b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x]) - (32*Sqrt[b*Sqrt[x] + a*x])/(7*b^2*x^2
) + (192*a*Sqrt[b*Sqrt[x] + a*x])/(35*b^3*x^(3/2)) - (256*a^2*Sqrt[b*Sqrt[x
] + a*x])/(35*b^4*x) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(35*b^5*Sqrt[x])
```

---

**Rubi [A]** time = 0.201879, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{512a^3\sqrt{ax+b\sqrt{x}}}{35b^5\sqrt{x}}-\frac{256a^2\sqrt{ax+b\sqrt{x}}}{35b^4x}+\frac{192a\sqrt{ax+b\sqrt{x}}}{35b^3x^{3/2}}-\frac{32\sqrt{ax+b\sqrt{x}}}{7b^2x^2}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x]) - (32*Sqrt[b*Sqrt[x] + a*x])/(7*b^2*x^2
) + (192*a*Sqrt[b*Sqrt[x] + a*x])/(35*b^3*x^(3/2)) - (256*a^2*Sqrt[b*Sqrt[x
] + a*x])/(35*b^4*x) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(35*b^5*Sqrt[x])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}} + \frac{8\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}} - \frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2} - \frac{(48a)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{7b^2}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}} - \frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2} + \frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}} + \frac{\left(192a^2\right)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{35b^3}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}} - \frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2} + \frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}} - \frac{256a^2\sqrt{b\sqrt{x}+ax}}{35b^4x} - \frac{\left(128a^3\right)\int \frac{1}{x\sqrt{b\sqrt{x}+}}}{35b^4}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}} - \frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2} + \frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}} - \frac{256a^2\sqrt{b\sqrt{x}+ax}}{35b^4x} + \frac{512a^3\sqrt{b\sqrt{x}+a}}{35b^5\sqrt{x}}$$

**Mathematica [A]** time = 0.0594126, size = 72, normalized size = 0.53

$$\frac{4\left(-16a^2b^2x+64a^3bx^{3/2}+128a^4x^2+8ab^3\sqrt{x}-5b^4\right)}{35b^5x^{3/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (4*(-5*b^4 + 8*a*b^3*Sqrt[x] - 16*a^2*b^2*x + 64*a^3*b*x^(3/2) + 128*a^4*x^
2))/(35*b^5*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])
```

---

**Maple [C]** time = 0.01, size = 570, normalized size = 4.2

$$\frac{1}{35\,b^6}\sqrt{b\sqrt{x}+ax}\left(560\,\left(b\sqrt{x}+ax\right)^{3/2}a^{11/2}x^{9/2}-210\,\sqrt{b\sqrt{x}+ax}a^{13/2}x^{11/2}-105\,\ln\left(1/2\,\frac{2\,a\sqrt{x}+2\,\sqrt{b\sqrt{x}+ax}\sqrt{a}+b}{\sqrt{a}}\right)x^1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(1/2)+a*x)^(3/2),x)
```

```
[Out] 1/35*(b*x^(1/2)+a*x)^(1/2)*(560*(b*x^(1/2)+a*x)^(3/2)*a^(11/2)*x^(9/2)-210*
(b*x^(1/2)+a*x)^(1/2)*a^(13/2)*x^(11/2)-105*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2
)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(11/2)*a^6*b-210*a^(13/2)*(x^(1/2)*(b+a*
x^(1/2)))^(1/2)*x^(11/2)+105*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2
)+2*a*x^(1/2)+b)/a^(1/2))*x^(11/2)*a^6*b+256*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*
x^(7/2)*b^2+932*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)*x^4*b-420*(b*x^(1/2)+a*x)^(1/
2)*a^(11/2)*x^5*b-210*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b
)/a^(1/2))*x^5*a^5*b^2-140*a^(11/2)*(x^(1/2)*(b+a*x^(1/2)))^(3/2)*x^(9/2)-4
20*a^(11/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*x^5*b+210*ln(1/2*(2*(x^(1/2)*(b+a
*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^5*a^5*b^2-64*(b*x^(1/2)+
```

```
a*x)^(3/2)*a^(5/2)*x^3*b^3-210*(b*x^(1/2)+a*x)^(1/2)*a^(9/2)*x^(9/2)*b^2-10
5*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(9/2)*a
^4*b^3-210*a^(9/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*x^(9/2)*b^2+105*ln(1/2*(2*
(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(9/2)*a^4*b
^3+32*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*x^(5/2)*b^4-20*(b*x^(1/2)+a*x)^(3/2)*a^
(1/2)*x^2*b^5)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b^6/a^(1/2)/x^(9/2)/(b+a*x^(1/
2))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)
```

---

**Fricas [A]** time = 1.78855, size = 190, normalized size = 1.39

$$-\frac{4\left(64\,a^4bx^2-24\,a^2b^3x-5\,b^5-\left(128\,a^5x^2-80\,a^3b^2x-13\,ab^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{35\left(a^2b^5x^3-b^7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -4/35*(64*a^4*b*x^2 - 24*a^2*b^3*x - 5*b^5 - (128*a^5*x^2 - 80*a^3*b^2*x -
13*a*b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^5*x^3 - b^7*x^2)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a*x + b*sqrt(x))**(3/2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)
```

## 3.116 $\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=195

$$\frac{512a^3\sqrt{ax+b\sqrt{x}}}{77b^5x^{3/2}}-\frac{1280a^2\sqrt{ax+b\sqrt{x}}}{231b^4x^2}+\frac{4096a^5\sqrt{ax+b\sqrt{x}}}{231b^7\sqrt{x}}-\frac{2048a^4\sqrt{ax+b\sqrt{x}}}{231b^6x}+\frac{160a\sqrt{ax+b\sqrt{x}}}{33b^3x^{5/2}}-\frac{48\sqrt{ax+b}}{11b^2x^3}$$

```
[Out] 4/(b*x^(5/2)*Sqrt[b*Sqrt[x] + a*x]) - (48*Sqrt[b*Sqrt[x] + a*x])/(11*b^2*x^
3) + (160*a*Sqrt[b*Sqrt[x] + a*x])/(33*b^3*x^(5/2)) - (1280*a^2*Sqrt[b*Sqrt
[x] + a*x])/(231*b^4*x^2) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(77*b^5*x^(3/2)
) - (2048*a^4*Sqrt[b*Sqrt[x] + a*x])/(231*b^6*x) + (4096*a^5*Sqrt[b*Sqrt[x]
 + a*x])/(231*b^7*Sqrt[x])
```

---

**Rubi [A]** time = 0.299999, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{512a^3\sqrt{ax+b\sqrt{x}}}{77b^5x^{3/2}}-\frac{1280a^2\sqrt{ax+b\sqrt{x}}}{231b^4x^2}+\frac{4096a^5\sqrt{ax+b\sqrt{x}}}{231b^7\sqrt{x}}-\frac{2048a^4\sqrt{ax+b\sqrt{x}}}{231b^6x}+\frac{160a\sqrt{ax+b\sqrt{x}}}{33b^3x^{5/2}}-\frac{48\sqrt{ax+b}}{11b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^(5/2)*Sqrt[b*Sqrt[x] + a*x]) - (48*Sqrt[b*Sqrt[x] + a*x])/(11*b^2*x^
3) + (160*a*Sqrt[b*Sqrt[x] + a*x])/(33*b^3*x^(5/2)) - (1280*a^2*Sqrt[b*Sqrt
[x] + a*x])/(231*b^4*x^2) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(77*b^5*x^(3/2)
) - (2048*a^4*Sqrt[b*Sqrt[x] + a*x])/(231*b^6*x) + (4096*a^5*Sqrt[b*Sqrt[x]
 + a*x])/(231*b^7*Sqrt[x])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
```

```
j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}+\frac{12\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}-\frac{(120a)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{11b^2}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}}+\frac{\left(320a^2\right)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{33b^3}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}}-\frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2}-\frac{\left(640a^3\right)\int \frac{1}{x^2\sqrt{b\sqrt{}}}}{77b^4}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}}-\frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2}+\frac{512a^3\sqrt{b\sqrt{x}+}}{77b^5x^{3/2}}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}}-\frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2}+\frac{512a^3\sqrt{b\sqrt{x}+}}{77b^5x^{3/2}}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}}-\frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3}+\frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}}-\frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2}+\frac{512a^3\sqrt{b\sqrt{x}+}}{77b^5x^{3/2}}$$

**Mathematica [A]** time = 0.0627457, size = 96, normalized size = 0.49

$$\frac{4\left(-128a^4b^2x^2+64a^3b^3x^{3/2}-40a^2b^4x+512a^5bx^{5/2}+1024a^6x^3+28ab^5\sqrt{x}-21b^6\right)}{231b^7x^{5/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (4*(-21*b^6 + 28*a*b^5*Sqrt[x] - 40*a^2*b^4*x + 64*a^3*b^3*x^(3/2) - 128*a^
4*b^2*x^2 + 512*a^5*b*x^(5/2) + 1024*a^6*x^3))/(231*b^7*x^(5/2)*Sqrt[b*Sqrt
[x] + a*x])
```

---

**Maple [C]** time = 0.012, size = 614, normalized size = 3.2

$$\frac{1}{231\,b^8}\sqrt{b\sqrt{x}+ax}\left(1155\,\ln\left(1/2\,\frac{2\,\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}\sqrt{a}+2\,a\sqrt{x}+b}{\sqrt{a}}\right)x^{15/2}a^8b-2310\,\ln\left(1/2\,\frac{2\,a\sqrt{x}+2\,\sqrt{b\sqrt{x}+ax}\sqrt{a}+b}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(1/2)+a*x)^(3/2),x)
```

```
[Out] 1/231*(b*x^(1/2)+a*x)^(1/2)*(1155*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a
^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(15/2)*a^8*b-2310*ln(1/2*(2*a*x^(1/2)+2*(b
*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^7*a^7*b^2-924*a^(15/2)*x^(13/2)*(
x^(1/2)*(b+a*x^(1/2)))^(3/2)+2310*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a
^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^7*a^7*b^2+256*(b*x^(1/2)+a*x)^(3/2)*a^(7/2
)*x^(9/2)*b^4-1155*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a
^(1/2))*x^(13/2)*a^6*b^3+1155*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/
2)+2*a*x^(1/2)+b)/a^(1/2))*x^(13/2)*a^6*b^3-160*(b*x^(1/2)+a*x)^(3/2)*a^(5/
2)*x^4*b^5+112*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*x^(7/2)*b^6-84*(b*x^(1/2)+a*x)
^(3/2)*a^(1/2)*x^3*b^7-512*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)*x^5*b^3+2048*(b*x^
(1/2)+a*x)^(3/2)*a^(11/2)*x^(11/2)*b^2-4620*a^(15/2)*x^7*(x^(1/2)*(b+a*x^(1
/2)))^(1/2)*b+8716*(b*x^(1/2)+a*x)^(3/2)*a^(13/2)*x^6*b-4620*(b*x^(1/2)+a*x
)^(1/2)*a^(15/2)*x^7*b-2310*(b*x^(1/2)+a*x)^(1/2)*a^(13/2)*x^(13/2)*b^2-231
0*a^(13/2)*x^(13/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^2+5544*(b*x^(1/2)+a*x)^
(3/2)*a^(15/2)*x^(13/2)-2310*(b*x^(1/2)+a*x)^(1/2)*a^(17/2)*x^(15/2)-2310*a
^(17/2)*x^(15/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)-1155*ln(1/2*(2*a*x^(1/2)+2*(
b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(15/2)*a^8*b)/(x^(1/2)*(b+a*x^(1
/2)))^(1/2)/b^8/a^(1/2)/x^(13/2)/(b+a*x^(1/2))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)
```

---

**Fricas [A]** time = 1.89979, size = 246, normalized size = 1.26

$$-\frac{4\left(512\,a^6bx^3 - 192\,a^4b^3x^2 - 68\,a^2b^5x - 21\,b^7 - \left(1024\,a^7x^3 - 640\,a^5b^2x^2 - 104\,a^3b^4x - 49\,ab^6\right)\sqrt{x}\right)\sqrt{ax + b\sqrt{x}}}{231\left(a^2b^7x^4 - b^9x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -4/231*(512*a^6*b*x^3 - 192*a^4*b^3*x^2 - 68*a^2*b^5*x - 21*b^7 - (1024*a^7
*x^3 - 640*a^5*b^2*x^2 - 104*a^3*b^4*x - 49*a*b^6)*sqrt(x))*sqrt(a*x + b*sq
rt(x))/(a^2*b^7*x^4 - b^9*x^3)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a*x + b*sqrt(x))**(3/2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)
```

## 3.117 $\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

**Optimal**. Leaf size=204

$$\frac{33b^2x^{3/2}\sqrt{ax+b\sqrt{x}}}{80a^3}-\frac{231b^5\sqrt{ax+b\sqrt{x}}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{ax+b\sqrt{x}}}{128a^5}-\frac{77b^3x\sqrt{ax+b\sqrt{x}}}{160a^4}+\frac{231b^6\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{256a^{13/2}}-11$$

```
[Out] (-231*b^5*Sqrt[b*Sqrt[x] + a*x])/(256*a^6) + (77*b^4*Sqrt[x]*Sqrt[b*Sqrt[x]
 + a*x])/(128*a^5) - (77*b^3*x*Sqrt[b*Sqrt[x] + a*x])/(160*a^4) + (33*b^2*x
^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(80*a^3) - (11*b*x^2*Sqrt[b*Sqrt[x] + a*x])/(
30*a^2) + (x^(5/2)*Sqrt[b*Sqrt[x] + a*x])/(3*a) + (231*b^6*ArcTanh[(Sqrt[a]
*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(256*a^(13/2))
```

---

**Rubi [A]** time = 0.17155, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2018, 670, 640, 620, 206}

$$\frac{33b^2x^{3/2}\sqrt{ax+b\sqrt{x}}}{80a^3}-\frac{231b^5\sqrt{ax+b\sqrt{x}}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{ax+b\sqrt{x}}}{128a^5}-\frac{77b^3x\sqrt{ax+b\sqrt{x}}}{160a^4}+\frac{231b^6\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{256a^{13/2}}-11$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-231*b^5*Sqrt[b*Sqrt[x] + a*x])/(256*a^6) + (77*b^4*Sqrt[x]*Sqrt[b*Sqrt[x]
 + a*x])/(128*a^5) - (77*b^3*x*Sqrt[b*Sqrt[x] + a*x])/(160*a^4) + (33*b^2*x
^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(80*a^3) - (11*b*x^2*Sqrt[b*Sqrt[x] + a*x])/(
30*a^2) + (x^(5/2)*Sqrt[b*Sqrt[x] + a*x])/(3*a) + (231*b^6*ArcTanh[(Sqrt[a]
*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(256*a^(13/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(11b)\,\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{6a}$$

$$= -\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2} + \frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a} + \frac{\left(33b^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{20a^2}$$

$$= \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3} - \frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2} + \frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a} - \frac{\left(231b^3\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{160a^3}$$

$$= -\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4} + \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3} - \frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2} + \frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a} + \frac{\left(77b^4\right)\text{Subst}\left(\right.}{}$$

$$= \frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5} - \frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4} + \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3} - \frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2} + \frac{x^{5/2}\sqrt{b\sqrt{x}+}}{3a}$$

$$= -\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6} + \frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5} - \frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4} + \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3} - \frac{11bx^2\sqrt{b\sqrt{}}}{30a^2}$$

$$= -\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6} + \frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5} - \frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4} + \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3} - \frac{11bx^2\sqrt{b\sqrt{}}}{30a^2}$$

$$= -\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6} + \frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5} - \frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4} + \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3} - \frac{11bx^2\sqrt{b\sqrt{}}}{30a^2}$$

**Mathematica [A]** time = 0.179312, size = 164, normalized size = 0.8

$$\frac{\left(a\sqrt{x}+b\right)\left(\sqrt{a}\sqrt{x}\sqrt{\frac{a\sqrt{x}}{b}+1}\left(1584a^3b^2x^{3/2}-1848a^2b^3x-1408a^4bx^2+1280a^5x^{5/2}+2310ab^4\sqrt{x}-3465b^5\right)+3465b^{11/2}\sqrt[4]{}\right.}{3840a^{13/2}\sqrt{\frac{a\sqrt{x}}{b}+1}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] ((b + a*Sqrt[x])*(Sqrt[a]*Sqrt[1 + (a*Sqrt[x])/b]*Sqrt[x]*(-3465*b^5 + 2310
*a*b^4*Sqrt[x] - 1848*a^2*b^3*x + 1584*a^3*b^2*x^(3/2) - 1408*a^4*b*x^2 + 1
280*a^5*x^(5/2)) + 3465*b^(11/2)*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]]
))/(3840*a^(13/2)*Sqrt[1 + (a*Sqrt[x])/b]*Sqrt[b*Sqrt[x] + a*x])
```

---

**Maple [A]** time = 0.01, size = 245, normalized size = 1.2

$$\frac{1}{7680}\sqrt{b\sqrt{x}+ax}\left(2560\,x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}a^{11/2}+8544\left(b\sqrt{x}+ax\right)^{3/2}a^{7/2}\sqrt{x}b^{2}-5376\left(b\sqrt{x}+ax\right)^{3/2}a^{9/2}xb-12240\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] 1/7680*(b*x^(1/2)+a*x)^(1/2)*(2560*x^(3/2)*(b*x^(1/2)+a*x)^(3/2)*a^(11/2)+8
544*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*x^(1/2)*b^2-5376*(b*x^(1/2)+a*x)^(3/2)*a^
(9/2)*x*b-12240*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*b^3+16860*(b*x^(1/2)+a*x)^(1/
2)*a^(5/2)*x^(1/2)*b^4+8430*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b^5-15360*a^(3/2)
*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^5+7680*a*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))
^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^6-4215*ln(1/2*(2*a*x^(1/2)+2*(b*x^
(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^6)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/a
^(15/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/sqrt(a*x + b*sqrt(x)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(x**(5/2)/sqrt(a*x + b*sqrt(x)), x)
```

---

**Giac [A]** time = 1.32523, size = 169, normalized size = 0.83

$$\frac{1}{3840}\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\left(8\sqrt{x}\left(\frac{10\sqrt{x}}{a}-\frac{11\,b}{a^{2}}\right)+\frac{99\,b^{2}}{a^{3}}\right)\sqrt{x}-\frac{231\,b^{3}}{a^{4}}\right)\sqrt{x}+\frac{1155\,b^{4}}{a^{5}}\right)\sqrt{x}-\frac{3465\,b^{5}}{a^{6}}\right)-\frac{231\,b^{6}\log\Bigl(\Bigl|-2\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*sqrt(a*x + b*sqrt(x))*(2*(4*(2*(8*sqrt(x)*(10*sqrt(x)/a - 11*b/a^2)
+ 99*b^2/a^3)*sqrt(x) - 231*b^3/a^4)*sqrt(x) + 1155*b^4/a^5)*sqrt(x) - 3465
*b^5/a^6) - 231/512*b^6*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*
sqrt(x))) - b))/a^(13/2)
```

## 3.118 $\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

**Optimal**. Leaf size=146

$$-\frac{35b^3\sqrt{ax+b\sqrt{x}}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{48a^3}+\frac{35b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{9/2}}-\frac{7bx\sqrt{ax+b\sqrt{x}}}{12a^2}+\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a}$$

```
[Out] (-35*b^3*Sqrt[b*Sqrt[x] + a*x])/(32*a^4) + (35*b^2*Sqrt[x]*Sqrt[b*Sqrt[x] +
 a*x])/(48*a^3) - (7*b*x*Sqrt[b*Sqrt[x] + a*x])/(12*a^2) + (x^(3/2)*Sqrt[b*
Sqrt[x] + a*x])/(2*a) + (35*b^4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] +
a*x]])/(32*a^(9/2))
```

---

**Rubi [A]** time = 0.119912, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2018, 670, 640, 620, 206}

$$-\frac{35b^3\sqrt{ax+b\sqrt{x}}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{48a^3}+\frac{35b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{9/2}}-\frac{7bx\sqrt{ax+b\sqrt{x}}}{12a^2}+\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-35*b^3*Sqrt[b*Sqrt[x] + a*x])/(32*a^4) + (35*b^2*Sqrt[x]*Sqrt[b*Sqrt[x] +
 a*x])/(48*a^3) - (7*b*x*Sqrt[b*Sqrt[x] + a*x])/(12*a^2) + (x^(3/2)*Sqrt[b*
Sqrt[x] + a*x])/(2*a) + (35*b^4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] +
a*x]])/(32*a^(9/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a}-\frac{(7b)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{4a}\\
&= -\frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2}+\frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a}+\frac{\left(35b^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{24a^2}\\
&= \frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3}-\frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2}+\frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a}-\frac{\left(35b^3\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{32a^3}\\
&= -\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3}-\frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2}+\frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a}+\frac{\left(35b^4\right)\text{Subst}\left(\int\right.}{64a}\\
&= -\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3}-\frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2}+\frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a}+\frac{\left(35b^4\right)\text{Subst}\left(\int\right.}{32}\\
&= -\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3}-\frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2}+\frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a}+\frac{35b^4\tanh^{-1}\left(\right.}{32a^{9/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.137471, size = 142, normalized size = 0.97

$$
-\frac{35b^5\left(\frac{a\sqrt{x}}{b}+1\right)\left(-\frac{32a^4x^2}{35b^4}+\frac{16a^3x^{3/2}}{15b^3}-\frac{4a^2x}{3b^2}+\frac{2a\sqrt{x}}{b}-\frac{2\sqrt{a}\sqrt[4]{x}\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{\frac{a\sqrt{x}}{b}+1}}\right)}{64a^5\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-35*b^5*(1 + (a*Sqrt[x])/b)*((2*a*Sqrt[x])/b - (4*a^2*x)/(3*b^2) + (16*a^3
*x^(3/2))/(15*b^3) - (32*a^4*x^2)/(35*b^4) - (2*Sqrt[a]*x^(1/4)*ArcSinh[(Sq
rt[a]*x^(1/4))/Sqrt[b]])/(Sqrt[b]*Sqrt[1 + (a*Sqrt[x])/b])))/(64*a^5*Sqrt[(
b + a*Sqrt[x])*Sqrt[x]])
```

---

**Maple [A]** time = 0.006, size = 203, normalized size = 1.4

$$
\frac{1}{192}\sqrt{b\sqrt{x}+ax}\left(96\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}a^{7/2}-208\left(b\sqrt{x}+ax\right)^{3/2}a^{5/2}b+348\sqrt{b\sqrt{x}+axa^{5/2}\sqrt{x}b^2}+174\sqrt{b\sqrt{x}+axa^{3/2}b^3}-\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] 1/192*(b*x^(1/2)+a*x)^(1/2)*(96*x^(1/2)*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)-208*(
b*x^(1/2)+a*x)^(3/2)*a^(5/2)*b+348*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*x^(1/2)*b^
2+174*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b^3-384*a^(3/2)*(x^(1/2)*(b+a*x^(1/2)))
^(1/2)*b^3+192*a*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2
)+b)/a^(1/2))*b^4-87*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)
/a^(1/2))*a*b^4)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/a^(11/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{ax + b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(a*x + b*sqrt(x)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{ax + b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(a*x + b*sqrt(x)), x)
```

---

**Giac [A]** time = 1.31092, size = 131, normalized size = 0.9

$$\frac{1}{96}\sqrt{ax + b\sqrt{x}}\left(2\left(4\sqrt{x}\left(\frac{6\sqrt{x}}{a} - \frac{7b}{a^2}\right) + \frac{35b^2}{a^3}\right)\sqrt{x} - \frac{105b^3}{a^4}\right) - \frac{35b^4\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x} - \sqrt{ax + b\sqrt{x}}\right) - b\right|\right)}{64a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*sqrt(a*x + b*sqrt(x))*(2*(4*sqrt(x)*(6*sqrt(x)/a - 7*b/a^2) + 35*b^2/a
^3)*sqrt(x) - 105*b^3/a^4) - 35/64*b^4*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x)
- sqrt(a*x + b*sqrt(x))) - b))/a^(9/2)
```

## 3.119 $\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx$

**Optimal**. Leaf size=87

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{5/2}}-\frac{3b\sqrt{ax+b\sqrt{x}}}{2a^2}+\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a}$$

```
[Out] (-3*b*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) + (Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/a +
(3*b^2*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(2*a^(5/2))
```

---

**Rubi [A]** time = 0.0788138, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2018, 670, 640, 620, 206}

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{5/2}}-\frac{3b\sqrt{ax+b\sqrt{x}}}{2a^2}+\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-3*b*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) + (Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/a +
(3*b^2*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(2*a^(5/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} - \frac{(3b)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{2a}\\
&= -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^2}\\
&= -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^2}\\
&= -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{5/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.0799227, size = 102, normalized size = 1.17

$$\frac{\sqrt{a}\sqrt{x}\left(2a^2x - ab\sqrt{x} - 3b^2\right) + 3b^{5/2}\sqrt[4]{x}\sqrt{\frac{a\sqrt{x}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{2a^{5/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (Sqrt[a]*Sqrt[x]*(-3*b^2 - a*b*Sqrt[x] + 2*a^2*x) + 3*b^(5/2)*Sqrt[1 + (a*S
qrt[x])/b]*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]])/(2*a^(5/2)*Sqrt[b*Sq
rt[x] + a*x])
```

---

**Maple [B]** time = 0.007, size = 160, normalized size = 1.8

$$\frac{1}{4}\sqrt{b\sqrt{x}+ax}\left(4\sqrt{b\sqrt{x}+ax}a^{5/2}\sqrt{x} + 2\sqrt{b\sqrt{x}+ax}a^{3/2}b - 8\,a^{3/2}\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}b + 4\,a\ln\left(1/2\,\frac{2\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}\sqrt{a}+2\,a}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] 1/4*(b*x^(1/2)+a*x)^(1/2)*(4*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*x^(1/2)+2*(b*x^(
1/2)+a*x)^(1/2)*a^(3/2)*b-8*a^(3/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b+4*a*ln(
1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^2-b^
2*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a)/(x^(1/
```

```
2)*(b+a*x^(1/2)))^(1/2)/a^(7/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(a*x + b*sqrt(x)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(a*x + b*sqrt(x)), x)
```

---

**Giac [A]** time = 1.34643, size = 93, normalized size = 1.07

$$\frac{1}{2}\sqrt{ax+b\sqrt{x}}\left(\frac{2\sqrt{x}}{a}-\frac{3\,b}{a^2}\right)-\frac{3\,b^2\log\left(\left|-2\,\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{4\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)/a - 3*b/a^2) - 3/4*b^2*log(abs(-2*sqrt
(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(5/2)
```

## 3.120 $\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx$

**Optimal**. Leaf size=34

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}$$

```
[Out] (4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]
```

---

**Rubi [A]** time = 0.0490875, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2013, 620, 206}

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)\\
&= 4\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)\\
&= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{\sqrt{a}}
\end{aligned}
$$

**Mathematica [A]** time = 0.0415114, size = 65, normalized size = 1.91

$$
\frac{4\sqrt{b}\sqrt[4]{x}\sqrt{\frac{a\sqrt{x}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{ax+b\sqrt{x}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*Sqrt[b]*Sqrt[1 + (a*Sqrt[x])/b]*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b
]])/(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])
```

---

**Maple [B]** time = 0.009, size = 133, normalized size = 3.9

$$
\frac{1}{b}\sqrt{b\sqrt{x}+ax}\Bigg(2\sqrt{b\sqrt{x}+ax}\sqrt{a}+b\ln\left(\frac{1}{2}\left(2a\sqrt{x}+2\sqrt{b\sqrt{x}+ax}\sqrt{a}+b\right)\frac{1}{\sqrt{a}}\right)-2\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}\sqrt{a}+b\ln\Bigg(\frac{1}{2}\Bigg(2
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] (b*x^(1/2)+a*x)^(1/2)*(2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b*ln(1/2*(2*a*x^(1/2
)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))-2*(x^(1/2)*(b+a*x^(1/2)))^(1/
2)*a^(1/2)+b*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)
/a^(1/2)))/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b/a^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{\sqrt{ax+b\sqrt{x}}\sqrt{x}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*sqrt(x)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(a*x + b*sqrt(x))), x)
```

---

**Giac [A]** time = 1.29107, size = 50, normalized size = 1.47

$$-\frac{2\,\log\left(\left|-2\,\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/sqrt(
a)
```

## 3.121 $\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx$

**Optimal**. Leaf size=54

$$\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}$$

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqr
t[x])
```

---

**Rubi [A]** time = 0.073614, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqr
t[x])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx &= -\frac{4\sqrt{b\sqrt{x}+ax}}{3bx}-\frac{(2a)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{3b}\\ &= -\frac{4\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{8a\sqrt{b\sqrt{x}+ax}}{3b^2\sqrt{x}}\end{aligned}$$

**Mathematica [A]** time = 0.0466093, size = 37, normalized size = 0.69

$$\frac{4\left(2a\sqrt{x}-b\right)\sqrt{ax+b\sqrt{x}}}{3b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*(-b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x)
```

---

**Maple [C]** time = 0.009, size = 194, normalized size = 3.6

$$\frac{1}{3\,b^3}\sqrt{b\sqrt{x}+ax}\left(12\,\left(b\sqrt{x}+ax\right)^{3/2}a^{3/2}x^{3/2}-6\,\sqrt{b\sqrt{x}+ax}a^{5/2}x^{5/2}-3\,\ln\left(1/2\,\frac{2\,a\sqrt{x}+2\,\sqrt{b\sqrt{x}+ax}\sqrt{a}+b}{\sqrt{a}}\right)x^{5/2}a^2b-6\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] 1/3*(b*x^(1/2)+a*x)^(1/2)*(12*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*x^(3/2)-6*(b*x^
(1/2)+a*x)^(1/2)*a^(5/2)*x^(5/2)-3*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1
/2)*a^(1/2)+b)/a^(1/2))*x^(5/2)*a^2*b-6*a^(5/2)*x^(5/2)*(x^(1/2)*(b+a*x^(1/
2)))^(1/2)+3*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)
/a^(1/2))*x^(5/2)*a^2*b-4*(b*x^(1/2)+a*x)^(3/2)*b*a^(1/2)*x)/(x^(1/2)*(b+a*
x^(1/2)))^(1/2)/b^3/a^(1/2)/x^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(3/2)), x)
```

---

**Fricas [A]** time = 2.30499, size = 72, normalized size = 1.33

$$\frac{4\,\sqrt{ax+b\sqrt{x}}\left(2\,a\sqrt{x}-b\right)}{3\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3*sqrt(a*x + b*sqrt(x))*(2*a*sqrt(x) - b)/(b^2*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(a*x + b*sqrt(x))), x)
```

---

**Giac [A]** time = 1.27765, size = 72, normalized size = 1.33

$$\frac{4\left(3\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)}{3\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)/(sqrt(a)*sqrt
(x) - sqrt(a*x + b*sqrt(x)))^3
```

## 3.122 $\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx$

**Optimal**. Leaf size=112

$$\frac{64a^3\sqrt{ax+b\sqrt{x}}}{35b^4\sqrt{x}}-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{35b^3x}+\frac{24a\sqrt{ax+b\sqrt{x}}}{35b^2x^{3/2}}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2) + (24*a*Sqrt[b*Sqrt[x] + a*x])/(35*b^2
*x^(3/2)) - (32*a^2*Sqrt[b*Sqrt[x] + a*x])/(35*b^3*x) + (64*a^3*Sqrt[b*Sqrt
[x] + a*x])/(35*b^4*Sqrt[x])
```

---

**Rubi [A]** time = 0.153552, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{64a^3\sqrt{ax+b\sqrt{x}}}{35b^4\sqrt{x}}-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{35b^3x}+\frac{24a\sqrt{ax+b\sqrt{x}}}{35b^2x^{3/2}}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2) + (24*a*Sqrt[b*Sqrt[x] + a*x])/(35*b^2
*x^(3/2)) - (32*a^2*Sqrt[b*Sqrt[x] + a*x])/(35*b^3*x) + (64*a^3*Sqrt[b*Sqrt
[x] + a*x])/(35*b^4*Sqrt[x])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} - \frac{(6a)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{7b}$$
$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} + \frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}} + \frac{\left(24a^2\right)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{35b^2}$$
$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} + \frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}} - \frac{32a^2\sqrt{b\sqrt{x}+ax}}{35b^3x} - \frac{\left(16a^3\right)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{35b^3}$$
$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} + \frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}} - \frac{32a^2\sqrt{b\sqrt{x}+ax}}{35b^3x} + \frac{64a^3\sqrt{b\sqrt{x}+ax}}{35b^4\sqrt{x}}$$

**Mathematica [A]** time = 0.0499462, size = 59, normalized size = 0.53

$$\frac{4\sqrt{ax+b\sqrt{x}}\left(-8a^2bx+16a^3x^{3/2}+6ab^2\sqrt{x}-5b^3\right)}{35b^4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*Sqrt[b*Sqrt[x] + a*x]*(-5*b^3 + 6*a*b^2*Sqrt[x] - 8*a^2*b*x + 16*a^3*x^(
3/2)))/(35*b^4*x^2)
```

---

**Maple [C]** time = 0.008, size = 240, normalized size = 2.1

$$\frac{1}{35\,b^5}\sqrt{b\sqrt{x}+ax}\left(140\,\left(b\sqrt{x}+ax\right)^{3/2}a^{7/2}x^{7/2}-70\,\sqrt{b\sqrt{x}+ax}a^{9/2}x^{9/2}-35\,\ln\left(1/2\,\frac{2\,a\sqrt{x}+2\,\sqrt{b\sqrt{x}+ax}\sqrt{a}+b}{\sqrt{a}}\right)x^{9/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] 1/35*(b*x^(1/2)+a*x)^(1/2)*(140*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*x^(7/2)-70*(b
*x^(1/2)+a*x)^(1/2)*a^(9/2)*x^(9/2)-35*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x
)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(9/2)*a^4*b-70*a^(9/2)*x^(9/2)*(x^(1/2)*(b+a*
x^(1/2)))^(1/2)+35*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1
/2)+b)/a^(1/2))*x^(9/2)*a^4*b+44*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*x^(5/2)*b^2-
76*a^(5/2)*(b*x^(1/2)+a*x)^(3/2)*b*x^3-20*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*x^2
*b^3)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b^5/a^(1/2)/x^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(5/2)), x)
```

---

**Fricas [A]** time = 2.36806, size = 123, normalized size = 1.1

$$-\frac{4\left(8\,a^{2}bx+5\,b^{3}-2\left(8\,a^{3}x+3\,ab^{2}\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{35\,b^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4/35*(8*a^2*b*x + 5*b^3 - 2*(8*a^3*x + 3*a*b^2)*sqrt(x))*sqrt(a*x + b*sqrt
(x))/(b^4*x^2)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^{\frac{5}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(a*x + b*sqrt(x))), x)
```

---

**Giac [A]** time = 1.22808, size = 155, normalized size = 1.38

$$\frac{4\left(70\,a^{\frac{3}{2}}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{3}+84\,ab\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{2}+35\,\sqrt{a}b^{2}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+5\,b^{3}\right)}{35\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/35*(70*a^(3/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 84*a*b*(sqrt
(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 35*sqrt(a)*b^2*(sqrt(a)*sqrt(x) -
sqrt(a*x + b*sqrt(x))) + 5*b^3)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^7
```

## 3.123 $\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx$

**Optimal**. Leaf size=170

$$\frac{128a^3\sqrt{ax+b\sqrt{x}}}{231b^4x^{3/2}}-\frac{320a^2\sqrt{ax+b\sqrt{x}}}{693b^3x^2}+\frac{1024a^5\sqrt{ax+b\sqrt{x}}}{693b^6\sqrt{x}}-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{693b^5x}+\frac{40a\sqrt{ax+b\sqrt{x}}}{99b^2x^{5/2}}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(11*b*x^3) + (40*a*Sqrt[b*Sqrt[x] + a*x])/(99*b^
2*x^(5/2)) - (320*a^2*Sqrt[b*Sqrt[x] + a*x])/(693*b^3*x^2) + (128*a^3*Sqrt[
b*Sqrt[x] + a*x])/(231*b^4*x^(3/2)) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(693*
b^5*x) + (1024*a^5*Sqrt[b*Sqrt[x] + a*x])/(693*b^6*Sqrt[x])
```

---

**Rubi [A]** time = 0.244644, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{128a^3\sqrt{ax+b\sqrt{x}}}{231b^4x^{3/2}}-\frac{320a^2\sqrt{ax+b\sqrt{x}}}{693b^3x^2}+\frac{1024a^5\sqrt{ax+b\sqrt{x}}}{693b^6\sqrt{x}}-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{693b^5x}+\frac{40a\sqrt{ax+b\sqrt{x}}}{99b^2x^{5/2}}-\frac{4\sqrt{ax+b\sqrt{x}}}{11bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(11*b*x^3) + (40*a*Sqrt[b*Sqrt[x] + a*x])/(99*b^
2*x^(5/2)) - (320*a^2*Sqrt[b*Sqrt[x] + a*x])/(693*b^3*x^2) + (128*a^3*Sqrt[
b*Sqrt[x] + a*x])/(231*b^4*x^(3/2)) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(693*
b^5*x) + (1024*a^5*Sqrt[b*Sqrt[x] + a*x])/(693*b^6*Sqrt[x])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} - \frac{(10a)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{11b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} + \frac{\left(80a^2\right)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{99b^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} - \frac{\left(160a^3\right)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{231b^3}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} + \frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}} + \frac{\left(128a^4\right)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}}}}{231b^4}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} + \frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}} - \frac{512a^4\sqrt{b\sqrt{x}+ax}}{693b^5x}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} + \frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}} - \frac{512a^4\sqrt{b\sqrt{x}+ax}}{693b^5x}$$

**Mathematica [A]** time = 0.0548261, size = 83, normalized size = 0.49

$$\frac{4\sqrt{ax+b\sqrt{x}}\left(96a^3b^2x^{3/2}-80a^2b^3x-128a^4bx^2+256a^5x^{5/2}+70ab^4\sqrt{x}-63b^5\right)}{693b^6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*Sqrt[b*Sqrt[x] + a*x]*(-63*b^5 + 70*a*b^4*Sqrt[x] - 80*a^2*b^3*x + 96*a^
3*b^2*x^(3/2) - 128*a^4*b*x^2 + 256*a^5*x^(5/2)))/(693*b^6*x^3)
```

---

**Maple [C]** time = 0.01, size = 284, normalized size = 1.7

$$\frac{1}{693\,b^7}\sqrt{b\sqrt{x}+ax}\left(2772\,\left(b\sqrt{x}+ax\right)^{3/2}a^{11/2}x^{11/2}-1386\,\sqrt{b\sqrt{x}+ax}a^{13/2}x^{13/2}-693\,\ln\left(1/2\,\frac{2\,a\sqrt{x}+2\,\sqrt{b\sqrt{x}+ax}\sqrt{a}+}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] 1/693*(b*x^(1/2)+a*x)^(1/2)*(2772*(b*x^(1/2)+a*x)^(3/2)*a^(11/2)*x^(11/2)-1
386*(b*x^(1/2)+a*x)^(1/2)*a^(13/2)*x^(13/2)-693*ln(1/2*(2*a*x^(1/2)+2*(b*x^
(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(13/2)*a^6*b-1386*a^(13/2)*x^(13/2)*
(x^(1/2)*(b+a*x^(1/2)))^(1/2)+693*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a
^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(13/2)*a^6*b+1236*(b*x^(1/2)+a*x)^(3/2)*a^
(7/2)*x^(9/2)*b^2+532*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*x^(7/2)*b^4-1748*a^(9/2
)*(b*x^(1/2)+a*x)^(3/2)*b*x^5-852*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*x^4*b^3-252
*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*x^3*b^5)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b^7/a
^(1/2)/x^(13/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(7/2)), x)
```

---

**Fricas [A]** time = 2.13488, size = 178, normalized size = 1.05

$$-\frac{4\left(128\,a^4bx^2+80\,a^2b^3x+63\,b^5-2\left(128\,a^5x^2+48\,a^3b^2x+35\,ab^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{693\,b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4/693*(128*a^4*b*x^2 + 80*a^2*b^3*x + 63*b^5 - 2*(128*a^5*x^2 + 48*a^3*b^2
*x + 35*a*b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^6*x^3)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.18865, size = 239, normalized size = 1.41

$$\frac{4\left(3696\,a^{\frac{5}{2}}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5+7920\,a^2b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^4+6930\,a^{\frac{3}{2}}b^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3+3080\,ab^{\cdots}\right.}{693\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/693*(3696*a^(5/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5 + 7920*a^2*
b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 6930*a^(3/2)*b^2*(sqrt(a)*s
qrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 3080*a*b^3*(sqrt(a)*sqrt(x) - sqrt(a*x
+ b*sqrt(x)))^2 + 693*sqrt(a)*b^4*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))
 + 63*b^5)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^11
```

## 3.124 $\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=171

$$-\frac{315b^3\sqrt{ax+b\sqrt{x}}}{32a^5}+\frac{105b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{16a^4}+\frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{11/2}}+\frac{9x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a^2}-\frac{21bx\sqrt{ax+b\sqrt{x}}}{4a^3}-\frac{4x^{5}}{a\sqrt{ax+}}$$

```
[Out] (-4*x^(5/2))/(a*Sqrt[b*Sqrt[x] + a*x]) - (315*b^3*Sqrt[b*Sqrt[x] + a*x])/(3
2*a^5) + (105*b^2*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(16*a^4) - (21*b*x*Sqrt[b*
Sqrt[x] + a*x])/(4*a^3) + (9*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) + (315*
b^4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(32*a^(11/2))
```

---

**Rubi [A]** time = 0.146974, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2018, 668, 670, 640, 620, 206}

$$-\frac{315b^3\sqrt{ax+b\sqrt{x}}}{32a^5}+\frac{105b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{16a^4}+\frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{11/2}}+\frac{9x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a^2}-\frac{21bx\sqrt{ax+b\sqrt{x}}}{4a^3}-\frac{4x^{5}}{a\sqrt{ax+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^(5/2))/(a*Sqrt[b*Sqrt[x] + a*x]) - (315*b^3*Sqrt[b*Sqrt[x] + a*x])/(3
2*a^5) + (105*b^2*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(16*a^4) - (21*b*x*Sqrt[b*
Sqrt[x] + a*x])/(4*a^3) + (9*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) + (315*
b^4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(32*a^(11/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
```

```
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^6}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{18\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}\\
&= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} - \frac{(63b)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^2}\\
&= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\left(105b^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{8a^3}\\
&= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} - \left(315b^3\right)\text{Subs}\\
&= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}}}{2a^2}\\
&= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}}}{2a^2}\\
&= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}}}{2a^2}
\end{aligned}
$$

**Mathematica [C]** time = 0.0517102, size = 62, normalized size = 0.36

$$\frac{4x^3\sqrt{\frac{a\sqrt{x}}{b}+1}\,{}_2F_1\left(\frac{3}{2},\frac{11}{2};\frac{13}{2};-\frac{a\sqrt{x}}{b}\right)}{11b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^3*Hypergeometric2F1[3/2, 11/2, 13/2, -((a*Sqrt
[x])/b)])/(11*b*Sqrt[b*Sqrt[x] + a*x])
```

---

**Maple [B]** time = 0.009, size = 527, normalized size = 3.1

$$\frac{1}{64}\sqrt{b\sqrt{x}+ax}\left(32\,x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}a^{11/2}-48\left(b\sqrt{x}+ax\right)^{3/2}a^{9/2}xb+276\sqrt{b\sqrt{x}+ax}a^{9/2}x^{3/2}b^2-192\left(b\sqrt{x}+ax\right)^{3/2}a^{7/}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x)
```

```
[Out] 1/64*(b*x^(1/2)+a*x)^(1/2)/a^(13/2)*(32*x^(3/2)*(b*x^(1/2)+a*x)^(3/2)*a^(11
/2)-48*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)*x*b+276*(b*x^(1/2)+a*x)^(1/2)*a^(9/2)*
x^(3/2)*b^2-192*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*x^(1/2)*b^2+690*(b*x^(1/2)+a*
x)^(1/2)*a^(7/2)*x*b^3-768*a^(7/2)*x*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^3+384*
a^3*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))
*x*b^4-112*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*b^3+552*(b*x^(1/2)+a*x)^(1/2)*a^(5
/2)*x^(1/2)*b^4-1536*a^(5/2)*x^(1/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^4+256*
a^(5/2)*(x^(1/2)*(b+a*x^(1/2)))^(3/2)*b^3+768*a^2*ln(1/2*(2*(x^(1/2)*(b+a*x
^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(1/2)*b^5-69*ln(1/2*(2*a*x
^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x*a^3*b^4+138*(b*x^(1/2)
+a*x)^(1/2)*a^(3/2)*b^5-768*a^(3/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^5+384*a
*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^
6-138*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(1/
2)*a^2*b^5-69*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2
))*a*b^6)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/(b+a*x^(1/2))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(a*x + b*sqrt(x))^(3/2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x**(5/2)/(a*x + b*sqrt(x))**(3/2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.125 $\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=113

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{7/2}}-\frac{15b\sqrt{ax+b\sqrt{x}}}{2a^3}+\frac{5\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^2}-\frac{4x^{3/2}}{a\sqrt{ax+b\sqrt{x}}}$$

```
[Out] (-4*x^(3/2))/(a*Sqrt[b*Sqrt[x] + a*x]) - (15*b*Sqrt[b*Sqrt[x] + a*x])/(2*a^
3) + (5*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/a^2 + (15*b^2*ArcTanh[(Sqrt[a]*Sqrt[
x])/Sqrt[b*Sqrt[x] + a*x]])/(2*a^(7/2))
```

---

**Rubi [A]** time = 0.102718, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2018, 668, 670, 640, 620, 206}

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{7/2}}-\frac{15b\sqrt{ax+b\sqrt{x}}}{2a^3}+\frac{5\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^2}-\frac{4x^{3/2}}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^(3/2))/(a*Sqrt[b*Sqrt[x] + a*x]) - (15*b*Sqrt[b*Sqrt[x] + a*x])/(2*a^
3) + (5*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/a^2 + (15*b^2*ArcTanh[(Sqrt[a]*Sqrt[
x])/Sqrt[b*Sqrt[x] + a*x]])/(2*a^(7/2))
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^4}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}+\frac{10\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}\\
&= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}+\frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{(15b)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{2a^2}\\
&= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}-\frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3}+\frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2}+\frac{\left(15b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^3}\\
&= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}-\frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3}+\frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2}+\frac{\left(15b^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^3}\\
&= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}-\frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3}+\frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2}+\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{7/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.049401, size = 62, normalized size = 0.55

$$\frac{4x^2\sqrt{\frac{a\sqrt{x}}{b}+1}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};-\frac{a\sqrt{x}}{b}\right)}{7b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^2*Hypergeometric2F1[3/2, 7/2, 9/2, -((a*Sqrt[x
])/b)])/(7*b*Sqrt[b*Sqrt[x] + a*x])
```

**Maple [B]** time = 0.008, size = 440, normalized size = 3.9

$$\frac{1}{4}\sqrt{b\sqrt{x}+ax}\left(4\sqrt{b\sqrt{x}+ax}a^{9/2}x^{3/2}+10\sqrt{b\sqrt{x}+ax}a^{7/2}xb-32\,a^{7/2}x\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}b+16\,a^{3}\ln\left(1/2\,\frac{2\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x)
```

```
[Out] 1/4*(b*x^(1/2)+a*x)^(1/2)/a^(9/2)*(4*(b*x^(1/2)+a*x)^(1/2)*a^(9/2)*x^(3/2)+
10*(b*x^(1/2)+a*x)^(1/2)*a^(7/2)*x*b-32*a^(7/2)*x*(x^(1/2)*(b+a*x^(1/2)))^(
1/2)*b+16*a^3*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b
)/a^(1/2))*x*b^2+8*(b*x^(1/2)+a*x)^(1/2)*a^(5/2)*x^(1/2)*b^2-64*a^(5/2)*x^(
1/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^2+16*a^(5/2)*(x^(1/2)*(b+a*x^(1/2)))^(
3/2)*b+32*a^2*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b
)/a^(1/2))*x^(1/2)*b^3-ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+
b)/a^(1/2))*x*a^3*b^2+2*(b*x^(1/2)+a*x)^(1/2)*a^(3/2)*b^3-32*a^(3/2)*(x^(1/
2)*(b+a*x^(1/2)))^(1/2)*b^3+16*a*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^
(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^4-2*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(
1/2)*a^(1/2)+b)/a^(1/2))*x^(1/2)*a^2*b^3-ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a
*x)^(1/2)*a^(1/2)+b)/a^(1/2))*a*b^4)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/(b+a*x^(
1/2))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(a*x + b*sqrt(x))^(3/2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x**(3/2)/(a*x + b*sqrt(x))**(3/2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.126 $\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=60

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}-\frac{4\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}$$

```
[Out] (-4*Sqrt[x])/(a*Sqrt[b*Sqrt[x] + a*x]) + (4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[
b*Sqrt[x] + a*x]])/a^(3/2)
```

---

**Rubi [A]** time = 0.0671288, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2018, 652, 620, 206}

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}-\frac{4\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*Sqrt[x])/(a*Sqrt[b*Sqrt[x] + a*x]) + (4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[
b*Sqrt[x] + a*x]])/a^(3/2)
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(e*(d + e*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[
(e^2*(p + 2))/(c*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= 2\,\mathrm{Subst}\left(\int \frac{x^2}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}\\
&= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}+\frac{4\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a}\\
&= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}}+\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.105056, size = 79, normalized size = 1.32

$$
\frac{4\sqrt[4]{x}\left(\sqrt{b}\sqrt{\frac{a\sqrt{x}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)-\sqrt{a}\sqrt[4]{x}\right)}{a^{3/2}\sqrt{ax+b\sqrt{x}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*x^(1/4)*(-(Sqrt[a]*x^(1/4)) + Sqrt[b]*Sqrt[1 + (a*Sqrt[x])/b]*ArcSinh[(S
qrt[a]*x^(1/4))/Sqrt[b]]))/(a^(3/2)*Sqrt[b*Sqrt[x] + a*x])
```

---

**Maple [B]** time = 0.006, size = 238, normalized size = 4.

$$
2\,\frac{\sqrt{b\sqrt{x}+ax}}{a^{3/2}\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}b\left(b+a\sqrt{x}\right)^{2}}\Bigg(\ln\Bigg(1/2\,\frac{2\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}\sqrt{a}+2\,a\sqrt{x}+b}{\sqrt{a}}\Bigg)xa^{2}b-2\,a^{5/2}x\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}+2\,\ln\Bigg(1
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x)
```

```
[Out] 2*(b*x^(1/2)+a*x)^(1/2)/a^(3/2)*(ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^
(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x*a^2*b-2*a^(5/2)*x*(x^(1/2)*(b+a*x^(1/2)))^(
1/2)+2*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/
2))*x^(1/2)*a*b^2-4*a^(3/2)*x^(1/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b+2*a^(3/
2)*(x^(1/2)*(b+a*x^(1/2)))^(3/2)+ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^
(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^3-2*a^(1/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b
^2)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b/(b+a*x^(1/2))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{x}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(a*x + b*sqrt(x))^(3/2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(x)/(a*x + b*sqrt(x))**(3/2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.127 $\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=30

$$-\frac{4\left(2a\sqrt{x}+b\right)}{b^2\sqrt{ax+b\sqrt{x}}}$$

```
[Out] (-4*(b + 2*a*Sqrt[x]))/(b^2*Sqrt[b*Sqrt[x] + a*x])
```

---

**Rubi [A]** time = 0.0471669, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2013, 613}

$$-\frac{4\left(2a\sqrt{x}+b\right)}{b^2\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*(b + 2*a*Sqrt[x]))/(b^2*Sqrt[b*Sqrt[x] + a*x])
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)\\ &= -\frac{4\left(b+2a\sqrt{x}\right)}{b^2\sqrt{b\sqrt{x}+ax}}\end{aligned}$$

**Mathematica [A]** time = 0.0373334, size = 45, normalized size = 1.5

$$-\frac{4\left(2a\sqrt{x}+b\right)\sqrt{ax+b\sqrt{x}}}{ab^2x+b^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*(b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(b^3*Sqrt[x] + a*b^2*x)
```

---

**Maple [B]** time = 0.01, size = 111, normalized size = 3.7

$$-4\,\frac{\sqrt{b\sqrt{x}+ax}\left(\left(b\sqrt{x}+ax\right)^{3/2}xa^{2}+2\left(b\sqrt{x}+ax\right)^{3/2}\sqrt{x}ab-\left(\sqrt{x}\left(b+a\sqrt{x}\right)\right)^{3/2}xa^{2}+\left(b\sqrt{x}+ax\right)^{3/2}b^{2}\right)}{\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}b^{3}\left(b+a\sqrt{x}\right)^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x)
```

```
[Out] -4*(b*x^(1/2)+a*x)^(1/2)*((b*x^(1/2)+a*x)^(3/2)*x*a^2+2*(b*x^(1/2)+a*x)^(3/
2)*x^(1/2)*a*b-(x^(1/2)*(b+a*x^(1/2)))^(3/2)*x*a^2+(b*x^(1/2)+a*x)^(3/2)*b^
2)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b^3/(b+a*x^(1/2))^2/x
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*sqrt(x)), x)
```

---

**Fricas [B]** time = 2.2923, size = 109, normalized size = 3.63

$$\frac{4\left(abx-\left(2\,a^{2}x-b^{2}\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{a^{2}b^{2}x^{2}-b^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4*(a*b*x - (2*a^2*x - b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^2*x^2 - b^
4*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(a*x + b*sqrt(x))**(3/2)), x)
```

---

**Giac [A]** time = 1.21724, size = 35, normalized size = 1.17

$$-\frac{4\left(\frac{2a\sqrt{x}}{b^2}+\frac{1}{b}\right)}{\sqrt{ax+b\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -4*(2*a*sqrt(x)/b^2 + 1/b)/sqrt(a*x + b*sqrt(x))
```

## 3.128 $\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=107

$$-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{5b^4\sqrt{x}}-\frac{24\sqrt{ax+b\sqrt{x}}}{5b^2x^{3/2}}+\frac{32a\sqrt{ax+b\sqrt{x}}}{5b^3x}+\frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

```
[Out] 4/(b*x*Sqrt[b*Sqrt[x] + a*x]) - (24*Sqrt[b*Sqrt[x] + a*x])/(5*b^2*x^(3/2))
+ (32*a*Sqrt[b*Sqrt[x] + a*x])/(5*b^3*x) - (64*a^2*Sqrt[b*Sqrt[x] + a*x])/(
5*b^4*Sqrt[x])
```

---

**Rubi [A]** time = 0.156028, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2015, 2016, 2014}

$$-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{5b^4\sqrt{x}}-\frac{24\sqrt{ax+b\sqrt{x}}}{5b^2x^{3/2}}+\frac{32a\sqrt{ax+b\sqrt{x}}}{5b^3x}+\frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x*Sqrt[b*Sqrt[x] + a*x]) - (24*Sqrt[b*Sqrt[x] + a*x])/(5*b^2*x^(3/2))
+ (32*a*Sqrt[b*Sqrt[x] + a*x])/(5*b^3*x) - (64*a^2*Sqrt[b*Sqrt[x] + a*x])/(
5*b^4*Sqrt[x])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= \frac{4}{bx\sqrt{b\sqrt{x}+ax}}+\frac{6\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{b}\\
&= \frac{4}{bx\sqrt{b\sqrt{x}+ax}}-\frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}}-\frac{(24a)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{5b^2}\\
&= \frac{4}{bx\sqrt{b\sqrt{x}+ax}}-\frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}}+\frac{32a\sqrt{b\sqrt{x}+ax}}{5b^3x}+\frac{\left(16a^2\right)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{5b^3}\\
&= \frac{4}{bx\sqrt{b\sqrt{x}+ax}}-\frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}}+\frac{32a\sqrt{b\sqrt{x}+ax}}{5b^3x}-\frac{64a^2\sqrt{b\sqrt{x}+ax}}{5b^4\sqrt{x}}\end{aligned}$$

**Mathematica [A]** time = 0.0506263, size = 57, normalized size = 0.53

$$-\frac{4\left(8a^2bx+16a^3x^{3/2}-2ab^2\sqrt{x}+b^3\right)}{5b^4x\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*(b^3 - 2*a*b^2*Sqrt[x] + 8*a^2*b*x + 16*a^3*x^(3/2)))/(5*b^4*x*Sqrt[b*S
qrt[x] + a*x])
```

---

**Maple [C]** time = 0.011, size = 548, normalized size = 5.1

$$-\frac{2}{5\,b^5}\sqrt{b\sqrt{x}+ax}\left(30\,\left(b\sqrt{x}+ax\right)^{3/2}a^{9/2}x^{7/2}-10\,\sqrt{b\sqrt{x}+ax}a^{11/2}x^{9/2}-5\,\ln\left(1/2\,\frac{2\,a\sqrt{x}+2\,\sqrt{b\sqrt{x}+ax}\sqrt{a}+b}{\sqrt{a}}\right)x^{9/2}a^{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x)
```

```
[Out] -2/5*(b*x^(1/2)+a*x)^(1/2)*(30*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)*x^(7/2)-10*(b*
x^(1/2)+a*x)^(1/2)*a^(11/2)*x^(9/2)-5*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)
^(1/2)*a^(1/2)+b)/a^(1/2))*x^(9/2)*a^5*b-10*a^(11/2)*x^(9/2)*(x^(1/2)*(b+a*
x^(1/2)))^(1/2)+5*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/
2)+b)/a^(1/2))*x^(9/2)*a^5*b+16*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*x^(5/2)*b^2+5
2*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*x^3*b-20*(b*x^(1/2)+a*x)^(1/2)*a^(9/2)*x^4*
b-10*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^4*a^
4*b^2-20*a^(9/2)*x^4*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b-10*a^(9/2)*x^(7/2)*(x^
(1/2)*(b+a*x^(1/2)))^(3/2)+10*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/
2)+2*a*x^(1/2)+b)/a^(1/2))*x^4*a^4*b^2-4*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*x^2*
b^3-10*(b*x^(1/2)+a*x)^(1/2)*a^(7/2)*x^(7/2)*b^2-5*ln(1/2*(2*a*x^(1/2)+2*(b
*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(7/2)*a^3*b^3-10*a^(7/2)*x^(7/2)*
(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b^2+5*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)
*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(7/2)*a^3*b^3+2*(b*x^(1/2)+a*x)^(3/2)*a^
```

```
(1/2)*x^(3/2)*b^4)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b^5/a^(1/2)/x^(7/2)/(b+a*x
^(1/2))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)
```

---

**Fricas [A]** time = 2.34268, size = 163, normalized size = 1.52

$$\frac{4\left(8\,a^3bx^2-3\,ab^3x-\left(16\,a^4x^2-10\,a^2b^2x-b^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{5\left(a^2b^4x^3-b^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4/5*(8*a^3*b*x^2 - 3*a*b^3*x - (16*a^4*x^2 - 10*a^2*b^2*x - b^4)*sqrt(x))*s
qrt(a*x + b*sqrt(x))/(a^2*b^4*x^3 - b^6*x^2)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)
```

## 3.129 $\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=165

$$-\frac{128a^2\sqrt{ax+b\sqrt{x}}}{21b^4x^{3/2}}-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{63b^6\sqrt{x}}+\frac{512a^3\sqrt{ax+b\sqrt{x}}}{63b^5x}+\frac{320a\sqrt{ax+b\sqrt{x}}}{63b^3x^2}-\frac{40\sqrt{ax+b\sqrt{x}}}{9b^2x^{5/2}}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

```
[Out] 4/(b*x^2*Sqrt[b*Sqrt[x] + a*x]) - (40*Sqrt[b*Sqrt[x] + a*x])/(9*b^2*x^(5/2)
) + (320*a*Sqrt[b*Sqrt[x] + a*x])/(63*b^3*x^2) - (128*a^2*Sqrt[b*Sqrt[x] +
a*x])/(21*b^4*x^(3/2)) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(63*b^5*x) - (1024
*a^4*Sqrt[b*Sqrt[x] + a*x])/(63*b^6*Sqrt[x])
```

---

**Rubi [A]** time = 0.257146, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2015, 2016, 2014}

$$-\frac{128a^2\sqrt{ax+b\sqrt{x}}}{21b^4x^{3/2}}-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{63b^6\sqrt{x}}+\frac{512a^3\sqrt{ax+b\sqrt{x}}}{63b^5x}+\frac{320a\sqrt{ax+b\sqrt{x}}}{63b^3x^2}-\frac{40\sqrt{ax+b\sqrt{x}}}{9b^2x^{5/2}}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^2*Sqrt[b*Sqrt[x] + a*x]) - (40*Sqrt[b*Sqrt[x] + a*x])/(9*b^2*x^(5/2)
) + (320*a*Sqrt[b*Sqrt[x] + a*x])/(63*b^3*x^2) - (128*a^2*Sqrt[b*Sqrt[x] +
a*x])/(21*b^4*x^(3/2)) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(63*b^5*x) - (1024
*a^4*Sqrt[b*Sqrt[x] + a*x])/(63*b^6*Sqrt[x])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}}+\frac{10\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{b}\\
&= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}}-\frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}}-\frac{(80a)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{9b^2}\\
&= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}}-\frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}}+\frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2}+\frac{\left(160a^2\right)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{21b^3}\\
&= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}}-\frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}}+\frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2}-\frac{128a^2\sqrt{b\sqrt{x}+ax}}{21b^4x^{3/2}}-\frac{\left(128a^3\right)\int \frac{1}{x^{3/2}\sqrt{b}}}{21b^4}\\
&= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}}-\frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}}+\frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2}-\frac{128a^2\sqrt{b\sqrt{x}+ax}}{21b^4x^{3/2}}+\frac{512a^3\sqrt{b\sqrt{x}+a}}{63b^5x}\\
&= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}}-\frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}}+\frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2}-\frac{128a^2\sqrt{b\sqrt{x}+ax}}{21b^4x^{3/2}}+\frac{512a^3\sqrt{b\sqrt{x}+a}}{63b^5x}\end{aligned}$$

**Mathematica [A]** time = 0.0522184, size = 83, normalized size = 0.5

$$-\frac{4\left(-32a^3b^2x^{3/2}+16a^2b^3x+128a^4bx^2+256a^5x^{5/2}-10ab^4\sqrt{x}+7b^5\right)}{63b^6x^2\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*(7*b^5 - 10*a*b^4*Sqrt[x] + 16*a^2*b^3*x - 32*a^3*b^2*x^(3/2) + 128*a^4
*b*x^2 + 256*a^5*x^(5/2)))/(63*b^6*x^2*Sqrt[b*Sqrt[x] + a*x])
```

---

**Maple [C]** time = 0.01, size = 592, normalized size = 3.6

$$-\frac{4}{63\,b^7}\sqrt{b\sqrt{x}+ax}\left(63\,\ln\left(1/2\,\frac{2\,\sqrt{\sqrt{x}\left(b+a\sqrt{x}\right)}\sqrt{a}+2\,a\sqrt{x}+b}{\sqrt{a}}\right)x^{11/2}a^5b^3-10\,\left(b\sqrt{x}+ax\right)^{3/2}a^{3/2}x^3b^5+63\,\ln\left(1/2\,\frac{2\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x)
```

```
[Out] -4/63*(b*x^(1/2)+a*x)^(1/2)*(63*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(
1/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(11/2)*a^5*b^3-10*(b*x^(1/2)+a*x)^(3/2)*a^(3
/2)*x^3*b^5+63*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+
b)/a^(1/2))*x^(13/2)*a^7*b-126*(b*x^(1/2)+a*x)^(1/2)*a^(15/2)*x^(13/2)-63*l
n(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a^(1/2))*x^(13/2)*a^7
```

```
*b-126*a^(15/2)*x^(13/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)-126*(b*x^(1/2)+a*x)^
(1/2)*a^(11/2)*x^(11/2)*b^2+128*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)*x^(9/2)*b^2+5
08*(b*x^(1/2)+a*x)^(3/2)*a^(11/2)*x^5*b-252*(b*x^(1/2)+a*x)^(1/2)*a^(13/2)*
x^6*b-252*a^(13/2)*x^6*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*b-32*(b*x^(1/2)+a*x)^(
3/2)*a^(7/2)*x^4*b^3+7*(b*x^(1/2)+a*x)^(3/2)*a^(1/2)*x^(5/2)*b^6+315*(b*x^(
1/2)+a*x)^(3/2)*a^(13/2)*x^(11/2)-126*a^(11/2)*x^(11/2)*(x^(1/2)*(b+a*x^(1/
2)))^(1/2)*b^2-126*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a
^(1/2))*x^6*a^6*b^2-63*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+
b)/a^(1/2))*x^(11/2)*a^5*b^3-63*a^(13/2)*x^(11/2)*(x^(1/2)*(b+a*x^(1/2)))^(
3/2)+126*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(
1/2))*x^6*a^6*b^2+16*(b*x^(1/2)+a*x)^(3/2)*a^(5/2)*x^(7/2)*b^4)/(x^(1/2)*(b
+a*x^(1/2)))^(1/2)/b^7/a^(1/2)/x^(11/2)/(b+a*x^(1/2))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)
```

---

**Fricas [A]** time = 2.39902, size = 220, normalized size = 1.33

$$\frac{4\left(128\,a^{5}bx^{3}-48\,a^{3}b^{3}x^{2}-17\,ab^{5}x-\left(256\,a^{6}x^{3}-160\,a^{4}b^{2}x^{2}-26\,a^{2}b^{4}x-7\,b^{6}\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{63\left(a^{2}b^{6}x^{4}-b^{8}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4/63*(128*a^5*b*x^3 - 48*a^3*b^3*x^2 - 17*a*b^5*x - (256*a^6*x^3 - 160*a^4*
b^2*x^2 - 26*a^2*b^4*x - 7*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^6*x^4
 - b^8*x^3)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{5}{2}}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**(5/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)
```

## 3.130 $\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=223

$$-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{143b^6x^{3/2}}+\frac{2560a^3\sqrt{ax+b\sqrt{x}}}{429b^5x^2}-\frac{2240a^2\sqrt{ax+b\sqrt{x}}}{429b^4x^{5/2}}-\frac{8192a^6\sqrt{ax+b\sqrt{x}}}{429b^8\sqrt{x}}+\frac{4096a^5\sqrt{ax+b\sqrt{x}}}{429b^7x}+$$

```
[Out] 4/(b*x^3*Sqrt[b*Sqrt[x] + a*x]) - (56*Sqrt[b*Sqrt[x] + a*x])/(13*b^2*x^(7/2
)) + (672*a*Sqrt[b*Sqrt[x] + a*x])/(143*b^3*x^3) - (2240*a^2*Sqrt[b*Sqrt[x]
 + a*x])/(429*b^4*x^(5/2)) + (2560*a^3*Sqrt[b*Sqrt[x] + a*x])/(429*b^5*x^2)
 - (1024*a^4*Sqrt[b*Sqrt[x] + a*x])/(143*b^6*x^(3/2)) + (4096*a^5*Sqrt[b*Sq
rt[x] + a*x])/(429*b^7*x) - (8192*a^6*Sqrt[b*Sqrt[x] + a*x])/(429*b^8*Sqrt[
x])
```

---

**Rubi [A]** time = 0.353146, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2015, 2016, 2014}

$$-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{143b^6x^{3/2}}+\frac{2560a^3\sqrt{ax+b\sqrt{x}}}{429b^5x^2}-\frac{2240a^2\sqrt{ax+b\sqrt{x}}}{429b^4x^{5/2}}-\frac{8192a^6\sqrt{ax+b\sqrt{x}}}{429b^8\sqrt{x}}+\frac{4096a^5\sqrt{ax+b\sqrt{x}}}{429b^7x}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^3*Sqrt[b*Sqrt[x] + a*x]) - (56*Sqrt[b*Sqrt[x] + a*x])/(13*b^2*x^(7/2
)) + (672*a*Sqrt[b*Sqrt[x] + a*x])/(143*b^3*x^3) - (2240*a^2*Sqrt[b*Sqrt[x]
 + a*x])/(429*b^4*x^(5/2)) + (2560*a^3*Sqrt[b*Sqrt[x] + a*x])/(429*b^5*x^2)
 - (1024*a^4*Sqrt[b*Sqrt[x] + a*x])/(143*b^6*x^(3/2)) + (4096*a^5*Sqrt[b*Sq
rt[x] + a*x])/(429*b^7*x) - (8192*a^6*Sqrt[b*Sqrt[x] + a*x])/(429*b^8*Sqrt[
x])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
```

```
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}+\frac{14\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx}{b}\\
&= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}-\frac{(168a)\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx}{13b^2}\\
&= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}+\frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3}+\frac{\left(1680a^2\right)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{143b^3}\\
&= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}+\frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3}-\frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}}-\frac{\left(4480a^3\right)\int \frac{}{x^{5/2}}}{429b^4}\\
&= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}+\frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3}-\frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}}+\frac{2560a^3\sqrt{b\sqrt{x}+}}{429b^5x^2}\\
&= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}+\frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3}-\frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}}+\frac{2560a^3\sqrt{b\sqrt{x}+}}{429b^5x^2}\\
&= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}+\frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3}-\frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}}+\frac{2560a^3\sqrt{b\sqrt{x}+}}{429b^5x^2}\\
&= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}}-\frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}}+\frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3}-\frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}}+\frac{2560a^3\sqrt{b\sqrt{x}+}}{429b^5x^2}
\end{aligned}
$$

**Mathematica [A]** time = 0.0629794, size = 107, normalized size = 0.48

$$
-\frac{4\left(-256a^5b^2x^{5/2}+128a^4b^3x^2-80a^3b^4x^{3/2}+56a^2b^5x+1024a^6bx^3+2048a^7x^{7/2}-42ab^6\sqrt{x}+33b^7\right)}{429b^8x^3\sqrt{ax+b\sqrt{x}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*(33*b^7 - 42*a*b^6*Sqrt[x] + 56*a^2*b^5*x - 80*a^3*b^4*x^(3/2) + 128*a^
4*b^3*x^2 - 256*a^5*b^2*x^(5/2) + 1024*a^6*b*x^3 + 2048*a^7*x^(7/2)))/(429*
b^8*x^3*Sqrt[b*Sqrt[x] + a*x])
```

---

**Maple [C]** time = 0.011, size = 636, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x)
```

```
[Out] -2/429*(b*x^(1/2)+a*x)^(1/2)*(2048*(b*x^(1/2)+a*x)^(3/2)*a^(13/2)*x^(13/2)*
b^2+9244*(b*x^(1/2)+a*x)^(3/2)*a^(15/2)*x^7*b+66*(b*x^(1/2)+a*x)^(3/2)*a^(1
/2)*x^(7/2)*b^8-160*(b*x^(1/2)+a*x)^(3/2)*a^(7/2)*x^5*b^5-2574*(b*x^(1/2)+a
*x)^(1/2)*a^(15/2)*x^(15/2)*b^2-2574*a^(15/2)*x^(15/2)*(x^(1/2)*(b+a*x^(1/2
)))^(1/2)*b^2-1287*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/a
^(1/2))*x^(17/2)*a^9*b-2574*a^(19/2)*x^(17/2)*(x^(1/2)*(b+a*x^(1/2)))^(1/2)
+1287*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2
))*x^(17/2)*a^9*b-2574*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+
b)/a^(1/2))*x^8*a^8*b^2-858*a^(17/2)*x^(15/2)*(x^(1/2)*(b+a*x^(1/2)))^(3/2)
+2574*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2
))*x^8*a^8*b^2-1287*ln(1/2*(2*a*x^(1/2)+2*(b*x^(1/2)+a*x)^(1/2)*a^(1/2)+b)/
a^(1/2))*x^(15/2)*a^7*b^3+1287*ln(1/2*(2*(x^(1/2)*(b+a*x^(1/2)))^(1/2)*a^(1
/2)+2*a*x^(1/2)+b)/a^(1/2))*x^(15/2)*a^7*b^3+112*(b*x^(1/2)+a*x)^(3/2)*a^(5
/2)*x^(9/2)*b^6-84*(b*x^(1/2)+a*x)^(3/2)*a^(3/2)*x^4*b^7+6006*(b*x^(1/2)+a*
x)^(3/2)*a^(17/2)*x^(15/2)-2574*(b*x^(1/2)+a*x)^(1/2)*a^(19/2)*x^(17/2)-514
8*(b*x^(1/2)+a*x)^(1/2)*a^(17/2)*x^8*b-5148*a^(17/2)*x^8*(x^(1/2)*(b+a*x^(1
/2)))^(1/2)*b+256*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)*x^(11/2)*b^4-512*(b*x^(1/2)
+a*x)^(3/2)*a^(11/2)*x^6*b^3)/(x^(1/2)*(b+a*x^(1/2)))^(1/2)/b^9/a^(1/2)/x^(
15/2)/(b+a*x^(1/2))^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}} x^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)
```

---

**Fricas [A]** time = 2.36309, size = 278, normalized size = 1.25

$$\frac{4\left(1024\,a^{7}bx^{4} - 384\,a^{5}b^{3}x^{3} - 136\,a^{3}b^{5}x^{2} - 75\,ab^{7}x - \left(2048\,a^{8}x^{4} - 1280\,a^{6}b^{2}x^{3} - 208\,a^{4}b^{4}x^{2} - 98\,a^{2}b^{6}x - 33\,b^{8}\right)\sqrt{x}\right)\sqrt{}}{429\left(a^{2}b^{8}x^{5} - b^{10}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4/429*(1024*a^7*b*x^4 - 384*a^5*b^3*x^3 - 136*a^3*b^5*x^2 - 75*a*b^7*x - (2
048*a^8*x^4 - 1280*a^6*b^2*x^3 - 208*a^4*b^4*x^2 - 98*a^2*b^6*x - 33*b^8)*s
qrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^8*x^5 - b^10*x^4)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)
```

## 3.131 $\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx$

**Optimal**. Leaf size=301

$$\frac{442b^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{14421a^{25/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{884b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{ax+b}}{216315a^4}$$

```
[Out] (-884*b^6*Sqrt[b*x^(1/3) + a*x])/(14421*a^6) + (884*b^5*x^(2/3)*Sqrt[b*x^(1
/3) + a*x])/(24035*a^5) - (6188*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(216315*
a^4) + (476*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(19665*a^3) - (28*b^2*x^(8/3)*Sq
rt[b*x^(1/3) + a*x])/(1311*a^2) + (4*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/(207
*a) + (2*x^4*Sqrt[b*x^(1/3) + a*x])/9 + (442*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^
(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*Elliptic
F[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(14421*a^(25/4)*Sqrt[b*x^(1/3)
 + a*x])
```

---

**Rubi [A]** time = 0.509048, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2021, 2024, 2011, 329, 220}

$$\frac{884b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{216315a^4}+\frac{476b^3x^2\sqrt{ax+b\sqrt[3]{x}}}{19665a^3}-\frac{28b^2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{1311a^2}+\frac{442b^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-884*b^6*Sqrt[b*x^(1/3) + a*x])/(14421*a^6) + (884*b^5*x^(2/3)*Sqrt[b*x^(1
/3) + a*x])/(24035*a^5) - (6188*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(216315*
a^4) + (476*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(19665*a^3) - (28*b^2*x^(8/3)*Sq
rt[b*x^(1/3) + a*x])/(1311*a^2) + (4*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/(207
*a) + (2*x^4*Sqrt[b*x^(1/3) + a*x])/9 + (442*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^
(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*Elliptic
F[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(14421*a^(25/4)*Sqrt[b*x^(1/3)
 + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rubi steps

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx = 3\,\text{Subst}\left(\int x^{11}\sqrt{bx+ax^3}\,dx,x,\sqrt[3]{x}\right)$$

$$= \frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax}+\frac{1}{9}(2b)\,\text{Subst}\left(\int \frac{x^{12}}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)$$

$$= \frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a}+\frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(14b^2\right)\text{Subst}\left(\int \frac{x^{10}}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{69a}$$

$$= -\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2}+\frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a}+\frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax}+\frac{\left(238b^3\right)\text{Subst}(\int \frac{x^8}{\sqrt{bx+ax^3}}\,dx,}{1311a^2}$$

$$= \frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2}+\frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a}+\frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax}-(3094b^4$$

$$= -\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}+\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2}+\frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a}+$$

$$= \frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}+\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2}$$

$$= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}+\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-$$

$$= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}+\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-$$

$$= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}+\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-$$

$$= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4}+\frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3}-$$

**Mathematica [C]** time = 0.160714, size = 155, normalized size = 0.51

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\,(\sqrt{\frac{ax^{2/3}}{b}+1}\,(-2310a^4b^2x^{8/3}+2618a^3b^3x^2-3094a^2b^4x^{4/3}+2090a^5bx^{10/3}+24035a^6x^4+3978ab^5x^{2/3}-}{216315a^6\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(-9945*b^6 + 3978*a*b^5*x
^(2/3) - 3094*a^2*b^4*x^(4/3) + 2618*a^3*b^3*x^2 - 2310*a^4*b^2*x^(8/3) + 2
090*a^5*b*x^(10/3) + 24035*a^6*x^4) + 9945*b^6*Hypergeometric2F1[-1/2, 1/4,
 5/4, -((a*x^(2/3))/b)]))/(216315*a^6*Sqrt[1 + (a*x^(2/3))/b])
```

---

**Maple [A]** time = 0.033, size = 264, normalized size = 0.9

$$\frac{2\,x^4}{9}\sqrt{b\sqrt[3]{x}+ax}+\frac{4\,b}{207\,a}x^{\frac{10}{3}}\sqrt{b\sqrt[3]{x}+ax}-\frac{28\,b^2}{1311\,a^2}x^{\frac{8}{3}}\sqrt{b\sqrt[3]{x}+ax}+\frac{476\,b^3x^2}{19665\,a^3}\sqrt{b\sqrt[3]{x}+ax}-\frac{6188\,b^4}{216315\,a^4}x^{\frac{4}{3}}\sqrt{b\sqrt[3]{x}+ax}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^(1/3)+a*x)^(1/2),x)
```

```
[Out] 2/9*x^4*(b*x^(1/3)+a*x)^(1/2)+4/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a-28/1
311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+476/19665*b^3*x^2*(b*x^(1/3)+a*x)
^(1/2)/a^3-6188/216315*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+884/24035*b^5*
x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-884/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a^6+4
42/14421*b^7/a^7*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(
1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b
)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*
a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax+bx^{\frac{1}{3}}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^3, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ax+bx^{\frac{1}{3}}}x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))*x^3, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{ax+b\sqrt[3]{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*x + b*x**(1/3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax+bx^{\frac{1}{3}}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^3, x)
```

## 3.132 $\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$

**Optimal**. Leaf size=411

$$\frac{22b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{221a^{19/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{44b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221a^{9/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{60b^2x^{5/3}\sqrt{}}{1547}$$

```
[Out] (44*b^5*(b + a*x^(2/3))*x^(1/3))/(221*a^(9/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*S
qrt[b*x^(1/3) + a*x]) - (44*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(663*a^4) +
(220*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^3) - (60*b^2*x^(5/3)*Sqrt[b*x^(1/
3) + a*x])/(1547*a^2) + (4*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(119*a) + (2*x^
3*Sqrt[b*x^(1/3) + a*x])/7 - (44*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[
(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(
a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(221*a^(19/4)*Sqrt[b*x^(1/3) + a*x]) + (22
*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[
a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])
/(221*a^(19/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.577651, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2021, 2024, 2032, 329, 305, 220, 1196}

$$\frac{44b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221a^{9/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{60b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a^2}+\frac{22b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{221a^{19/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (44*b^5*(b + a*x^(2/3))*x^(1/3))/(221*a^(9/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*S
qrt[b*x^(1/3) + a*x]) - (44*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(663*a^4) +
(220*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^3) - (60*b^2*x^(5/3)*Sqrt[b*x^(1/
3) + a*x])/(1547*a^2) + (4*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(119*a) + (2*x^
3*Sqrt[b*x^(1/3) + a*x])/7 - (44*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[
(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(
a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(221*a^(19/4)*Sqrt[b*x^(1/3) + a*x]) + (22
*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[
a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])
/(221*a^(19/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
```

```
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx = 3\,\text{Subst}\left(\int x^8\sqrt{bx+ax^3}\,dx,x,\sqrt[3]{x}\right)$$

$$= \frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax}+\frac{1}{7}(2b)\,\text{Subst}\left(\int \frac{x^9}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)$$

$$= \frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a}+\frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(30b^2\right)\text{Subst}\left(\int \frac{x^7}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{119a}$$

$$= -\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a}+\frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax}+\frac{\left(330b^3\right)\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^3}}\,dx,x,\right.}{1547a^2}$$

$$= \frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3}-\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a}+\frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax}-\left(110b^4\right)\text{Subs}$$

$$= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3}-\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a}+\frac{2}{7}x^3\sqrt{b}$$

$$= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3}-\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a}+\frac{2}{7}x^3\sqrt{b}$$

$$= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3}-\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a}+\frac{2}{7}x^3\sqrt{b}$$

$$= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3}-\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a}+\frac{2}{7}x^3\sqrt{b}$$

$$= \frac{44b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{9/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3}-\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}}}{1547a^2}$$

**Mathematica [C]** time = 0.113741, size = 136, normalized size = 0.33

$$\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\left(\sqrt{\frac{ax^{2/3}}{b}+1}\left(-90a^2b^2x^{4/3}+78a^3bx^2+663a^4x^{8/3}+110ab^3x^{2/3}-385b^4\right)+385b^4\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)\right)}{4641a^4\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(-385*b^4 + 110*a
*b^3*x^(2/3) - 90*a^2*b^2*x^(4/3) + 78*a^3*b*x^2 + 663*a^4*x^(8/3)) + 385*b
^4*Hypergeometric2F1[-1/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(4641*a^4*Sqrt[1 +
 (a*x^(2/3))/b])
```

---

**Maple [A]** time = 0.011, size = 273, normalized size = 0.7

$$\frac{2x^3}{7}\sqrt{b\sqrt[3]{x}+ax}+\frac{4b}{119a}x^{\frac{7}{3}}\sqrt{b\sqrt[3]{x}+ax}-\frac{60b^2}{1547a^2}x^{\frac{5}{3}}\sqrt{b\sqrt[3]{x}+ax}+\frac{220b^3x}{4641a^3}\sqrt{b\sqrt[3]{x}+ax}-\frac{44b^4}{663a^4}\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}+\frac{22b^5}{221a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^(1/3)+a*x)^(1/2),x)
```

```
[Out] 2/7*x^3*(b*x^(1/3)+a*x)^(1/2)+4/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a-60/15
47*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+220/4641*b^3*x*(b*x^(1/3)+a*x)^(1/
2)/a^3-44/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+22/221*b^5/a^5*(-a*b)^(
1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a
*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)
+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*
b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b
)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{1}{3}}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^2, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{ax + bx^{\frac{1}{3}}}x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))*x^2, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{ax + b\sqrt[3]{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*x + b*x**(1/3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{1}{3}}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^2, x)
```

# 3.133 $\int x\sqrt{b\sqrt[3]{x}+ax}\,dx$

**Optimal**. Leaf size=213

$$-\frac{6b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77a^{13/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{36b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^2}+\frac{12b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^3}+4$$

```
[Out] (12*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^3) - (36*b^2*x^(2/3)*Sqrt[b*x^(1/3) +
a*x])/(385*a^2) + (4*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(55*a) + (2*x^2*Sqrt[
b*x^(1/3) + a*x])/5 - (6*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x
^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*
x^(1/6))/b^(1/4)], 1/2])/(77*a^(13/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.299155, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2018, 2021, 2024, 2011, 329, 220}

$$-\frac{36b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^2}-\frac{6b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{77a^{13/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{12b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^3}+\frac{4bx^{4/3}\sqrt{ax}}{55a}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (12*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^3) - (36*b^2*x^(2/3)*Sqrt[b*x^(1/3) +
a*x])/(385*a^2) + (4*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(55*a) + (2*x^2*Sqrt[
b*x^(1/3) + a*x])/5 - (6*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x
^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*
x^(1/6))/b^(1/4)], 1/2])/(77*a^(13/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
```

```
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx = 3\,\text{Subst}\left(\int x^5\sqrt{bx+ax^3}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} + \frac{1}{5}(2b)\,\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{\left(18b^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{55a}$$

$$= -\frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} + \frac{\left(18b^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{77a^2}$$

$$= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{\left(6b^4\right)\text{Subst}\left(\int\right.}{77}$$

$$= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{\left(6b^4\sqrt{b+ax^{2/3}}\right.}{}$$

$$= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{\left(12b^4\sqrt{b+ax^{2/3}}\right.}{7}$$

$$= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{6b^{15/4}\left(\sqrt{b}+\sqrt{a}\right.}{}$$

**Mathematica [C]** time = 0.098597, size = 118, normalized size = 0.55

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(\sqrt{\frac{ax^{2/3}}{b}+1}\left(14a^2bx^{4/3}+77a^3x^2-18ab^2x^{2/3}+45b^3\right)-45b^3\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)\right)}{385a^3\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(45*b^3 - 18*a*b^2*x^(2/3
) + 14*a^2*b*x^(4/3) + 77*a^3*x^2) - 45*b^3*Hypergeometric2F1[-1/2, 1/4, 5/
4, -((a*x^(2/3))/b)]))/(385*a^3*Sqrt[1 + (a*x^(2/3))/b])
```

---

**Maple [A]** time = 0.013, size = 198, normalized size = 0.9

$$\frac{2x^{2}}{5}\sqrt{b\sqrt[3]{x}+ax}+\frac{4b}{55a}x^{\frac{4}{3}}\sqrt{b\sqrt[3]{x}+ax}-\frac{36b^{2}}{385a^{2}}x^{\frac{2}{3}}\sqrt{b\sqrt[3]{x}+ax}+\frac{12b^{3}}{77a^{3}}\sqrt{b\sqrt[3]{x}+ax}-\frac{6b^{4}}{77a^{4}}\sqrt{-ab}\sqrt{a\left(\sqrt[3]{x}+\frac{1}{a}\sqrt{-ab}\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^(1/3)+a*x)^(1/2),x)
```

```
[Out] 2/5*x^2*(b*x^(1/3)+a*x)^(1/2)+4/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a-36/385
*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+12/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^3-
6/77/a^4*b^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)
*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1
/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\sqrt{ax+bx^{\frac{1}{3}}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{ax+bx^{\frac{1}{3}}}x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))*x, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{ax+b\sqrt[3]{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*x + b*x**(1/3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{1}{3}}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x, x)
```

## 3.134 $\int \sqrt{b\sqrt[3]{x}+ax}\,dx$

**Optimal**. Leaf size=323

$$-\frac{2b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{4b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5a^{3/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{4b^{9/4}\sqrt[6]{x}\,(}{}$$

```
[Out] (-4*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(3/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[b*x^(1/3) + a*x]) + (4*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(15*a) + (2*x*Sqr
t[b*x^(1/3) + a*x])/3 + (4*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*
x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)
*x^(1/6))/b^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[b*x^(1/3) + a*x]) - (2*b^(9/4)*(S
qrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^
2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(7/4)*
Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.325712, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2004, 2018, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{4b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5a^{3/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{2b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{4b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-4*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(3/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[b*x^(1/3) + a*x]) + (4*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(15*a) + (2*x*Sqr
t[b*x^(1/3) + a*x])/3 + (4*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*
x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)
*x^(1/6))/b^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[b*x^(1/3) + a*x]) - (2*b^(9/4)*(S
qrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^
2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(7/4)*
Sqrt[b*x^(1/3) + a*x])
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

### Rubi steps

$$\begin{aligned}\int \sqrt{b\sqrt[3]{x}+ax}\,dx &= \frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}+\frac{1}{9}(2b)\int \frac{\sqrt[3]{x}}{\sqrt{b\sqrt[3]{x}+ax}}\,dx\\
&= \frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}+\frac{1}{3}(2b)\,\mathrm{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(2b^2\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{5a}\\
&= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(2b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{5a\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(4b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{5a\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(4b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{5a^{3/2}\sqrt{b\sqrt[3]{x}+ax}}+\frac{\left(4b^{5/2}\right.}{}\\
&= -\frac{4b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{3/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}+\frac{4b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{}}{5}\end{aligned}$$

**Mathematica [C]** time = 0.0537649, size = 94, normalized size = 0.29

$$\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\left(\left(ax^{2/3}+b\right)\sqrt{\frac{ax^{2/3}}{b}+1}-b\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)\right)}{3a\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))*Sqrt[1 + (a*x^(2/3))/b] -
 b*Hypergeometric2F1[-1/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(3*a*Sqrt[1 + (a*x
^(2/3))/b])
```

---

**Maple [A]** time = 0.012, size = 207, normalized size = 0.6

$$\frac{2x}{3}\sqrt{b\sqrt[3]{x}+ax}+\frac{4b}{15a}\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}-\frac{2b^2}{5a^2}\sqrt{-ab}\sqrt{a\left(\sqrt[3]{x}+\frac{1}{a}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\frac{a}{\sqrt{-ab}}\left(\sqrt[3]{x}-\frac{\sqrt{-ab}}{a}\right)}\sqrt{-a\sqrt[3]{x}\frac{1}{\sqrt{-ab}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(1/2),x)
```

```
[Out] 2/3*x*(b*x^(1/3)+a*x)^(1/2)+4/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a-2/5/a^2*
b^2*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(
1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2
```

```
)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*(-a*b)^(
1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF(((x^(1/
3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ax + bx^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3)), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + b\sqrt[3]{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x + b*x**(1/3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3)), x)
```

## 3.135 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx$

**Optimal**. Leaf size=123

$$\frac{2b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{ax+b\sqrt[3]{x}}}+2\sqrt{ax+b\sqrt[3]{x}}$$

```
[Out] 2*Sqrt[b*x^(1/3) + a*x] + (2*b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b +
a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/
4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.146223, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 2021, 2011, 329, 220}

$$\frac{2b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{ax+b\sqrt[3]{x}}}+2\sqrt{ax+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x,x]
```

```
[Out] 2*Sqrt[b*x^(1/3) + a*x] + (2*b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b +
a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/
4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx &= 3\,\mathrm{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x}\,dx, x, \sqrt[3]{x}\right)\\
&= 2\sqrt{b\sqrt[3]{x}+ax}+(2b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= 2\sqrt{b\sqrt[3]{x}+ax}+\frac{\left(2b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{b\sqrt[3]{x}+ax}}\\
&= 2\sqrt{b\sqrt[3]{x}+ax}+\frac{\left(4b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{b\sqrt[3]{x}+ax}}\\
&= 2\sqrt{b\sqrt[3]{x}+ax}+\frac{2b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

**Mathematica [C]** time = 0.0439971, size = 54, normalized size = 0.44

$$\frac{6\sqrt{ax+b\sqrt[3]{x}}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)}{\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x,x]
```

```
[Out] (6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((a*x^(2/3))/b)
])/Sqrt[1 + (a*x^(2/3))/b]
```

---

**Maple [A]** time = 0.012, size = 132, normalized size = 1.1

$$2\sqrt{b\sqrt[3]{x}+ax}+2\,\frac{b\sqrt{-ab}}{a\sqrt{b\sqrt[3]{x}+ax}}\sqrt{\frac{a}{\sqrt{-ab}}\left(\sqrt[3]{x}+\frac{\sqrt{-ab}}{a}\right)}\sqrt{-2\,\frac{a}{\sqrt{-ab}}\left(\sqrt[3]{x}-\frac{\sqrt{-ab}}{a}\right)}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{a}{\sqrt{-ab}}\left(\sqrt[3]{x}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(1/2)/x,x)
```

```
[Out] 2*(b*x^(1/3)+a*x)^(1/2)+2*b/a*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-
a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^
(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+1/a*(
-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))/x, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + b\sqrt[3]{x}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(1/3))/x, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x, x)
```

## 3.136 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx$

**Optimal**. Leaf size=325

$$\frac{6a^{5/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{12a^{5/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] (12*a^(3/2)*(b + a*x^(2/3))*x^(1/3))/(5*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[
b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(5*x) - (12*a*Sqrt[b*x^(1/3)
+ a*x])/(5*b*x^(1/3)) - (12*a^(5/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a
*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4
)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^(1/3) + a*x]) + (6*a^(5/4)*(
Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))
^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)
*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.352673, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{6a^{5/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{12a^{5/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right.}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^2,x]
```

```
[Out] (12*a^(3/2)*(b + a*x^(2/3))*x^(1/3))/(5*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[
b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(5*x) - (12*a*Sqrt[b*x^(1/3)
+ a*x])/(5*b*x^(1/3)) - (12*a^(5/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a
*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4
)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^(1/3) + a*x]) + (6*a^(5/4)*(
Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))
^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)
*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

### Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

### Rubi steps

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} + \frac{1}{5}(6a)\,\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} + \frac{\left(6a^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{5b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} + \frac{\left(6a^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{5b\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} + \frac{\left(12a^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5b\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} + \frac{\left(12a^{3/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(12a^{3/2}\sqrt{b}}{}$$

$$= \frac{12a^{3/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{12a^{5/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right.}}}{5b^{3/4}\sqrt{b\sqrt[3]{x}}}$$

**Mathematica [C]** time = 0.046491, size = 59, normalized size = 0.18

$$-\frac{6\sqrt{ax+b\sqrt[3]{x}}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{ax^{2/3}}{b}\right)}{5x\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^2,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((a*x^(2/3))
/b)])/(5*Sqrt[1 + (a*x^(2/3))/b]*x)
```

---

**Maple [A]** time = 0.018, size = 213, normalized size = 0.7

$$-\frac{6}{5\,x}\sqrt{b\sqrt[3]{x}+ax} - \frac{12\,a}{5\,b}\left(b+ax^{\frac{2}{3}}\right)\frac{1}{\sqrt{\sqrt[3]{x}\left(b+ax^{\frac{2}{3}}\right)}} + \frac{6\,a}{5\,b}\sqrt{-ab}\sqrt{a\left(\sqrt[3]{x}+\frac{1}{a}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{a}{\sqrt{-ab}}\left(\sqrt[3]{x}-\frac{\sqrt{-ab}}{a}\right)}\sqrt{-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(1/2)/x^2,x)
```

```
[Out] -6/5*(b*x^(1/3)+a*x)^(1/2)/x-12/5*(b+a*x^(2/3))/b*a/(x^(1/3)*(b+a*x^(2/3)))
^(1/2)+6/5/b*a*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/
2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^
```

```
(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1
/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*Ellipt
icF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^2, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))/x^2, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(1/3))/x**2, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^2, x)
```

## 3.137 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx$

**Optimal**. Leaf size=188

$$\frac{10a^{11/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{20a^2\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}}-\frac{12a\sqrt{ax+b\sqrt[3]{x}}}{77bx^{4/3}}-\frac{6\sqrt{ax+b}}{11x^2}$$

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(11*x^2) - (12*a*Sqrt[b*x^(1/3) + a*x])/(77*b*x^
(4/3)) + (20*a^2*Sqrt[b*x^(1/3) + a*x])/(77*b^2*x^(2/3)) + (10*a^(11/4)*(Sq
rt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2
]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(77*b^(9/4)*
Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.243934, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2020, 2025, 2011, 329, 220}

$$\frac{20a^2\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}}+\frac{10a^{11/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{12a\sqrt{ax+b\sqrt[3]{x}}}{77bx^{4/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{11x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^3,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(11*x^2) - (12*a*Sqrt[b*x^(1/3) + a*x])/(77*b*x^
(4/3)) + (20*a^2*Sqrt[b*x^(1/3) + a*x])/(77*b^2*x^(2/3)) + (10*a^(11/4)*(Sq
rt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2
]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(77*b^(9/4)*
Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx &= 3\,\mathrm{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^7}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} + \frac{1}{11}(6a)\,\mathrm{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} - \frac{\left(30a^2\right)\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{77b}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} + \frac{\left(10a^3\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{77b^2}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} + \frac{\left(10a^3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}}{77b^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} + \frac{\left(20a^3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx}{77b^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} + \frac{10a^{11/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F}{77b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0501204, size = 59, normalized size = 0.31

$$
-\frac{6\sqrt{ax+b\sqrt[3]{x}}\,{}_2F_1\left(-\frac{11}{4},-\frac{1}{2};-\frac{7}{4};-\frac{ax^{2/3}}{b}\right)}{11x^2\sqrt{\frac{ax^{2/3}}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^3,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-11/4, -1/2, -7/4, -((a*x^(2/3)
)/b)])/(11*Sqrt[1 + (a*x^(2/3))/b]*x^2)
```

---

**Maple [A]** time = 0.017, size = 179, normalized size = 1.

$$-\frac{6}{11\,x^{2}}\sqrt{b\sqrt[3]{x}+ax}-\frac{12\,a}{77\,b}\sqrt{b\sqrt[3]{x}+ax}x^{-\frac{4}{3}}+\frac{20\,a^{2}}{77\,b^{2}}\sqrt{b\sqrt[3]{x}+ax}x^{-\frac{2}{3}}+\frac{10\,a^{2}}{77\,b^{2}}\sqrt{-ab}\sqrt{a\left(\sqrt[3]{x}+\frac{1}{a}\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-2\,\frac{a}{\sqrt{-ab}}\Bigg(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(1/2)/x^3,x)
```

```
[Out] -6/11*(b*x^(1/3)+a*x)^(1/2)/x^2-12/77*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+20/
77*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)+10/77*a^2/b^2*(-a*b)^(1/2)*((x^(1/
3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a
/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*
EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^3, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))/x^3, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(1/3))/x**3, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^3, x)
```

## 3.138 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx$

**Optimal**. Leaf size=413

$$-\frac{154a^{17/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{308a^{9/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{1105b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{44a^2\sqrt{a}}{663b}$$

```
[Out] (-308*a^(9/2)*(b + a*x^(2/3))*x^(1/3))/(1105*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3)
)*Sqrt[b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(17*x^3) - (12*a*Sqrt[
b*x^(1/3) + a*x])/(221*b*x^(7/3)) + (44*a^2*Sqrt[b*x^(1/3) + a*x])/(663*b^2
*x^(5/3)) - (308*a^3*Sqrt[b*x^(1/3) + a*x])/(3315*b^3*x) + (308*a^4*Sqrt[b*
x^(1/3) + a*x])/(1105*b^4*x^(1/3)) + (308*a^(17/4)*(Sqrt[b] + Sqrt[a]*x^(1/
3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2
*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(1105*b^(15/4)*Sqrt[b*x^(1/3) + a
*x]) - (154*a^(17/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt
[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1
/4)], 1/2])/(1105*b^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.543533, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2020, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{308a^{9/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{1105b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{44a^2\sqrt{ax+b\sqrt[3]{x}}}{663b^2x^{5/3}}-\frac{154a^{17/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^4,x]
```

```
[Out] (-308*a^(9/2)*(b + a*x^(2/3))*x^(1/3))/(1105*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3)
)*Sqrt[b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(17*x^3) - (12*a*Sqrt[
b*x^(1/3) + a*x])/(221*b*x^(7/3)) + (44*a^2*Sqrt[b*x^(1/3) + a*x])/(663*b^2
*x^(5/3)) - (308*a^3*Sqrt[b*x^(1/3) + a*x])/(3315*b^3*x) + (308*a^4*Sqrt[b*
x^(1/3) + a*x])/(1105*b^4*x^(1/3)) + (308*a^(17/4)*(Sqrt[b] + Sqrt[a]*x^(1/
3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2
*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(1105*b^(15/4)*Sqrt[b*x^(1/3) + a
*x]) - (154*a^(17/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt
[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1
/4)], 1/2])/(1105*b^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
```

```
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = 3\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} + \frac{1}{17}(6a)\,\text{Subst}\left(\int \frac{1}{x^7\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} - \frac{(66a^2)\,\text{Subst}\left(\int \frac{1}{x^5\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{221b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} + \frac{(154a^3)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{663b^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} - \frac{(154a^4)\,\text{Subst}\left(\int \frac{1}{x\sqrt{bx}}\right.}{1105b^3}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{308a^4\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} - (1$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{308a^4\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} - (1$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{308a^4\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} - (3$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{308a^4\sqrt{b\sqrt[3]{x}+ax}}{1105b^4\sqrt[3]{x}} - (3$$

$$= -\frac{308a^{9/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3}{33}$$

**Mathematica [C]** time = 0.0470283, size = 59, normalized size = 0.14

$$-\frac{6\sqrt{ax+b\sqrt[3]{x}}\,{}_2F_1\left(-\frac{17}{4},-\frac{1}{2};-\frac{13}{4};-\frac{ax^{2/3}}{b}\right)}{17x^3\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^4,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-17/4, -1/2, -13/4, -((a*x^(2/3
))/b)])/(17*Sqrt[1 + (a*x^(2/3))/b]*x^3)
```

---

**Maple [A]** time = 0.018, size = 281, normalized size = 0.7

$$-\frac{6}{17\,x^3}\sqrt{b\sqrt[3]{x}+ax} - \frac{12\,a}{221\,b}\sqrt{b\sqrt[3]{x}+ax}x^{-\frac{7}{3}} + \frac{44\,a^2}{663\,b^2}\sqrt{b\sqrt[3]{x}+ax}x^{-\frac{5}{3}} - \frac{308\,a^3}{3315\,b^3x}\sqrt{b\sqrt[3]{x}+ax} + \frac{308\,a^4}{1105\,b^4}\left(b+ax^{\frac{2}{3}}\right)\frac{1}{\sqrt{\sqrt[3]{x}\left(b\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(1/2)/x^4,x)
```

```
[Out] -6/17*(b*x^(1/3)+a*x)^(1/2)/x^3-12/221*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+44
/663*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(5/3)-308/3315*a^3*(b*x^(1/3)+a*x)^(1/
2)/b^3/x+308/1105*(b+a*x^(2/3))*a^4/b^4/(x^(1/3)*(b+a*x^(2/3)))^(1/2)-154/1
105*a^4/b^4*(-a*b)^(1/2)*((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*
(-2*(x^(1/3)-1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/
2))^(1/2)/(b*x^(1/3)+a*x)^(1/2)*(-2/a*(-a*b)^(1/2)*EllipticE(((x^(1/3)+1/a*
(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/a*(-a*b)^(1/2)*EllipticF
(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^4, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))/x^4, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + b\sqrt[3]{x}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(1/3))/x**4, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^4, x)
```

## 3.139 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx$

**Optimal**. Leaf size=276

$$-\frac{1326a^{23/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{33649b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{2652a^5\sqrt{ax+b\sqrt[3]{x}}}{33649b^5x^{2/3}}+\frac{7956a^4\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}}$$

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(23*x^4) - (12*a*Sqrt[b*x^(1/3) + a*x])/(437*b*x
^(10/3)) + (68*a^2*Sqrt[b*x^(1/3) + a*x])/(2185*b^2*x^(8/3)) - (884*a^3*Sqr
t[b*x^(1/3) + a*x])/(24035*b^3*x^2) + (7956*a^4*Sqrt[b*x^(1/3) + a*x])/(168
245*b^4*x^(4/3)) - (2652*a^5*Sqrt[b*x^(1/3) + a*x])/(33649*b^5*x^(2/3)) - (
1326*a^(23/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + S
qrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1
/2])/(33649*b^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.412899, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2020, 2025, 2011, 329, 220}

$$-\frac{2652a^5\sqrt{ax+b\sqrt[3]{x}}}{33649b^5x^{2/3}}+\frac{7956a^4\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}}-\frac{884a^3\sqrt{ax+b\sqrt[3]{x}}}{24035b^3x^2}+\frac{68a^2\sqrt{ax+b\sqrt[3]{x}}}{2185b^2x^{8/3}}-\frac{1326a^{23/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)}{33649b^{21/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^5,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(23*x^4) - (12*a*Sqrt[b*x^(1/3) + a*x])/(437*b*x
^(10/3)) + (68*a^2*Sqrt[b*x^(1/3) + a*x])/(2185*b^2*x^(8/3)) - (884*a^3*Sqr
t[b*x^(1/3) + a*x])/(24035*b^3*x^2) + (7956*a^4*Sqrt[b*x^(1/3) + a*x])/(168
245*b^4*x^(4/3)) - (2652*a^5*Sqrt[b*x^(1/3) + a*x])/(33649*b^5*x^(2/3)) - (
1326*a^(23/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + S
qrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1
/2])/(33649*b^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = 3\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{13}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} + \frac{1}{23}(6a)\,\text{Subst}\left(\int \frac{1}{x^{10}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} - \frac{(102a^2)\,\text{Subst}\left(\int \frac{1}{x^8\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{437b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} + \frac{(442a^3)\,\text{Subst}\left(\int \frac{1}{x^6\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2185b^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} - \frac{(3978a^4)\,\text{Subst}\left(\int\right.}{240}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b\sqrt[3]{x}+ax}}{168245b^4x^{4/3}}$$

**Mathematica [C]** time = 0.0546913, size = 59, normalized size = 0.21

$$-\frac{6\sqrt{ax+b\sqrt[3]{x}}\,{}_2F_1\left(-\frac{23}{4},-\frac{1}{2};-\frac{19}{4};-\frac{ax^{2/3}}{b}\right)}{23x^4\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^5,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-23/4, -1/2, -19/4, -((a*x^(2/3
))/b)])/(23*Sqrt[1 + (a*x^(2/3))/b]*x^4)
```

---

**Maple [A]** time = 0.019, size = 245, normalized size = 0.9

$$-\frac{6}{23\,x^4}\sqrt{b\sqrt[3]{x}+ax} - \frac{12\,a}{437\,b}\sqrt{b\sqrt[3]{x}+ax}x^{-\frac{10}{3}} + \frac{68\,a^2}{2185\,b^2}\sqrt{b\sqrt[3]{x}+ax}x^{-\frac{8}{3}} - \frac{884\,a^3}{24035\,b^3x^2}\sqrt{b\sqrt[3]{x}+ax} + \frac{7956\,a^4}{168245\,b^4}\sqrt{b\sqrt[3]{x}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(1/2)/x^5,x)
```

```
[Out] -6/23*(b*x^(1/3)+a*x)^(1/2)/x^4-12/437*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+6
8/2185*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)-884/24035*a^3*(b*x^(1/3)+a*x)^
(1/2)/b^3/x^2+7956/168245*a^4*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(4/3)-2652/33649*
a^5*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-1326/33649*a^5/b^5*(-a*b)^(1/2)*((x^(
1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2)*(-2*(x^(1/3)-1/a*(-a*b)^(1/2))
*a/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)/(b*x^(1/3)+a*x)^(1/2
)*EllipticF(((x^(1/3)+1/a*(-a*b)^(1/2))*a/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^5, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))/x^5, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^5, x)
```

## 3.140 $\int x^2 \left(b\sqrt[3]{x}+ax\right)^{3/2} dx$

**Optimal**. Leaf size=298

$$-\frac{884 b^{27/4} \sqrt[6]{x} \left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right) \sqrt{\frac{a x^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}} \text{EllipticF}\left(2 \tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{100947 a^{21/4} \sqrt{a x+b\sqrt[3]{x}}}-\frac{1768 b^5 x^{2/3} \sqrt{a x+b\sqrt[3]{x}}}{168245 a^4}+\frac{1768 b^4 x^{4/3} \sqrt{a x+b}}{216315 a^3}$$

```
[Out] (1768*b^6*Sqrt[b*x^(1/3) + a*x])/(100947*a^5) - (1768*b^5*x^(2/3)*Sqrt[b*x^
(1/3) + a*x])/(168245*a^4) + (1768*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(2163
15*a^3) - (136*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(19665*a^2) + (8*b^2*x^(8/3)*
Sqrt[b*x^(1/3) + a*x])/(1311*a) + (4*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/69 +
 (2*x^3*(b*x^(1/3) + a*x)^(3/2))/9 - (884*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^(1/
3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2
*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(100947*a^(21/4)*Sqrt[b*x^(1/3) +
 a*x])
```

---

**Rubi [A]** time = 0.50061, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2021, 2024, 2011, 329, 220}

$$-\frac{1768 b^5 x^{2/3} \sqrt{a x+b\sqrt[3]{x}}}{168245 a^4}+\frac{1768 b^4 x^{4/3} \sqrt{a x+b\sqrt[3]{x}}}{216315 a^3}-\frac{136 b^3 x^2 \sqrt{a x+b\sqrt[3]{x}}}{19665 a^2}-\frac{884 b^{27/4} \sqrt[6]{x} \left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right) \sqrt{\frac{a x^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}} F\left(2\right.}{100947 a^{21/4} \sqrt{a x+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (1768*b^6*Sqrt[b*x^(1/3) + a*x])/(100947*a^5) - (1768*b^5*x^(2/3)*Sqrt[b*x^
(1/3) + a*x])/(168245*a^4) + (1768*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(2163
15*a^3) - (136*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(19665*a^2) + (8*b^2*x^(8/3)*
Sqrt[b*x^(1/3) + a*x])/(1311*a) + (4*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/69 +
 (2*x^3*(b*x^(1/3) + a*x)^(3/2))/9 - (884*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^(1/
3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2
*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(100947*a^(21/4)*Sqrt[b*x^(1/3) +
 a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rubi steps

$$
\begin{aligned}
\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}dx &= 3\,\text{Subst}\left(\int x^8\left(bx+ax^3\right)^{3/2}dx,x,\sqrt[3]{x}\right)\\
&= \frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{3}(2b)\,\text{Subst}\left(\int x^9\sqrt{bx+ax^3}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{69}\left(4b^2\right)\text{Subst}\left(\int\frac{x^{10}}{\sqrt{bx+ax^3}}dx,x,\sqrt[3]{x}\right)\\
&= \frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(68b^3\right)\text{Subst}\left(\int\frac{x^8}{\sqrt{bx+ax^3}}dx,\right.}{1311a}\\
&= -\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}+(88\\
&= \frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}+\frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{9}\\
&= -\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a}\\
&= \frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\\
&= \frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\\
&= \frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+\\
&= \frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3}-\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2}+
\end{aligned}
$$

**Mathematica [C]** time = 0.137646, size = 142, normalized size = 0.48

$$
\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(\left(ax^{2/3}+b\right)^2\sqrt{\frac{ax^{2/3}}{b}+1}\left(12155a^2b^2x^{4/3}-17765a^3bx^2+24035a^4x^{8/3}-7293ab^3x^{2/3}+3315b^4\right)-3315b^6\,{}_2F_1\right.}{216315a^5\sqrt{\frac{ax^{2/3}}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))^2*Sqrt[1 + (a*x^(2/3))/b]*(3315*b
^4 - 7293*a*b^3*x^(2/3) + 12155*a^2*b^2*x^(4/3) - 17765*a^3*b*x^2 + 24035*a
^4*x^(8/3)) - 3315*b^6*Hypergeometric2F1[-3/2, 1/4, 5/4, -((a*x^(2/3))/b)])
)/(216315*a^5*Sqrt[1 + (a*x^(2/3))/b])
```

---

**Maple [A]** time = 0.027, size = 197, normalized size = 0.7

$$
-\frac{2}{1514205\,a^6}\left(-216755\,x^{11/3}a^6b^2-380380\,x^{13/3}a^7b+616\,x^3a^5b^3+6630\,b^7\sqrt{-ab}\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^(1/3)+a*x)^(3/2),x)
```

```
[Out] -2/1514205*(-216755*x^(11/3)*a^6*b^2-380380*x^(13/3)*a^7*b+616*x^3*a^5*b^3+
6630*b^7*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)
*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(
1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1
768*x^(5/3)*a^3*b^5-952*x^(7/3)*a^4*b^4-168245*x^5*a^8-5304*x*a^2*b^6-13260
*x^(1/3)*a*b^7)/a^6/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)*x^2, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ax^{3}+bx^{\frac{7}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x^3 + b*x^(7/3))*sqrt(a*x + b*x^(1/3)), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)*x^2, x)
```

# 3.141 $\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

**Optimal**. Leaf size=408

$$-\frac{44b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{88b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{1105a^{7/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{88b^{21}}{}$$

```
[Out] (-88*b^5*(b + a*x^(2/3))*x^(1/3))/(1105*a^(7/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))
*Sqrt[b*x^(1/3) + a*x]) + (88*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(3315*a^3)
 - (88*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^2) + (24*b^2*x^(5/3)*Sqrt[b*x^(
1/3) + a*x])/(1547*a) + (12*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/119 + (2*x^2*(
b*x^(1/3) + a*x)^(3/2))/7 + (88*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(
b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a
^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(1105*a^(15/4)*Sqrt[b*x^(1/3) + a*x]) - (44
*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[
a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])
/(1105*a^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.554065, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2018, 2021, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{88b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{1105a^{7/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{44b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{1105a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{88b^{21/4}\sqrt[6]{x}\left(\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-88*b^5*(b + a*x^(2/3))*x^(1/3))/(1105*a^(7/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))
*Sqrt[b*x^(1/3) + a*x]) + (88*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(3315*a^3)
 - (88*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^2) + (24*b^2*x^(5/3)*Sqrt[b*x^(
1/3) + a*x])/(1547*a) + (12*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/119 + (2*x^2*(
b*x^(1/3) + a*x)^(3/2))/7 + (88*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(
b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a
^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(1105*a^(15/4)*Sqrt[b*x^(1/3) + a*x]) - (44
*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[
a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])
/(1105*a^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
```

```
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx &= 3\,\mathrm{Subst}\left(\int x^5\left(bx+ax^3\right)^{3/2}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{7}(6b)\,\mathrm{Subst}\left(\int x^6\sqrt{bx+ax^3}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{119}\left(12b^2\right)\mathrm{Subst}\left(\int\frac{x^7}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(132b^3\right)\mathrm{Subst}\left(\int\frac{x^5}{\sqrt{bx+ax}}\right.}{1547a}\\
&= -\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{(4}{}\\
&= \frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}\\
&= \frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}\\
&= \frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}\\
&= \frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}\\
&= -\frac{88b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}}{15}
\end{aligned}
$$

**Mathematica [C]** time = 0.10018, size = 123, normalized size = 0.3

$$
\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\left(\left(ax^{2/3}+b\right)^2\sqrt{\frac{ax^{2/3}}{b}+1}\left(221a^2x^{4/3}-143abx^{2/3}+77b^2\right)-77b^4\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)\right)}{1547a^3\sqrt{\frac{ax^{2/3}}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))^2*Sqrt[1 + (a*x^(2/3))/b]
*(77*b^2 - 143*a*b*x^(2/3) + 221*a^2*x^(4/3)) - 77*b^4*Hypergeometric2F1[-3
/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(1547*a^3*Sqrt[1 + (a*x^(2/3))/b])
```

---

**Maple [A]** time = 0.018, size = 263, normalized size = 0.6

$$
-\frac{2}{23205\,a^4}\left(-4665\,x^{8/3}a^4b^2-7800\,x^{10/3}a^5b+40\,x^2a^3b^3+924\,b^6\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\mathrm{Ellipt}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^(1/3)+a*x)^(3/2),x)
```

```
[Out] -2/23205/a^4*(-4665*x^(8/3)*a^4*b^2-7800*x^(10/3)*a^5*b+40*x^2*a^3*b^3+924*
b^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticE(((a
*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-462*b^6*((a*x^(1/3)
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-3315*x^4*a^6-308*x^(2/3)*a*b^5-88*
x^(4/3)*a^2*b^4)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)*x, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(ax^{2} + bx^{\frac{4}{3}}\right)\sqrt{ax + bx^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x^2 + b*x^(4/3))*sqrt(a*x + b*x^(1/3)), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x*(a*x + b*x**(1/3))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)*x, x)
```

## 3.142 $\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

**Optimal**. Leaf size=208

$$\frac{4b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77a^{9/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a}+\frac{12}{55}bx^{4/3}$$

```
[Out] (-8*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^2) + (24*b^2*x^(2/3)*Sqrt[b*x^(1/3) +
a*x])/(385*a) + (12*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/55 + (2*x*(b*x^(1/3) +
 a*x)^(3/2))/5 + (4*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3
))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(77*a^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.270748, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2004, 2018, 2021, 2024, 2011, 329, 220}

$$\frac{4b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77a^{9/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{ax+b}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-8*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^2) + (24*b^2*x^(2/3)*Sqrt[b*x^(1/3) +
a*x])/(385*a) + (12*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/55 + (2*x*(b*x^(1/3) +
 a*x)^(3/2))/5 + (4*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3
))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(77*a^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx &= \frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{5}(2b)\int \sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\,dx\\
&= \frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{5}(6b)\,\text{Subst}\left(\int x^3\sqrt{bx+ax^3}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{55}\left(12b^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(12b^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^3}}\,dx,\right.}{77a}\\
&= -\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{\left(4b^4\right)\text{Su}}{}\\
&= -\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{\left(4b^4\sqrt{b}\right.}{}\\
&= -\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{\left(8b^4\sqrt{b}\right.}{}\\
&= -\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{4b^{15/4}\left(\sqrt{}\right.}{}
\end{aligned}$$

**Mathematica [C]** time = 0.0809067, size = 106, normalized size = 0.51

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(5b^3\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)-\left(5b-11ax^{2/3}\right)\left(ax^{2/3}+b\right)^2\sqrt{\frac{ax^{2/3}}{b}+1}\right)}{55a^2\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(-((5*b - 11*a*x^(2/3))*(b + a*x^(2/3))^2*Sqrt[1 +
 (a*x^(2/3))/b]) + 5*b^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((a*x^(2/3))/b)
]))/(55*a^2*Sqrt[1 + (a*x^(2/3))/b])
```

---

**Maple [A]** time = 0.017, size = 164, normalized size = 0.8

$$\frac{2}{385\,a^3}\left(10\,b^4\sqrt{-ab}\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)+131\,x^{5/3}a^3b^2+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(3/2),x)
```

```
[Out] 2/385*(10*b^4*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^
(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/
2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/
2))+131*x^(5/3)*a^3*b^2+196*x^(7/3)*a^4*b-8*x*a^2*b^3+77*x^3*a^5-20*x^(1/3)
*a*b^4)/a^3/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**(1/3))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2), x)
```

## 3.143 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx$

**Optimal**. Leaf size=319

$$\frac{4b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{5a^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] (8*b^2*(b + a*x^(2/3))*x^(1/3))/(5*Sqrt[a]*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt
[b*x^(1/3) + a*x]) + (4*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/5 + (2*(b*x^(1/3)
+ a*x)^(3/2))/3 - (8*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3
))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[b*x^(1/3) + a*x]) + (4*b^(9/4)*(Sqrt[b]
 + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(
1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[b
*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.32934, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2021, 2004, 2032, 329, 305, 220, 1196}

$$\frac{4b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x,x]
```

```
[Out] (8*b^2*(b + a*x^(2/3))*x^(1/3))/(5*Sqrt[a]*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt
[b*x^(1/3) + a*x]) + (4*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/5 + (2*(b*x^(1/3)
+ a*x)^(3/2))/3 - (8*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3
))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[b*x^(1/3) + a*x]) + (4*b^(9/4)*(Sqrt[b]
 + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(
1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[b
*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

### Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx = 3\,\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + (2b)\,\text{Subst}\left(\int \sqrt{bx+ax^3}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{1}{5}\left(4b^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{\left(4b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{5\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{\left(8b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{\left(8b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5\sqrt{a}\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(8b^{5/2}\right.}{}$$

$$= \frac{8b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5\sqrt{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} + \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} - \frac{8b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{}}{5a}$$

**Mathematica [C]** time = 0.0457798, size = 60, normalized size = 0.19

$$\frac{2b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)}{\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x,x]
```

```
[Out] (2*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((a*x
^(2/3))/b)])/Sqrt[1 + (a*x^(2/3))/b]
```

---

**Maple [A]** time = 0.017, size = 230, normalized size = 0.7

$$\frac{2}{15a}\left(12b^3\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)-6b^3\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x,x)
```

```
[Out] 2/15/a*(12*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x
^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*El
lipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-6*b^3*((
a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/
```

```
3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+11*x^(2/3)*a*b^2+16*x^(4/
3)*a^2*b+5*x^2*a^3)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x,x)
```

```
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x, x)
```

## 3.144 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx$

**Optimal**. Leaf size=144

$$\frac{4a^{3/4}b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x}+4a\sqrt{ax+b\sqrt[3]{x}}$$

```
[Out] 4*a*Sqrt[b*x^(1/3) + a*x] - (2*(b*x^(1/3) + a*x)^(3/2))/x + (4*a^(3/4)*b^(3
/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(
1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/Sqrt[
b*x^(1/3) + a*x]
```

---

**Rubi [A]** time = 0.20206, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2020, 2021, 2011, 329, 220}

$$\frac{4a^{3/4}b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x}+4a\sqrt{ax+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] 4*a*Sqrt[b*x^(1/3) + a*x] - (2*(b*x^(1/3) + a*x)^(3/2))/x + (4*a^(3/4)*b^(3
/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(
1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/Sqrt[
b*x^(1/3) + a*x]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx &= 3\,\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^4}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + (6a)\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x}\,dx, x, \sqrt[3]{x}\right)\\
&= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + (4ab)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + \frac{\left(4ab\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{b\sqrt[3]{x}+ax}}\\
&= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + \frac{\left(8ab\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{b\sqrt[3]{x}+ax}}\\
&= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + \frac{4a^{3/4}b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\right.}{\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0582228, size = 60, normalized size = 0.42

$$
-\frac{2b\sqrt{ax+b\sqrt[3]{x}}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{ax^{2/3}}{b}\right)}{x^{2/3}\sqrt{\frac{ax^{2/3}}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((a*x^(2/3)
)/b)])/(Sqrt[1 + (a*x^(2/3))/b]*x^(2/3))
```

---

**Maple [A]** time = 0.02, size = 130, normalized size = 0.9

$$2\frac{1}{\sqrt[3]{x}\sqrt{\sqrt[3]{x}\left(b+ax^{2/3}\right)}}\left(2\sqrt[3]{x}\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-2\frac{a\sqrt[3]{x}-\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}b+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x^2,x)
```

```
[Out] 2/x^(1/3)*(2*x^(1/3)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x
^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*El
lipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(
1/2)*b+x^(4/3)*a^2-b^2)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**2,x)
```

```
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x**2, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

## 3.145 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx$

**Optimal**. Leaf size=350

$$\frac{4a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] (8*a^(5/2)*(b + a*x^(2/3))*x^(1/3))/(5*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b
*x^(1/3) + a*x]) - (4*a*Sqrt[b*x^(1/3) + a*x])/(5*x) - (8*a^2*Sqrt[b*x^(1/3
) + a*x])/(5*b*x^(1/3)) - (2*(b*x^(1/3) + a*x)^(3/2))/(3*x^2) - (8*a^(9/4)*
(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3)
)^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4
)*Sqrt[b*x^(1/3) + a*x]) + (4*a^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b +
 a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1
/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.431339, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{4a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (8*a^(5/2)*(b + a*x^(2/3))*x^(1/3))/(5*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b
*x^(1/3) + a*x]) - (4*a*Sqrt[b*x^(1/3) + a*x])/(5*x) - (8*a^2*Sqrt[b*x^(1/3
) + a*x])/(5*b*x^(1/3)) - (2*(b*x^(1/3) + a*x)^(3/2))/(3*x^2) - (8*a^(9/4)*
(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3)
)^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4
)*Sqrt[b*x^(1/3) + a*x]) + (4*a^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b +
 a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1
/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

### Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

### Rubi steps

$$
\begin{aligned}
\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx &= 3\,\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^7}\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2}+(2a)\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^4}\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2}+\frac{1}{5}\left(4a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x}-\frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2}+\frac{\left(4a^3\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{5b}\\
&= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x}-\frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2}+\frac{\left(4a^3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx,\right.}{5b\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x}-\frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2}+\frac{\left(8a^3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx,\right.}{5b\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x}-\frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2}+\frac{\left(8a^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx\right.}{5\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{8a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x}-\frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2}-\frac{8a^{9/4}\left(\sqrt{b}\right.}{}
\end{aligned}
$$

**Mathematica [C]** time = 0.0529985, size = 62, normalized size = 0.18

$$
-\frac{2b\sqrt{ax+b\sqrt[3]{x}}\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{ax^{2/3}}{b}\right)}{3x^{5/3}\sqrt{\frac{ax^{2/3}}{b}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((a*x^(2/3
))/b)])/(3*Sqrt[1 + (a*x^(2/3))/b]*x^(5/3))
```

---

**Maple [A]** time = 0.022, size = 339, normalized size = 1.

$$
-\frac{2}{15\,bx^3}\left(-12\,a^2b\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-2\,\frac{a\sqrt[3]{x}-\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}x^{8/3}\sqrt{\sqrt[3]{x}\left(b+ax^{2/3}\right)}\text{EllipticE}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)+\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x^3,x)
```

```
[Out] -2/15*(-12*a^2*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/
3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*x^(8/3
```

```
)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^
(1/2))^(1/2),1/2*2^(1/2))+6*a^2*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1
/2))^(1/2)*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF(((a*x^(1/3)+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+12*x^(10/3)*(b*x^(1/3)+a*x)^(1/2
)*a^3+12*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)*a^2*b+16*x^2*(x^(1/3)*(b+a*x^(2/3)))
^(1/2)*a*b^2+11*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b+5*x^(4/3)*(x^(1
/3)*(b+a*x^(2/3)))^(1/2)*b^3)/b/x^3/(b+a*x^(2/3))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**3,x)
```

```
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x**3, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

## 3.146 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx$

**Optimal**. Leaf size=213

$$\frac{4a^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{8a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}}-\frac{24a^2\sqrt{ax+b\sqrt[3]{x}}}{385bx^{4/3}}-\frac{12a\sqrt{ax}}{55}$$

```
[Out] (-12*a*Sqrt[b*x^(1/3) + a*x])/(55*x^2) - (24*a^2*Sqrt[b*x^(1/3) + a*x])/(38
5*b*x^(4/3)) + (8*a^3*Sqrt[b*x^(1/3) + a*x])/(77*b^2*x^(2/3)) - (2*(b*x^(1/
3) + a*x)^(3/2))/(5*x^3) + (4*a^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b
+ a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(
1/4)*x^(1/6))/b^(1/4)], 1/2])/(77*b^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.308573, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2020, 2025, 2011, 329, 220}

$$\frac{8a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}}+\frac{4a^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{24a^2\sqrt{ax+b\sqrt[3]{x}}}{385bx^{4/3}}-\frac{12a\sqrt{ax+b\sqrt[3]{x}}}{55x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-12*a*Sqrt[b*x^(1/3) + a*x])/(55*x^2) - (24*a^2*Sqrt[b*x^(1/3) + a*x])/(38
5*b*x^(4/3)) + (8*a^3*Sqrt[b*x^(1/3) + a*x])/(77*b^2*x^(2/3)) - (2*(b*x^(1/
3) + a*x)^(3/2))/(5*x^3) + (4*a^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b
+ a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(
1/4)*x^(1/6))/b^(1/4)], 1/2])/(77*b^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx = 3\,\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^{10}}\,dx,x,\sqrt[3]{x}\right)$$

$$= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}+\frac{1}{5}(6a)\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^7}\,dx,x,\sqrt[3]{x}\right)$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}+\frac{1}{55}\left(12a^2\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}-\frac{\left(12a^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{77b}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}+\frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}+\frac{\left(4a^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+}}\right.}{77b^2}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}+\frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}+\frac{\left(4a^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)}{77b}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}+\frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}+\frac{\left(8a^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)}{77b^2}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2}-\frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}}+\frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}}-\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3}+\frac{4a^{15/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)}{}$$

**Mathematica [C]** time = 0.0661396, size = 62, normalized size = 0.29

$$-\frac{2b\sqrt{ax+b\sqrt[3]{x}}\,{}_2F_1\left(-\frac{15}{4},-\frac{3}{2};-\frac{11}{4};-\frac{ax^{2/3}}{b}\right)}{5x^{8/3}\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-15/4, -3/2, -11/4, -((a*x^(2
/3))/b)])/(5*Sqrt[1 + (a*x^(2/3))/b]*x^(8/3))
```

---

**Maple [A]** time = 0.023, size = 168, normalized size = 0.8

$$\frac{2}{385\,b^2}\left(10\,a^3\sqrt{-ab}\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-2\,\frac{a\sqrt[3]{x}-\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)x^{14/3}-131\,x^{11/3}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x^4,x)
```

```
[Out] 2/385*(10*a^3*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-
2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1
/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^
(14/3)-131*x^(11/3)*a^2*b^2+8*x^(13/3)*a^3*b-196*a*b^3*x^3+20*x^5*a^4-77*x^
(7/3)*b^4)/b^2/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^(14/3)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**4,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

## 3.147 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

**Optimal**. Leaf size=438

$$-\frac{44a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{88a^{11/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{1105b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{88a^3\sqrt{}}{464}$$

```
[Out] (-88*a^(11/2)*(b + a*x^(2/3))*x^(1/3))/(1105*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3)
)*Sqrt[b*x^(1/3) + a*x]) - (12*a*Sqrt[b*x^(1/3) + a*x])/(119*x^3) - (24*a^2
*Sqrt[b*x^(1/3) + a*x])/(1547*b*x^(7/3)) + (88*a^3*Sqrt[b*x^(1/3) + a*x])/(
4641*b^2*x^(5/3)) - (88*a^4*Sqrt[b*x^(1/3) + a*x])/(3315*b^3*x) + (88*a^5*S
qrt[b*x^(1/3) + a*x])/(1105*b^4*x^(1/3)) - (2*(b*x^(1/3) + a*x)^(3/2))/(7*x
^4) + (88*a^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b
] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4
)], 1/2])/(1105*b^(15/4)*Sqrt[b*x^(1/3) + a*x]) - (44*a^(21/4)*(Sqrt[b] + S
qrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)
*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(1105*b^(15/4)*Sqrt[b
*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.640416, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2020, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{88a^{11/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{1105b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{88a^3\sqrt{ax+b\sqrt[3]{x}}}{4641b^2x^{5/3}}-\frac{44a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] (-88*a^(11/2)*(b + a*x^(2/3))*x^(1/3))/(1105*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3)
)*Sqrt[b*x^(1/3) + a*x]) - (12*a*Sqrt[b*x^(1/3) + a*x])/(119*x^3) - (24*a^2
*Sqrt[b*x^(1/3) + a*x])/(1547*b*x^(7/3)) + (88*a^3*Sqrt[b*x^(1/3) + a*x])/(
4641*b^2*x^(5/3)) - (88*a^4*Sqrt[b*x^(1/3) + a*x])/(3315*b^3*x) + (88*a^5*S
qrt[b*x^(1/3) + a*x])/(1105*b^4*x^(1/3)) - (2*(b*x^(1/3) + a*x)^(3/2))/(7*x
^4) + (88*a^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b
] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4
)], 1/2])/(1105*b^(15/4)*Sqrt[b*x^(1/3) + a*x]) - (44*a^(21/4)*(Sqrt[b] + S
qrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)
*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(1105*b^(15/4)*Sqrt[b
*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

### Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

### Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx = 3\,\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^{13}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} + \frac{1}{7}(6a)\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} + \frac{1}{119}\left(12a^2\right)\text{Subst}\left(\int \frac{1}{x^7\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} - \frac{\left(132a^3\right)\text{Subst}\left(\int \frac{1}{x^5\sqrt{bx+ax^3}}\,dx, x,\right.}{1547b}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} + \frac{\left(44a^4\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{88a^5\sqrt{b\sqrt[3]{x}+a}}{1105b^4\sqrt[3]{x}}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{88a^5\sqrt{b\sqrt[3]{x}+a}}{1105b^4\sqrt[3]{x}}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{88a^5\sqrt{b\sqrt[3]{x}+a}}{1105b^4\sqrt[3]{x}}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{88a^5\sqrt{b\sqrt[3]{x}+a}}{1105b^4\sqrt[3]{x}}$$

$$= -\frac{88a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}}$$

**Mathematica [C]** time = 0.0548633, size = 62, normalized size = 0.14

$$-\frac{2b\sqrt{ax+b\sqrt[3]{x}}\,{}_2F_1\left(-\frac{21}{4},-\frac{3}{2};-\frac{17}{4};-\frac{ax^{2/3}}{b}\right)}{7x^{11/3}\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-21/4, -3/2, -17/4, -((a*x^(2
/3))/b)])/(7*Sqrt[1 + (a*x^(2/3))/b]*x^(11/3))
```

---

**Maple [A]** time = 0.026, size = 411, normalized size = 0.9

$$\frac{2}{23205\,b^4x^7}\left(-924\,a^5b\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-2\,\frac{a\sqrt[3]{x}-\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}x^{\frac{20}{3}}\sqrt{\sqrt[3]{x}\left(b+ax^{2/3}\right)}\text{EllipticE}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x^5,x)
```

```
[Out] 2/23205*(-924*a^5*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^
(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*x^(
20/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2),1/2*2^(1/2))+462*a^5*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a
*b)^(1/2))^(1/2)*x^(20/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF(((a*x^(1/
3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+924*x^(22/3)*(b*x^(1/3)+a
*x)^(1/2)*a^6+924*x^(20/3)*(b*x^(1/3)+a*x)^(1/2)*a^5*b-88*x^6*(x^(1/3)*(b+a
*x^(2/3)))^(1/2)*a^4*b^2-308*x^(20/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^5*b-4
665*x^(14/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b^4+40*x^(16/3)*(x^(1/3)*(b+
a*x^(2/3)))^(1/2)*a^3*b^3-7800*x^4*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a*b^5-3315
*x^(10/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*b^6)/b^4/x^7/(b+a*x^(2/3))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**5,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

## 3.148 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx$

**Optimal**. Leaf size=301

$$-\frac{884a^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{100947b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{1768a^6\sqrt{ax+b\sqrt[3]{x}}}{100947b^5x^{2/3}}+\frac{1768a^5\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}}-$$

```
[Out] (-4*a*Sqrt[b*x^(1/3) + a*x])/(69*x^4) - (8*a^2*Sqrt[b*x^(1/3) + a*x])/(1311
*b*x^(10/3)) + (136*a^3*Sqrt[b*x^(1/3) + a*x])/(19665*b^2*x^(8/3)) - (1768*
a^4*Sqrt[b*x^(1/3) + a*x])/(216315*b^3*x^2) + (1768*a^5*Sqrt[b*x^(1/3) + a*
x])/(168245*b^4*x^(4/3)) - (1768*a^6*Sqrt[b*x^(1/3) + a*x])/(100947*b^5*x^(
2/3)) - (2*(b*x^(1/3) + a*x)^(3/2))/(9*x^5) - (884*a^(27/4)*(Sqrt[b] + Sqrt
[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*El
lipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(100947*b^(21/4)*Sqrt[b*
x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.478874, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2020, 2025, 2011, 329, 220}

$$-\frac{1768a^6\sqrt{ax+b\sqrt[3]{x}}}{100947b^5x^{2/3}}+\frac{1768a^5\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}}-\frac{1768a^4\sqrt{ax+b\sqrt[3]{x}}}{216315b^3x^2}+\frac{136a^3\sqrt{ax+b\sqrt[3]{x}}}{19665b^2x^{8/3}}-\frac{884a^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)}{100947b^{21/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-4*a*Sqrt[b*x^(1/3) + a*x])/(69*x^4) - (8*a^2*Sqrt[b*x^(1/3) + a*x])/(1311
*b*x^(10/3)) + (136*a^3*Sqrt[b*x^(1/3) + a*x])/(19665*b^2*x^(8/3)) - (1768*
a^4*Sqrt[b*x^(1/3) + a*x])/(216315*b^3*x^2) + (1768*a^5*Sqrt[b*x^(1/3) + a*
x])/(168245*b^4*x^(4/3)) - (1768*a^6*Sqrt[b*x^(1/3) + a*x])/(100947*b^5*x^(
2/3)) - (2*(b*x^(1/3) + a*x)^(3/2))/(9*x^5) - (884*a^(27/4)*(Sqrt[b] + Sqrt
[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*El
lipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(100947*b^(21/4)*Sqrt[b*
x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

### Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

### Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx = 3\,\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^{16}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} + \frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{13}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} + \frac{1}{69}\left(4a^2\right)\text{Subst}\left(\int \frac{1}{x^{10}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} - \frac{\left(68a^3\right)\text{Subst}\left(\int \frac{1}{x^8\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1311b}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} + \frac{\left(884a^4\right)\text{Subst}\left(\int \frac{}{x^6}\right.}{1966}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} -$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x}+a}}{168245b^4x^{4/3}}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x}+a}}{168245b^4x^{4/3}}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x}+a}}{168245b^4x^{4/3}}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x}+a}}{168245b^4x^{4/3}}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{1768a^5\sqrt{b\sqrt[3]{x}+a}}{168245b^4x^{4/3}}$$

**Mathematica [C]** time = 0.06984, size = 62, normalized size = 0.21

$$-\frac{2b\sqrt{ax+b\sqrt[3]{x}}\,{}_2F_1\left(-\frac{27}{4},-\frac{3}{2};-\frac{23}{4};-\frac{ax^{2/3}}{b}\right)}{9x^{14/3}\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-27/4, -3/2, -23/4, -((a*x^(2
/3))/b)])/(9*Sqrt[1 + (a*x^(2/3))/b]*x^(14/3))
```

---

**Maple [A]** time = 0.026, size = 201, normalized size = 0.7

$$-\frac{2}{1514205\,b^5}\left(6630\,a^6\sqrt{-ab}\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-2\,\frac{a\sqrt[3]{x}-\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^{\frac{26}{3}}-1768\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/3)+a*x)^(3/2)/x^6,x)
```

```
[Out] -2/1514205*(6630*a^6*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1
/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*x^(26/3)-1768*x^(23/3)*a^5*b^2+5304*x^(25/3)*a^6*b+952*x^7*a^4*b^3+216
755*x^(17/3)*a^2*b^5-616*x^(19/3)*a^3*b^4+380380*x^5*a*b^6+13260*x^9*a^7+16
8245*x^(13/3)*b^7)/b^5/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^(26/3)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**6,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

## 3.149 $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

**Optimal**. Leaf size=304

$$-\frac{5525b^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{14421a^{29/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{2210b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{4807a^6}+\frac{15470b^4x^{4/3}\sqrt{a}}{43263a}$$

```
[Out] (11050*b^6*Sqrt[b*x^(1/3) + a*x])/(14421*a^7) - (2210*b^5*x^(2/3)*Sqrt[b*x^
(1/3) + a*x])/(4807*a^6) + (15470*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(43263
*a^5) - (1190*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(3933*a^4) + (350*b^2*x^(8/3)*
Sqrt[b*x^(1/3) + a*x])/(1311*a^3) - (50*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/(
207*a^2) + (2*x^4*Sqrt[b*x^(1/3) + a*x])/(9*a) - (5525*b^(27/4)*(Sqrt[b] +
Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6
)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(14421*a^(29/4)*Sqrt
[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.50605, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 2024, 2011, 329, 220}

$$-\frac{2210b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{4807a^6}+\frac{15470b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{43263a^5}-\frac{1190b^3x^2\sqrt{ax+b\sqrt[3]{x}}}{3933a^4}+\frac{350b^2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{1311a^3}-\frac{5525b^{27/4}\sqrt[6]{x}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (11050*b^6*Sqrt[b*x^(1/3) + a*x])/(14421*a^7) - (2210*b^5*x^(2/3)*Sqrt[b*x^
(1/3) + a*x])/(4807*a^6) + (15470*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(43263
*a^5) - (1190*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(3933*a^4) + (350*b^2*x^(8/3)*
Sqrt[b*x^(1/3) + a*x])/(1311*a^3) - (50*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/(
207*a^2) + (2*x^4*Sqrt[b*x^(1/3) + a*x])/(9*a) - (5525*b^(27/4)*(Sqrt[b] +
Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6
)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(14421*a^(29/4)*Sqrt
[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
```

```
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rubi steps

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\,\text{Subst}\left(\int \frac{x^{14}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} - \frac{(25b)\,\text{Subst}\left(\int \frac{x^{12}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{9a}$$

$$= -\frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} + \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} + \frac{\left(175b^2\right)\text{Subst}\left(\int \frac{x^{10}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{69a^2}$$

$$= \frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3} - \frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} + \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} - \frac{\left(2975b^3\right)\text{Subst}\left(\int \frac{x^8}{\sqrt{bx+ax^3}}\,dx\right.}{1311a^3}$$

$$= -\frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4} + \frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3} - \frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} + \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} + \left(773\right.$$

$$= \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4} + \frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3} - \frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2}$$

$$= -\frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4} + \frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+}}{1311a^3}$$

$$= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7} - \frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4}$$

$$= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7} - \frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4}$$

$$= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7} - \frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4}$$

$$= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7} - \frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4}$$

**Mathematica [C]** time = 0.0922186, size = 161, normalized size = 0.53

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(550a^5b^2x^{10/3} - 770a^4b^3x^{8/3} + 1190a^3b^4x^2 - 2210a^2b^5x^{4/3} - 418a^6bx^4 + 4807a^7x^{14/3} - 16575b^7\sqrt{\frac{ax^{2/3}}{b}+}\right.}{43263a^7\left(ax^{2/3}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(16575*b^7 + 6630*a*b^6*x^(2/3) - 2210*a^2*b^5*x^(
4/3) + 1190*a^3*b^4*x^2 - 770*a^4*b^3*x^(8/3) + 550*a^5*b^2*x^(10/3) - 418*
a^6*b*x^4 + 4807*a^7*x^(14/3) - 16575*b^7*Sqrt[1 + (a*x^(2/3))/b]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(43263*a^7*(b + a*x^(2/3)))
```

---

**Maple [A]** time = 0.032, size = 196, normalized size = 0.6

$$-\frac{1}{43263\,a^8}\left(-1100\,x^{11/3}a^6b^2 + 836\,x^{13/3}a^7b + 1540\,x^3a^5b^3 + 4420\,x^{5/3}a^3b^5 - 2380\,x^{7/3}a^4b^4 - 9614\,x^5a^8 + 16575\,b^7\sqrt{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^(1/3)+a*x)^(1/2),x)
```

```
[Out] -1/43263*(-1100*x^(11/3)*a^6*b^2+836*x^(13/3)*a^7*b+1540*x^3*a^5*b^3+4420*x
^(5/3)*a^3*b^5-2380*x^(7/3)*a^4*b^4-9614*x^5*a^8+16575*b^7*(-a*b)^(1/2)*((a
*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-13260*x*a^2*b^6-33150*x^(1/3)*a*
b^7)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^8
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{ax + bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(a*x + b*x^(1/3)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^2x^5 - abx^{\frac{13}{3}} + b^2x^{\frac{11}{3}}\right)\sqrt{ax + bx^{\frac{1}{3}}}}{a^3x^2 + b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^5 - a*b*x^(13/3) + b^2*x^(11/3))*sqrt(a*x + b*x^(1/3))/(a^3
*x^2 + b^3), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{ax + b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(a*x + b*x**(1/3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{ax + bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(a*x + b*x^(1/3)), x)
```

## 3.150 $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

<u>**Optimal**</u>. Leaf size=414

$$-\frac{209b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{221a^{23/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{418b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221a^{11/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+$$

```
[Out] (-418*b^5*(b + a*x^(2/3))*x^(1/3))/(221*a^(11/2)*(Sqrt[b] + Sqrt[a]*x^(1/3)
)*Sqrt[b*x^(1/3) + a*x]) + (418*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(663*a^5
) - (2090*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^4) + (570*b^2*x^(5/3)*Sqrt[b
*x^(1/3) + a*x])/(1547*a^3) - (38*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(119*a^2
) + (2*x^3*Sqrt[b*x^(1/3) + a*x])/(7*a) + (418*b^(21/4)*(Sqrt[b] + Sqrt[a]*
x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*Ellipt
icE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(221*a^(23/4)*Sqrt[b*x^(1/3)
 + a*x]) - (209*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(
Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/
b^(1/4)], 1/2])/(221*a^(23/4)*Sqrt[b*x^(1/3) + a*x])
```

---

<u>**Rubi**</u> **[A]** time = 0.566179, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2018, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{418b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221a^{11/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{570b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a^3}-\frac{209b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}}{\sqrt[4]{b}}\right.\right.}{221a^{23/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-418*b^5*(b + a*x^(2/3))*x^(1/3))/(221*a^(11/2)*(Sqrt[b] + Sqrt[a]*x^(1/3)
)*Sqrt[b*x^(1/3) + a*x]) + (418*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(663*a^5
) - (2090*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^4) + (570*b^2*x^(5/3)*Sqrt[b
*x^(1/3) + a*x])/(1547*a^3) - (38*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(119*a^2
) + (2*x^3*Sqrt[b*x^(1/3) + a*x])/(7*a) + (418*b^(21/4)*(Sqrt[b] + Sqrt[a]*
x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*Ellipt
icE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(221*a^(23/4)*Sqrt[b*x^(1/3)
 + a*x]) - (209*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(
Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/
b^(1/4)], 1/2])/(221*a^(23/4)*Sqrt[b*x^(1/3) + a*x])
```

<u>Rule 2018</u>

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

<u>Rule 2024</u>

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
```

```
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

### Rubi steps

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\,\text{Subst}\left(\int \frac{x^{11}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} - \frac{(19b)\,\text{Subst}\left(\int \frac{x^9}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{7a}$$

$$= -\frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} + \frac{\left(285b^2\right)\text{Subst}\left(\int \frac{x^7}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{119a^2}$$

$$= \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} - \frac{\left(3135b^3\right)\text{Subst}\Big(\int \frac{x^5}{\sqrt{bx+ax^3}}\,dx, x,}{1547a^3}$$

$$= -\frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} + \left(1045b^4\right)$$

$$= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + 2x^3$$

$$= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + 2x^3$$

$$= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + 2x^3$$

$$= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + 2x^3$$

$$= -\frac{418b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{11/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} + \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}}{1547}$$

**Mathematica [C]** time = 0.0707897, size = 143, normalized size = 0.35

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\Big(114a^3b^2x^{7/3} - 190a^2b^3x^{5/3} - 78a^4bx^3 + 663a^5x^{11/3} - 1463b^5\sqrt[3]{x}\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right) + 418ab^4x +}{4641a^5\left(ax^{2/3}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(1463*b^5*x^(1/3) + 418*a*b^4*x - 190*a^2*b^3*x^(5
/3) + 114*a^3*b^2*x^(7/3) - 78*a^4*b*x^3 + 663*a^5*x^(11/3) - 1463*b^5*Sqrt
[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^(2/3))/
b)]))/(4641*a^5*(b + a*x^(2/3)))
```

---

**Maple [A]** time = 0.024, size = 261, normalized size = 0.6

$$-\frac{1}{4641\,a^6}\Bigg(-228\,x^{8/3}a^4b^2 + 156\,x^{10/3}a^5b + 380\,x^2a^3b^3 + 8778\,b^6\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-2\,\frac{a\sqrt[3]{x}-\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\,\text{EllipticE}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(1/3)+a*x)^(1/2),x)
```

```
[Out] -1/4641/a^6*(-228*x^(8/3)*a^4*b^2+156*x^(10/3)*a^5*b+380*x^2*a^3*b^3+8778*b
^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticE(((a*x^(1/3
)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-4389*b^6*((a*x^(1/3)+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*
b)^(1/2))^(1/2),1/2*2^(1/2))-1326*x^4*a^6-2926*x^(2/3)*a*b^5-836*x^(4/3)*a^
2*b^4)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{ax + bx^{\frac{1}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(a*x + b*x^(1/3)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2x^4 - abx^{\frac{10}{3}} + b^2x^{\frac{8}{3}}\right)\sqrt{ax + bx^{\frac{1}{3}}}}{a^3x^2 + b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^4 - a*b*x^(10/3) + b^2*x^(8/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^2 + b^3), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{ax + b\sqrt[3]{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a*x + b*x**(1/3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{ax + bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(a*x + b*x^(1/3)), x)
```

## 3.151 $\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

**Optimal**. Leaf size=216

$$\frac{39b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{77a^{17/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{234b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^3}-\frac{78b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^4}-$$

```
[Out] (-78*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^4) + (234*b^2*x^(2/3)*Sqrt[b*x^(1/3)
+ a*x])/(385*a^3) - (26*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(55*a^2) + (2*x^2*
Sqrt[b*x^(1/3) + a*x])/(5*a) + (39*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan
[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(77*a^(17/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.318692, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 2024, 2011, 329, 220}

$$\frac{234b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^3}+\frac{39b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77a^{17/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{78b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^4}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-78*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^4) + (234*b^2*x^(2/3)*Sqrt[b*x^(1/3)
+ a*x])/(385*a^3) - (26*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(55*a^2) + (2*x^2*
Sqrt[b*x^(1/3) + a*x])/(5*a) + (39*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan
[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(77*a^(17/4)*Sqrt[b*x^(1/3) + a*x])
```

### Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

### Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
```

```
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx &= 3\,\mathrm{Subst}\left(\int \frac{x^8}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} - \frac{(13b)\,\mathrm{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{5a}\\
&= -\frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \frac{\left(117b^2\right)\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{55a^2}\\
&= \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} - \frac{\left(117b^3\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^3}}\,dx, x,\right.}{77a^3}\\
&= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \frac{\left(39b^4\right)\mathrm{Subst}}{}\\
&= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \frac{\left(39b^4\sqrt{b+a}\right.}{}\\
&= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \frac{\left(78b^4\sqrt{b+a}\right.}{}\\
&= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \frac{39b^{15/4}\left(\sqrt{b}+\right.}{}
\end{aligned}$$

**Mathematica [C]** time = 0.066839, size = 124, normalized size = 0.57

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(26a^2b^2x^{4/3}-14a^3bx^2+77a^4x^{8/3}+195b^4\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)-78ab^3x^{2/3}-195b^4\right)}{385a^4\left(ax^{2/3}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(-195*b^4 - 78*a*b^3*x^(2/3) + 26*a^2*b^2*x^(4/3)
- 14*a^3*b*x^2 + 77*a^4*x^(8/3) + 195*b^4*Sqrt[1 + (a*x^(2/3))/b]*Hypergeom
```

```
etric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(385*a^4*(b + a*x^(2/3)))
```

---

**Maple [A]** time = 0.007, size = 164, normalized size = 0.8

$$\frac{1}{385\,a^5}\left(195\,b^4\sqrt{-ab}\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)+52\,x^{5/3}a^3b^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/3)+a*x)^(1/2),x)
```

```
[Out] 1/385*(195*b^4*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2
^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1
/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))+52*x^(5/3)*a^3*b^2-28*x^(7/3)*a^4*b-156*x*a^2*b^3+154*x^3*a^5-390*x^(1
/3)*a*b^4)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^5
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a*x + b*x^(1/3)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^2x^3-abx^{\frac{7}{3}}+b^2x^{\frac{5}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^3x^2+b^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^3 - a*b*x^(7/3) + b^2*x^(5/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^2 + b^3), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*x + b*x**(1/3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{ax + bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(a*x + b*x^(1/3)), x)
```

# 3.152 $\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

**Optimal**. Leaf size=326

$$\frac{7b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{5a^{11/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{14b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5a^{5/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{14b^{9/4}\sqrt[6]{x}\left(\sqrt{\right.}{}$$

```
[Out] (14*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(5/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[b*x^(1/3) + a*x]) - (14*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(15*a^2) + (2*x*
Sqrt[b*x^(1/3) + a*x])/(3*a) - (14*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt
[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[
(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[b*x^(1/3) + a*x]) + (7*b
^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*
x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5
*a^(11/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.349393, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2018, 2024, 2032, 329, 305, 220, 1196}

$$\frac{14b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5a^{5/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{7b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{14b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (14*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(5/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[b*x^(1/3) + a*x]) - (14*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(15*a^2) + (2*x*
Sqrt[b*x^(1/3) + a*x])/(3*a) - (14*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt
[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[
(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[b*x^(1/3) + a*x]) + (7*b
^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*
x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5
*a^(11/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

### Rubi steps

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\,\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} - \frac{(7b)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{3a}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{5a^2}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{\left(7b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{5a^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{\left(14b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5a^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{\left(14b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5a^{5/2}\sqrt{b\sqrt[3]{x}+ax}} -$$

$$= \frac{14b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{5/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} - \frac{14b^{9/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)}{}$$

**Mathematica [C]** time = 0.0566559, size = 106, normalized size = 0.33

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(5a^2x^{5/3}+7b^2\sqrt[3]{x}\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)-2abx-7b^2\sqrt[3]{x}\right)}{15a^2\left(ax^{2/3}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(-7*b^2*x^(1/3) - 2*a*b*x + 5*a^2*x^(5/3) + 7*b^2*
Sqrt[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^(2/
3))/b)]))/(15*a^2*(b + a*x^(2/3)))
```

---

**Maple [A]** time = 0.008, size = 230, normalized size = 0.7

$$\frac{1}{15\,a^3}\left(42\,b^3\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)-21\,b^3\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/3)+a*x)^(1/2),x)
```

```
[Out] 1/15/a^3*(42*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a
*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*
EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-21*b^3
```

```
*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^
(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-14*x^(2/3)*a*b^2-4*x^(
4/3)*a^2*b+10*x^2*a^3)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(a*x + b*x^(1/3)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2x^2 - abx^{\frac{4}{3}} + b^2x^{\frac{2}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^3x^2+b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^2 + b^3), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*x + b*x**(1/3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(a*x + b*x^(1/3)), x)
```

## 3.153 $\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

**Optimal**. Leaf size=126

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{a^{5/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] (2*Sqrt[b*x^(1/3) + a*x])/a - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b
+ a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(
1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(5/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.11811, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2010, 2013, 2011, 329, 220}

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{5/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x])/a - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b
+ a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(
1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(5/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2010

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[(-2*Sqrt
[a*x^j + b*x^n])/(b*(n - 2)*x^(n - 1)), x] - Dist[(a*(2*n - j - 2))/(b*(n -
 2)), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx &= \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{b\int \frac{1}{x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}\,dx}{3a}\\
&= \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{a}\\
&= \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{\left(b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{a\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{\left(2b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{a\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a}-\frac{b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{5/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0363058, size = 80, normalized size = 0.63

$$
\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(-b\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)+ax^{2/3}+b\right)}{a\left(ax^{2/3}+b\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(b + a*x^(2/3) - b*Sqrt[1 + (a*x^(2/3))/b]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(a*(b + a*x^(2/3)))
```

---

**Maple [A]** time = 0.005, size = 128, normalized size = 1.

$$
-\frac{1}{a^2}\left(b\sqrt{-ab}\sqrt{\left(a\sqrt[3]{x}+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{2}\sqrt{\left(-a\sqrt[3]{x}+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}}\sqrt{-a\sqrt[3]{x}\frac{1}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\left(a\sqrt[3]{x}+\sqrt{-ab}\right)\frac{1}{\sqrt{-ab}}},\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a*x)^(1/2),x)
```

```
[Out] -(b*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a
*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*
EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-2*x^(1
/3)*a*b-2*a^2*x)/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/a^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x + b*x^(1/3)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2x^2-abx^{\frac{4}{3}}+b^2x^{\frac{2}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^3x^3+b^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^3 + b^3*x), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**(1/3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*x + b*x^(1/3)), x)
```

## 3.154 $\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx$

**Optimal**. Leaf size=294

$$\frac{3\sqrt[4]{a}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{6\sqrt[4]{a}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\tan^{-1}\right.}{b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] (6*Sqrt[a]*(b + a*x^(2/3))*x^(1/3))/(b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x
^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(b*x^(1/3)) - (6*a^(1/4)*(Sqrt[b
] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^
(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*
x^(1/3) + a*x]) + (3*a^(1/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3
))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.293531, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2018, 2025, 2032, 329, 305, 220, 1196}

$$\frac{3\sqrt[4]{a}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{6\sqrt[4]{a}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (6*Sqrt[a]*(b + a*x^(2/3))*x^(1/3))/(b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x
^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(b*x^(1/3)) - (6*a^(1/4)*(Sqrt[b
] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^
(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*
x^(1/3) + a*x]) + (3*a^(1/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3
))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

## Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

## Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

## Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

## Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

## Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{(3a)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{b}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{\left(3a\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{b\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{\left(6a\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{b\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{\left(6\sqrt{a}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(6\sqrt{a}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subs}}{\sqrt{b}\sqrt{b\sqrt[3]{x}}}\\
&= \frac{6\sqrt{a}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} - \frac{6\sqrt[4]{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-1}\left(\right.\right.}{b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0494866, size = 54, normalized size = 0.18

$$-\frac{6\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{ax^{2/3}}{b}\right)}{\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((a*x^(2/3))
/b)])/Sqrt[b*x^(1/3) + a*x]
```

---

**Maple [A]** time = 0.018, size = 253, normalized size = 0.9

$$-3\,\frac{1}{\sqrt[3]{x}\left(b+ax^{2/3}\right)b}\left(-2\sqrt{\sqrt[3]{x}\left(b+ax^{2/3}\right)}\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-2\,\frac{a\sqrt[3]{x}-\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\mathrm{EllipticE}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(1/3)+a*x)^(1/2),x)
```

```
[Out] -3*(-2*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)
^(1/2))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*b+(x^(1/3)*(b+a*x^(2/3)))^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a
*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2))*b+2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)*a+2*(b*x^(1/3)+a*x)^(1/2)*b)/x
^(1/3)/(b+a*x^(2/3))/b
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^2x^2-abx^{\frac{4}{3}}+b^2x^{\frac{2}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^3x^4+b^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^4 + b^3*x^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{ax + b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*x + b*x**(1/3))), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x), x)
```

## 3.155 $\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx$

**Optimal**. Leaf size=163

$$\frac{5a^{7/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{7b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{10a\sqrt{ax+b\sqrt[3]{x}}}{7b^2x^{2/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}$$

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(7*b*x^(4/3)) + (10*a*Sqrt[b*x^(1/3) + a*x])/(7*
b^2*x^(2/3)) + (5*a^(7/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/
(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))
/b^(1/4)], 1/2])/(7*b^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.200723, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 2025, 2011, 329, 220}

$$\frac{5a^{7/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{10a\sqrt{ax+b\sqrt[3]{x}}}{7b^2x^{2/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(7*b*x^(4/3)) + (10*a*Sqrt[b*x^(1/3) + a*x])/(7*
b^2*x^(2/3)) + (5*a^(7/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/
(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))
/b^(1/4)], 1/2])/(7*b^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx &= 3\,\mathrm{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}} - \frac{(15a)\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{7b}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}} + \frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}} + \frac{\left(5a^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{7b^2}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}} + \frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}} + \frac{\left(5a^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{7b^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}} + \frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}} + \frac{\left(10a^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{7b^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}} + \frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}} + \frac{5a^{7/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right.}{7b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

**Mathematica [C]** time = 0.0499106, size = 59, normalized size = 0.36

$$-\frac{6\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};-\frac{ax^{2/3}}{b}\right)}{7x\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-7/4, 1/2, -3/4, -((a*x^(2/3)
)/b)])/(7*x*Sqrt[b*x^(1/3) + a*x])
```

---

**Maple [A]** time = 0.022, size = 142, normalized size = 0.9

$$\frac{1}{7\,b^2}\left(5\,a\sqrt{-ab}\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-2\,\frac{a\sqrt[3]{x}-\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)x^{4/3}+4\,abx+10\,x^{5/3}a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(1/3)+a*x)^(1/2),x)
```

```
[Out] 1/7*(5*a*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*
x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*E
llipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^(4/3)
+4*a*b*x+10*x^(5/3)*a^2-6*x^(1/3)*b^2)/b^2/(x^(1/3)*(b+a*x^(2/3)))^(1/2)/x^
(4/3)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{2}x^{2} - abx^{\frac{4}{3}} + b^{2}x^{\frac{2}{3}}\right)\sqrt{ax + bx^{\frac{1}{3}}}}{a^{3}x^{5} + b^{3}x^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^5 + b^3*x^3), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}\sqrt{ax + b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*x + b*x**(1/3))), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^2), x)
```

## 3.156 $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

**Optimal**. Leaf size=388

$$-\frac{77a^{13/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{65b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{154a^{7/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{65b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+154a^{13/4}\sqrt[6]{x}$$

```
[Out] (-154*a^(7/2)*(b + a*x^(2/3))*x^(1/3))/(65*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3))*
Sqrt[b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(13*b*x^(7/3)) + (22*a*S
qrt[b*x^(1/3) + a*x])/(39*b^2*x^(5/3)) - (154*a^2*Sqrt[b*x^(1/3) + a*x])/(1
95*b^3*x) + (154*a^3*Sqrt[b*x^(1/3) + a*x])/(65*b^4*x^(1/3)) + (154*a^(13/4
)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/
3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(65*b^(
15/4)*Sqrt[b*x^(1/3) + a*x]) - (77*a^(13/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan
[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(65*b^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.475911, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2018, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{154a^{7/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{65b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{77a^{13/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+154a^{13/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-154*a^(7/2)*(b + a*x^(2/3))*x^(1/3))/(65*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3))*
Sqrt[b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(13*b*x^(7/3)) + (22*a*S
qrt[b*x^(1/3) + a*x])/(39*b^2*x^(5/3)) - (154*a^2*Sqrt[b*x^(1/3) + a*x])/(1
95*b^3*x) + (154*a^3*Sqrt[b*x^(1/3) + a*x])/(65*b^4*x^(1/3)) + (154*a^(13/4
)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/
3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(65*b^(
15/4)*Sqrt[b*x^(1/3) + a*x]) - (77*a^(13/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan
[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(65*b^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{x^7\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} - \frac{(33a)\,\text{Subst}\left(\int \frac{1}{x^5\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{13b}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} + \frac{\left(77a^2\right)\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{39b^2}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} - \frac{\left(77a^3\right)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{65b^3}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} + \frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}} - \frac{\left(77a^4\right)\text{Subst}\left(\int \frac{}{\sqrt{b}}\right.}{65b^4}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} + \frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}} - \frac{\left(77a^4\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right.}{65b}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} + \frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}} - \frac{\left(154a^4\sqrt{b+ax^{2/3}}\sqrt[6]{}\right.}{65}\\
&= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} + \frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}} - \frac{\left(154a^{7/2}\sqrt{b+ax^{2/3}}\right.}{65}\\
&= -\frac{154a^{7/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{65b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} + \frac{154a^3}{6}
\end{aligned}
$$

**Mathematica [C]** time = 0.049712, size = 59, normalized size = 0.15

$$
-\frac{6\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(-\frac{13}{4},\frac{1}{2};-\frac{9}{4};-\frac{ax^{2/3}}{b}\right)}{13x^2\sqrt{ax+b\sqrt[3]{x}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-13/4, 1/2, -9/4, -((a*x^(2/3
))/b)])/(13*x^2*Sqrt[b*x^(1/3) + a*x])
```

---

**Maple [A]** time = 0.025, size = 363, normalized size = 0.9

$$
-\frac{1}{195\,b^4}\left(462\,a^3b\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-2\,\frac{a\sqrt[3]{x}-\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}x^{10/3}\sqrt{\sqrt[3]{x}\left(b+ax^{2/3}\right)}\text{EllipticE}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)-\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(1/3)+a*x)^(1/2),x)
```

```
[Out] -1/195*(462*a^3*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1
/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*x^(10
/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2),1/2*2^(1/2))-231*a^3*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b
)^(1/2))^(1/2)*x^(10/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF(((a*x^(1/3)
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-462*x^(10/3)*(b*x^(1/3)+a*x
)^(1/2)*a^3*b+44*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b^2+154*x^(10/3)
*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^3*b-20*x^2*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a
*b^3-462*x^4*(b*x^(1/3)+a*x)^(1/2)*a^4+90*x^(4/3)*(x^(1/3)*(b+a*x^(2/3)))^(
1/2)*b^4)/x^(11/3)/(b+a*x^(2/3))/b^4
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^3), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2x^2 - abx^{\frac{4}{3}} + b^2x^{\frac{2}{3}}\right)\sqrt{ax + bx^{\frac{1}{3}}}}{a^3x^6 + b^3x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^6 + b^3*x^4), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{ax + b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a*x + b*x**(1/3))), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^3), x)
```

## 3.157 $\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx$

**Optimal**. Leaf size=251

$$-\frac{663a^{19/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{1463b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{1326a^4\sqrt{ax+b\sqrt[3]{x}}}{1463b^5x^{2/3}}+\frac{3978a^3\sqrt{ax+b\sqrt[3]{x}}}{7315b^4x^{4/3}}$$

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(19*b*x^(10/3)) + (34*a*Sqrt[b*x^(1/3) + a*x])/(
95*b^2*x^(8/3)) - (442*a^2*Sqrt[b*x^(1/3) + a*x])/(1045*b^3*x^2) + (3978*a^
3*Sqrt[b*x^(1/3) + a*x])/(7315*b^4*x^(4/3)) - (1326*a^4*Sqrt[b*x^(1/3) + a*
x])/(1463*b^5*x^(2/3)) - (663*a^(19/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b
+ a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(
1/4)*x^(1/6))/b^(1/4)], 1/2])/(1463*b^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.345415, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 2025, 2011, 329, 220}

$$-\frac{1326a^4\sqrt{ax+b\sqrt[3]{x}}}{1463b^5x^{2/3}}+\frac{3978a^3\sqrt{ax+b\sqrt[3]{x}}}{7315b^4x^{4/3}}-\frac{442a^2\sqrt{ax+b\sqrt[3]{x}}}{1045b^3x^2}-\frac{663a^{19/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\right.}{1463b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(19*b*x^(10/3)) + (34*a*Sqrt[b*x^(1/3) + a*x])/(
95*b^2*x^(8/3)) - (442*a^2*Sqrt[b*x^(1/3) + a*x])/(1045*b^3*x^2) + (3978*a^
3*Sqrt[b*x^(1/3) + a*x])/(7315*b^4*x^(4/3)) - (1326*a^4*Sqrt[b*x^(1/3) + a*
x])/(1463*b^5*x^(2/3)) - (663*a^(19/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b
+ a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(
1/4)*x^(1/6))/b^(1/4)], 1/2])/(1463*b^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
```

```
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\,\mathrm{Subst}\left(\int \frac{1}{x^{10}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} - \frac{(51a)\,\mathrm{Subst}\left(\int \frac{1}{x^8\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{19b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} + \frac{\left(221a^2\right)\mathrm{Subst}\left(\int \frac{1}{x^6\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{95b^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} - \frac{\left(1989a^3\right)\mathrm{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{1045b^3}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}} + \frac{\left(1989a^4\right)\mathrm{Subst}\left(\right.}{1}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}} - \frac{1326a^4\sqrt{b\sqrt[3]{x}+a}}{1463b^5x^{2/3}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}} - \frac{1326a^4\sqrt{b\sqrt[3]{x}+a}}{1463b^5x^{2/3}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}} - \frac{1326a^4\sqrt{b\sqrt[3]{x}+a}}{1463b^5x^{2/3}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}} - \frac{1326a^4\sqrt{b\sqrt[3]{x}+a}}{1463b^5x^{2/3}}$$

**Mathematica [C]** time = 0.056562, size = 59, normalized size = 0.24

$$-\frac{6\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(-\frac{19}{4},\frac{1}{2};-\frac{15}{4};-\frac{ax^{2/3}}{b}\right)}{19x^3\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-19/4, 1/2, -15/4, -((a*x^(2/
3))/b)])/(19*x^3*Sqrt[b*x^(1/3) + a*x])
```

---

**Maple [A]** time = 0.026, size = 179, normalized size = 0.7

$$-\frac{1}{7315\,b^{5}}\left(3315\,a^{4}\sqrt{-ab}\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-2\,\frac{a\sqrt[3]{x}-\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)x^{16/3}+2652\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^(1/3)+a*x)^(1/2),x)
```

```
[Out] -1/7315*(3315*a^4*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2)
)^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)
)*x^(16/3)+2652*x^5*a^4*b+6630*x^(17/3)*a^5+476*x^(11/3)*a^2*b^3-884*x^(13/
3)*a^3*b^2-308*x^3*a*b^4+2310*x^(7/3)*b^5)/b^5/(x^(1/3)*(b+a*x^(2/3)))^(1/2
)/x^(16/3)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}\,x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^4), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{2}x^{2}-abx^{\frac{4}{3}}+b^{2}x^{\frac{2}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^{3}x^{7}+b^{3}x^{5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^7 + b^3*x^5), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^4), x)
```

## 3.158 $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=437

$$-\frac{4807b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{442a^{27/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{4807b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221a^{13/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+65$$

```
[Out] (-4807*b^5*(b + a*x^(2/3))*x^(1/3))/(221*a^(13/2)*(Sqrt[b] + Sqrt[a]*x^(1/3
))*Sqrt[b*x^(1/3) + a*x]) - (3*x^4)/(a*Sqrt[b*x^(1/3) + a*x]) + (4807*b^4*x
^(1/3)*Sqrt[b*x^(1/3) + a*x])/(663*a^6) - (24035*b^3*x*Sqrt[b*x^(1/3) + a*x
])/(4641*a^5) + (6555*b^2*x^(5/3)*Sqrt[b*x^(1/3) + a*x])/(1547*a^4) - (437*
b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(119*a^3) + (23*x^3*Sqrt[b*x^(1/3) + a*x])
/(7*a^2) + (4807*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/
(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))
/b^(1/4)], 1/2])/(221*a^(27/4)*Sqrt[b*x^(1/3) + a*x]) - (4807*b^(21/4)*(Sqr
t[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]
*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(442*a^(27/4)
*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.673587, antiderivative size = 437, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2022, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{4807b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221a^{13/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{6555b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a^4}-\frac{4807b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\right.}{442a^{27/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-4807*b^5*(b + a*x^(2/3))*x^(1/3))/(221*a^(13/2)*(Sqrt[b] + Sqrt[a]*x^(1/3
))*Sqrt[b*x^(1/3) + a*x]) - (3*x^4)/(a*Sqrt[b*x^(1/3) + a*x]) + (4807*b^4*x
^(1/3)*Sqrt[b*x^(1/3) + a*x])/(663*a^6) - (24035*b^3*x*Sqrt[b*x^(1/3) + a*x
])/(4641*a^5) + (6555*b^2*x^(5/3)*Sqrt[b*x^(1/3) + a*x])/(1547*a^4) - (437*
b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(119*a^3) + (23*x^3*Sqrt[b*x^(1/3) + a*x])
/(7*a^2) + (4807*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/
(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))
/b^(1/4)], 1/2])/(221*a^(27/4)*Sqrt[b*x^(1/3) + a*x]) - (4807*b^(21/4)*(Sqr
t[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]
*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(442*a^(27/4)
*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

### Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

### Rubi steps

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\,\mathrm{Subst}\left(\int \frac{x^{14}}{\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{69\,\mathrm{Subst}\left(\int \frac{x^{11}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2a}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2} - \frac{(437b)\,\mathrm{Subst}\left(\int \frac{x^9}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{14a^2}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3} + \frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2} + \frac{\left(6555b^2\right)\mathrm{Subst}\left(\int \frac{x^7}{\sqrt{bx+ax^3}}\,dx, x,\right.}{238a^3}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3} + \frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2} - (72105$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3} + 23$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4} -$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4} -$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4} -$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4} -$$

$$= -\frac{4807b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{13/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6} - \frac{24035b^3x\sqrt{b}}{4641a}$$

**Mathematica [C]** time = 0.0933211, size = 131, normalized size = 0.3

$$\frac{2x^{2/3}\left(1311a^3b^2x^2 - 2185a^2b^3x^{4/3} - 897a^4bx^{8/3} + 663a^5x^{10/3} + 33649b^5\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{ax^{2/3}}{b}\right) + 4807ab^4x^{2/3} -\right.}{4641a^6\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(2/3)*(-33649*b^5 + 4807*a*b^4*x^(2/3) - 2185*a^2*b^3*x^(4/3) + 1311*a
^3*b^2*x^2 - 897*a^4*b*x^(8/3) + 663*a^5*x^(10/3) + 33649*b^5*Sqrt[1 + (a*x
^(2/3))/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^(2/3))/b)]))/(4641*a^6*S
```

```
qrt[b*x^(1/3) + a*x])
```

---

**Maple [A]** time = 0.026, size = 384, normalized size = 0.9

$$-\frac{1}{9282\,a^7}\left(-5244\,x^{8/3}\sqrt{\sqrt[3]{x}\left(b+ax^{2/3}\right)}a^4b^2+3588\,x^{10/3}\sqrt{\sqrt[3]{x}\left(b+ax^{2/3}\right)}a^5b+8740\,x^2\sqrt{\sqrt[3]{x}\left(b+ax^{2/3}\right)}a^3b^3+201894\,b^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^(1/3)+a*x)^(3/2),x)
```

```
[Out] -1/9282/a^7*(-5244*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^4*b^2+3588*x^(10
/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^5*b+8740*x^2*(x^(1/3)*(b+a*x^(2/3)))^(1
/2)*a^3*b^3+201894*b^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a
*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*
(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2),1/2*2^(1/2))-100947*b^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(
1/2))^(1/2)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-2652*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*x^4*
a^6-39452*x^(2/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a*b^5-27846*x^(2/3)*(b*x^(1
/3)+a*x)^(1/2)*a*b^5-19228*x^(4/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b^4)/x
^(1/3)/(b+a*x^(2/3))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4x^6+3\,a^2b^2x^{\frac{14}{3}}-2\,ab^3x^4-\left(2\,a^3bx^5-b^4x^3\right)x^{\frac{1}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^6x^4+2\,a^3b^3x^2+b^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^6 + 3*a^2*b^2*x^(14/3) - 2*a*b^3*x^4 - (2*a^3*b*x^5 - b^4*x
^3)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**4/(a*x + b*x**(1/3))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

## 3.159 $\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=239

$$\frac{663b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{154a^{21/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{1989b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^4}-\frac{663b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^5}-$$

```
[Out] (-3*x^3)/(a*Sqrt[b*x^(1/3) + a*x]) - (663*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^
5) + (1989*b^2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(385*a^4) - (221*b*x^(4/3)*Sq
rt[b*x^(1/3) + a*x])/(55*a^3) + (17*x^2*Sqrt[b*x^(1/3) + a*x])/(5*a^2) + (6
63*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqr
t[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2
])/(154*a^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.377479, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2022, 2024, 2011, 329, 220}

$$\frac{1989b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^4}+\frac{663b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{154a^{21/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{663b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^5}-\frac{221bx^{4/3}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-3*x^3)/(a*Sqrt[b*x^(1/3) + a*x]) - (663*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^
5) + (1989*b^2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(385*a^4) - (221*b*x^(4/3)*Sq
rt[b*x^(1/3) + a*x])/(55*a^3) + (17*x^2*Sqrt[b*x^(1/3) + a*x])/(5*a^2) + (6
63*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqr
t[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2
])/(154*a^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^{11}}{\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{51\,\text{Subst}\left(\int \frac{x^8}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2a}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2} - \frac{(221b)\,\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{10a^2}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3} + \frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2} + \frac{\left(1989b^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right.}{110a^3}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3} + \frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2} - \left(1989b^3\right)\text{S}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3} + \frac{17x^2\sqrt{b}}{5a^2}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3} + \frac{17x^2\sqrt{b}}{5a^2}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3} + \frac{17x^2\sqrt{b}}{5a^2}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5} + \frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4} - \frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3} + \frac{17x^2\sqrt{b}}{5a^2}$$

**Mathematica [C]** time = 0.0826685, size = 124, normalized size = 0.52

$$\frac{\sqrt{ax+b\sqrt[3]{x}}\left(442a^2b^2x^{4/3}-238a^3bx^2+154a^4x^{8/3}+3315b^4\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)-1326ab^3x^{2/3}-3315b^4\right)}{385a^5\left(ax^{2/3}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*x^(1/3) + a*x]*(-3315*b^4 - 1326*a*b^3*x^(2/3) + 442*a^2*b^2*x^(4/3
) - 238*a^3*b*x^2 + 154*a^4*x^(8/3) + 3315*b^4*Sqrt[1 + (a*x^(2/3))/b]*Hype
rgeometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(385*a^5*(b + a*x^(2/3)))
```

---

**Maple [A]** time = 0.024, size = 261, normalized size = 1.1

$$\frac{1}{770\,a^6}\left(3315\,\sqrt{-ab}\sqrt{\sqrt[3]{x}\left(b+ax^{2/3}\right)}\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)b^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(1/3)+a*x)^(3/2),x)
```

```
[Out] 1/770*(3315*(-a*b)^(1/2)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*((a*x^(1/3)+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2),1/2*2^(1/2))*b^4+884*x^(5/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/
2)*a^3*b^2-476*x^(7/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^4*b-2652*x*(x^(1/3)*
(b+a*x^(2/3)))^(1/2)*a^2*b^3+308*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*x^3*a^5-4320
*x^(1/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a*b^4-2310*x^(1/3)*(b*x^(1/3)+a*x)^(
1/2)*a*b^4)/x^(1/3)/(b+a*x^(2/3))/a^6
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4x^5+3\,a^2b^2x^{\frac{11}{3}}-2\,ab^3x^3-\left(2\,a^3bx^4-b^4x^2\right)x^{\frac{1}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^6x^4+2\,a^3b^3x^2+b^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^5 + 3*a^2*b^2*x^(11/3) - 2*a*b^3*x^3 - (2*a^3*b*x^4 - b^4*x
^2)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(a*x + b*x**(1/3))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

## 3.160 $\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=349

$$\frac{77b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{77b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5a^{7/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{77b^{9/4}\sqrt[6]{x}\,(}{}$$

```
[Out] (77*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(7/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[b*x^(1/3) + a*x]) - (3*x^2)/(a*Sqrt[b*x^(1/3) + a*x]) - (77*b*x^(1/3)*Sqr
t[b*x^(1/3) + a*x])/(15*a^3) + (11*x*Sqrt[b*x^(1/3) + a*x])/(3*a^2) - (77*b
^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*
x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5
*a^(15/4)*Sqrt[b*x^(1/3) + a*x]) + (77*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*
Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*Arc
Tan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(10*a^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.426236, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2022, 2024, 2032, 329, 305, 220, 1196}

$$\frac{77b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5a^{7/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{77b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{77b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (77*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(7/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[b*x^(1/3) + a*x]) - (3*x^2)/(a*Sqrt[b*x^(1/3) + a*x]) - (77*b*x^(1/3)*Sqr
t[b*x^(1/3) + a*x])/(15*a^3) + (11*x*Sqrt[b*x^(1/3) + a*x])/(3*a^2) - (77*b
^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*
x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5
*a^(15/4)*Sqrt[b*x^(1/3) + a*x]) + (77*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*
Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*Arc
Tan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(10*a^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
```

```
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^8}{\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{33\,\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2a}$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} - \frac{(77b)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{6a^2}$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} + \frac{\left(77b^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{10a^3}$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} + \frac{\left(77b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+a}}\right.}{10a^3\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} + \frac{\left(77b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+a}}\right.}{5a^3\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} + \frac{\left(77b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+}}\right.}{5a^{7/2}\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{77b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} - \frac{77b}{}$$

**Mathematica [C]** time = 0.0674355, size = 94, normalized size = 0.27

$$\frac{2x^{2/3}\left(5a^2x^{4/3} - 77b^2\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{ax^{2/3}}{b}\right) - 11abx^{2/3} + 77b^2\right)}{15a^3\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(2/3)*(77*b^2 - 11*a*b*x^(2/3) + 5*a^2*x^(4/3) - 77*b^2*Sqrt[1 + (a*x^
(2/3))/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^(2/3))/b)]))/(15*a^3*Sqrt
[b*x^(1/3) + a*x])
```

---

**Maple [A]** time = 0.011, size = 314, normalized size = 0.9

$$-\frac{1}{30\,a^4}\left(-462\,b^3\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\sqrt{\sqrt[3]{x}\left(b+ax^{2/3}\right)}\text{EllipticE}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/3)+a*x)^(3/2),x)
```

```
[Out] -1/30/a^4*(-462*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*(
(-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/
2)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2),1/2*2^(1/2))+231*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-
a*b)^(1/2))^(1/2)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+90*(b*x^(1/3)+a*x)^(1/2)*x^(2/3)*
a*b^2+64*x^(2/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a*b^2+44*x^(4/3)*(x^(1/3)*(b
+a*x^(2/3)))^(1/2)*a^2*b-20*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*x^2*a^3)/x^(1/3)/
(b+a*x^(2/3))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4x^4 + 3\,a^2b^2x^{\frac{8}{3}} - 2\,ab^3x^2 - \left(2\,a^3bx^3 - b^4x\right)x^{\frac{1}{3}}\right)\sqrt{ax + bx^{\frac{1}{3}}}}{a^6x^4 + 2\,a^3b^3x^2 + b^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 + 3*a^2*b^2*x^(8/3) - 2*a*b^3*x^2 - (2*a^3*b*x^3 - b^4*x)
*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(a*x + b*x**(1/3))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

## 3.161 $\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=149

$$-\frac{5b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{5\sqrt{ax+b\sqrt[3]{x}}}{a^2}-\frac{3x}{a\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] (-3*x)/(a*Sqrt[b*x^(1/3) + a*x]) + (5*Sqrt[b*x^(1/3) + a*x])/a^2 - (5*b^(3/
4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1
/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(
9/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.208698, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2018, 2022, 2024, 2011, 329, 220}

$$-\frac{5b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{5\sqrt{ax+b\sqrt[3]{x}}}{a^2}-\frac{3x}{a\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-3*x)/(a*Sqrt[b*x^(1/3) + a*x]) + (5*Sqrt[b*x^(1/3) + a*x])/a^2 - (5*b^(3/
4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1
/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(
9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
```

```
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx &= 3\,\mathrm{Subst}\left(\int \frac{x^5}{\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{15\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2a}\\
&= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2}-\frac{(5b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2a^2}\\
&= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2}-\frac{\left(5b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{2a^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2}-\frac{\left(5b\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{a^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2}-\frac{5b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{9/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0632735, size = 82, normalized size = 0.55

$$
\frac{\sqrt{ax+b\sqrt[3]{x}}\left(-5b\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)+2ax^{2/3}+5b\right)}{a^2\left(ax^{2/3}+b\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*x^(1/3) + a*x]*(5*b + 2*a*x^(2/3) - 5*b*Sqrt[1 + (a*x^(2/3))/b]*Hyp
ergeometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(a^2*(b + a*x^(2/3)))
```

---

**Maple [A]** time = 0.01, size = 185, normalized size = 1.2

$$\frac{1}{2\,a^3}\left(-5\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}\sqrt{\sqrt[3]{x}\left(b+ax^{2/3}\right)}b+6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(1/3)+a*x)^(3/2),x)
```

```
[Out] 1/2*(-5*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF
(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*(x
^(1/3)*(b+a*x^(2/3)))^(1/2)*b+6*(b*x^(1/3)+a*x)^(1/2)*x^(1/3)*a*b+4*x^(1/3)
*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a*b+4*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*x*a^2)/x
^(1/3)/(b+a*x^(2/3))/a^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(a*x + b*x^(1/3))^(3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^4x^3+3\,a^2b^2x^{\frac{5}{3}}-2\,ab^3x-\left(2\,a^3bx^2-b^4\right)x^{\frac{1}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^6x^4+2\,a^3b^3x^2+b^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x/(a*x + b*x**(1/3))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(a*x + b*x^(1/3))^(3/2), x)
```

## 3.162 $\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=296

$$-\frac{3\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{3\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)\right|}{a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] (-3*(b + a*x^(2/3))*x^(1/3))/(Sqrt[a]*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*
x^(1/3) + a*x]) + (3*x^(2/3))/(b*Sqrt[b*x^(1/3) + a*x]) + (3*(Sqrt[b] + Sqr
t[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*E
llipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(3/4)*b^(3/4)*Sqrt[b
*x^(1/3) + a*x]) - (3*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqr
t[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(
1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.25996, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2006, 2018, 2032, 329, 305, 220, 1196}

$$-\frac{3\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{3\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(-3/2),x]
```

```
[Out] (-3*(b + a*x^(2/3))*x^(1/3))/(Sqrt[a]*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*
x^(1/3) + a*x]) + (3*x^(2/3))/(b*Sqrt[b*x^(1/3) + a*x]) + (3*(Sqrt[b] + Sqr
t[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*E
llipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(3/4)*b^(3/4)*Sqrt[b
*x^(1/3) + a*x]) - (3*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqr
t[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(
1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
```

```
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx &= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}}-\frac{\int \frac{1}{\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}\,dx}{2b}\\
&= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}}-\frac{3\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{2b}\\
&= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}}-\frac{\left(3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{2b\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}}-\frac{\left(3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{b\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}}-\frac{\left(3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{\sqrt{a}\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}}+\frac{\left(3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{1-}{\sqrt{b}}\right.}{\sqrt{a}\sqrt{b}\sqrt{b\sqrt[3]{x}+a}}\\
&= -\frac{3\left(b+ax^{2/3}\right)\sqrt[3]{x}}{\sqrt{a}b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}}+\frac{3\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}E\left(2\tan^{-}\right.}{a^{3/4}b^{3/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0291438, size = 62, normalized size = 0.21

$$\frac{2x^{2/3}\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(-3/2),x]
```

```
[Out] (2*Sqrt[1 + (a*x^(2/3))/b]*x^(2/3)*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^
(2/3))/b)])/(b*Sqrt[b*x^(1/3) + a*x])
```

---

**Maple [A]** time = 0.005, size = 245, normalized size = 0.8

$$-\frac{3}{2\,ab}\left(2\sqrt{2}\sqrt{\frac{-a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\text{EllipticE}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\sqrt[3]{x}\left(b+ax^{2/3}\right)}b-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1/3)+a*x)^(3/2),x)
```

```
[Out] -3/2/a*(2*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*
a/(-a*b)^(1/2))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2),1/2*2^(1/2))*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(x^(1/3)*(b+a
*x^(2/3)))^(1/2)*b-2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(
-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2),1/2*2^(1/2))*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(x^(
1/3)*(b+a*x^(2/3)))^(1/2)*b-2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)*a)/x^(1/3)/(b+a
*x^(2/3))/b
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(-3/2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4x^3+3\,a^2b^2x^{\frac{5}{3}}-2\,ab^3x-\left(2\,a^3bx^2-b^4\right)x^{\frac{1}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^6x^5+2\,a^3b^3x^3+b^6x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^5 + 2*a^3*b^3*x^3 + b^6*x), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**(1/3))**(-3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(-3/2), x)
```

## 3.163 $\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=158

$$-\frac{5a^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{2b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{5\sqrt{ax+b\sqrt[3]{x}}}{b^2x^{2/3}}+\frac{3}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] 3/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]) - (5*Sqrt[b*x^(1/3) + a*x])/(b^2*x^(2/3
)) - (5*a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] +
 Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)],
 1/2])/(2*b^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.205232, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2023, 2025, 2011, 329, 220}

$$-\frac{5a^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{5\sqrt{ax+b\sqrt[3]{x}}}{b^2x^{2/3}}+\frac{3}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]) - (5*Sqrt[b*x^(1/3) + a*x])/(b^2*x^(2/3
)) - (5*a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] +
 Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)],
 1/2])/(2*b^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
```

```
 + j*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{x\left(bx+ax^3\right)^{3/2}}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}+\frac{15\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{2b}\\
&= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}-\frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}}-\frac{(5a)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{2b^2}\\
&= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}-\frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}}-\frac{\left(5a\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{2b^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}-\frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}}-\frac{\left(5a\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{b^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}-\frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}}-\frac{5a^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right.}{2b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0597793, size = 62, normalized size = 0.39

$$
-\frac{2\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{ax^{2/3}}{b}\right)}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((a*x^(2/3))
/b)])/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])
```

---

**Maple [A]** time = 0.012, size = 181, normalized size = 1.2

$$-\frac{1}{2\,b^2x}\left(5\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\text{EllipticF}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\,\sqrt{2}\right)\sqrt{-ab}x^{2/3}\sqrt{\sqrt[3]{x}\left(b+ax^{2/3}\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(1/3)+a*x)^(3/2),x)
```

```
[Out] -1/2*(5*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*EllipticF
(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x^
(2/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)+6*(b*x^(1/3)+a*x)^(1/2)*x*a+4*x^(1/3)*(
x^(1/3)*(b+a*x^(2/3)))^(1/2)*b+4*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*x*a)/b^2/x/(
b+a*x^(2/3))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4x^3 + 3\,a^2b^2x^{\frac{5}{3}} - 2\,ab^3x - \left(2\,a^3bx^2 - b^4\right)x^{\frac{1}{3}}\right)\sqrt{ax + bx^{\frac{1}{3}}}}{a^6x^6 + 2\,a^3b^3x^4 + b^6x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^6 + 2*a^3*b^3*x^4 + b^6*x^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(a*x + b*x**(1/3))**(3/2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x), x)
```

## 3.164 $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=383

$$\frac{77a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{10b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{77a^{5/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{77a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{}\right.}{}$$

```
[Out] 3/(b*x^(4/3)*Sqrt[b*x^(1/3) + a*x]) + (77*a^(5/2)*(b + a*x^(2/3))*x^(1/3))/
(5*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (11*Sqrt[b*x^(1
/3) + a*x])/(3*b^2*x^(5/3)) + (77*a*Sqrt[b*x^(1/3) + a*x])/(15*b^3*x) - (77
*a^2*Sqrt[b*x^(1/3) + a*x])/(5*b^4*x^(1/3)) - (77*a^(9/4)*(Sqrt[b] + Sqrt[a
]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*Elli
pticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(15/4)*Sqrt[b*x^(1/3)
 + a*x]) + (77*a^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sq
rt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^
(1/4)], 1/2])/(10*b^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.478506, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2023, 2025, 2032, 329, 305, 220, 1196}

$$\frac{77a^{5/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{77a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{77a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)}{5b^1}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(4/3)*Sqrt[b*x^(1/3) + a*x]) + (77*a^(5/2)*(b + a*x^(2/3))*x^(1/3))/
(5*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (11*Sqrt[b*x^(1
/3) + a*x])/(3*b^2*x^(5/3)) + (77*a*Sqrt[b*x^(1/3) + a*x])/(15*b^3*x) - (77
*a^2*Sqrt[b*x^(1/3) + a*x])/(5*b^4*x^(1/3)) - (77*a^(9/4)*(Sqrt[b] + Sqrt[a
]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*Elli
pticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(15/4)*Sqrt[b*x^(1/3)
 + a*x]) + (77*a^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sq
rt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^
(1/4)], 1/2])/(10*b^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

### Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

### Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
```

```
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

### Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

### Rubi steps

$$\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{1}{x^4\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{33\,\text{Subst}\left(\int \frac{1}{x^5\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} - \frac{(77a)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{6b^2}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} + \frac{\left(77a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{10b^3}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}} + \frac{\left(77a^3\right)\text{Subst}\left(\int\right.}{10b}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}} + \frac{\left(77a^3\sqrt{b+ax^{2/3}}\right.}{10b}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}} + \frac{\left(77a^3\sqrt{b+ax^{2/3}}\right.}{5b}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}} + \frac{\left(77a^{5/2}\sqrt{b+ax^{2/3}}\right.}{5b}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{77a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2}{5}$$

**Mathematica [C]** time = 0.0557277, size = 64, normalized size = 0.17

$$-\frac{2\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(-\frac{9}{4},\frac{3}{2};-\frac{5}{4};-\frac{ax^{2/3}}{b}\right)}{3bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-9/4, 3/2, -5/4, -((a*x^(2/3)
)/b)])/(3*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])
```

---

**Maple [A]** time = 0.011, size = 341, normalized size = 0.9

$$-\frac{1}{30\,x^3b^4}\left(-462\,a^2b\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}x^{8/3}\sqrt{\sqrt[3]{x}\left(b+ax^{2/3}\right)}\text{EllipticE}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(1/3)+a*x)^(3/2),x)
```

```
[Out] -1/30*(-462*a^2*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-
a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)
*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+231*a^2*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1
/3)*a/(-a*b)^(1/2))^(1/2)*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticF((
(a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+462*x^(10/3)*(b*x
^(1/3)+a*x)^(1/2)*a^3+372*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)*a^2*b-44*x^2*(x^(1/
3)*(b+a*x^(2/3)))^(1/2)*a*b^2-64*x^(8/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*
b+20*x^(4/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*b^3)/x^3/(b+a*x^(2/3))/b^4
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^4x^3 + 3\,a^2b^2x^{\frac{5}{3}} - 2\,ab^3x - \left(2\,a^3bx^2 - b^4\right)x^{\frac{1}{3}}\right)\sqrt{ax + bx^{\frac{1}{3}}}}{a^6x^7 + 2\,a^3b^3x^5 + b^6x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^7 + 2*a^3*b^3*x^5 + b^6*x^3), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a*x + b*x**(1/3))**(3/2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^2), x)
```

## 3.165 $\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=246

$$\frac{663a^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{154b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{663a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^5x^{2/3}}-\frac{1989a^2\sqrt{ax+b\sqrt[3]{x}}}{385b^4x^{4/3}}+$$

```
[Out] 3/(b*x^(7/3)*Sqrt[b*x^(1/3) + a*x]) - (17*Sqrt[b*x^(1/3) + a*x])/(5*b^2*x^(
8/3)) + (221*a*Sqrt[b*x^(1/3) + a*x])/(55*b^3*x^2) - (1989*a^2*Sqrt[b*x^(1/
3) + a*x])/(385*b^4*x^(4/3)) + (663*a^3*Sqrt[b*x^(1/3) + a*x])/(77*b^5*x^(2
/3)) + (663*a^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt
[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1
/4)], 1/2])/(154*b^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

---

**Rubi [A]** time = 0.364747, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2023, 2025, 2011, 329, 220}

$$\frac{663a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^5x^{2/3}}-\frac{1989a^2\sqrt{ax+b\sqrt[3]{x}}}{385b^4x^{4/3}}+\frac{663a^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{154b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(7/3)*Sqrt[b*x^(1/3) + a*x]) - (17*Sqrt[b*x^(1/3) + a*x])/(5*b^2*x^(
8/3)) + (221*a*Sqrt[b*x^(1/3) + a*x])/(55*b^3*x^2) - (1989*a^2*Sqrt[b*x^(1/
3) + a*x])/(385*b^4*x^(4/3)) + (663*a^3*Sqrt[b*x^(1/3) + a*x])/(77*b^5*x^(2
/3)) + (663*a^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt
[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1
/4)], 1/2])/(154*b^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

### Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rubi steps

$$\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\,\mathrm{Subst}\left(\int \frac{1}{x^7\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{51\,\mathrm{Subst}\left(\int \frac{1}{x^8\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} - \frac{(221a)\,\mathrm{Subst}\left(\int \frac{1}{x^6\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{10b^2}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} + \frac{\left(1989a^2\right)\mathrm{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x,\right.}{110b^3}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} - \frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}} - \frac{\left(1989a^3\right)\mathrm{Su}}{}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} - \frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}} + \frac{663a^3\sqrt{b\sqrt[3]{x}}}{77b^5x^{2/}}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} - \frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}} + \frac{663a^3\sqrt{b\sqrt[3]{x}}}{77b^5x^{2/}}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} - \frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}} + \frac{663a^3\sqrt{b\sqrt[3]{x}}}{77b^5x^{2/}}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} - \frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}} + \frac{663a^3\sqrt{b\sqrt[3]{x}}}{77b^5x^{2/}}$$

**Mathematica [C]** time = 0.0561036, size = 64, normalized size = 0.26

$$-\frac{2\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(-\frac{15}{4},\frac{3}{2};-\frac{11}{4};-\frac{ax^{2/3}}{b}\right)}{5bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-15/4, 3/2, -11/4, -((a*x^(2/
3))/b)])/(5*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])
```

---

**Maple [A]** time = 0.014, size = 262, normalized size = 1.1

$$\frac{1}{770\,b^5x^5}\left(3315\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\mathrm{EllipticF}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}},1/2\sqrt{2}\right)\sqrt{-ab}x^{14/3}\sqrt{\sqrt[3]{x}\left(b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(1/3)+a*x)^(3/2),x)
```

```
[Out] 1/770*(3315*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1
/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*Ellip
ticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2
)*x^(14/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^3-884*x^(11/3)*(x^(1/3)*(b+a*x^(
2/3)))^(1/2)*a^2*b^2+2652*x^(13/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^3*b+476*
x^3*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a*b^3+2310*(b*x^(1/3)+a*x)^(1/2)*x^5*a^4+
4320*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*x^5*a^4-308*x^(7/3)*(x^(1/3)*(b+a*x^(2/3
)))^(1/2)*b^4)/b^5/x^5/(b+a*x^(2/3))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^3), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^{4}x^{3}+3\,a^{2}b^{2}x^{\frac{5}{3}}-2\,ab^{3}x-\left(2\,a^{3}bx^{2}-b^{4}\right)x^{\frac{1}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^{6}x^{8}+2\,a^{3}b^{3}x^{6}+b^{6}x^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^8 + 2*a^3*b^3*x^6 + b^6*x^4), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a*x + b*x**(1/3))**(3/2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^3), x)
```

## 3.166 $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=471

$$-\frac{4807a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\operatorname{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right),\frac{1}{2}\right)}{442b^{27/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{4807a^{11/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221b^7\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{24035a^3}{464}$$

```
[Out] 3/(b*x^(10/3)*Sqrt[b*x^(1/3) + a*x]) - (4807*a^(11/2)*(b + a*x^(2/3))*x^(1/
3))/(221*b^7*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (23*Sqrt[
b*x^(1/3) + a*x])/(7*b^2*x^(11/3)) + (437*a*Sqrt[b*x^(1/3) + a*x])/(119*b^3
*x^3) - (6555*a^2*Sqrt[b*x^(1/3) + a*x])/(1547*b^4*x^(7/3)) + (24035*a^3*Sq
rt[b*x^(1/3) + a*x])/(4641*b^5*x^(5/3)) - (4807*a^4*Sqrt[b*x^(1/3) + a*x])/
(663*b^6*x) + (4807*a^5*Sqrt[b*x^(1/3) + a*x])/(221*b^7*x^(1/3)) + (4807*a^
(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*
x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2
21*b^(27/4)*Sqrt[b*x^(1/3) + a*x]) - (4807*a^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1
/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[
2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(442*b^(27/4)*Sqrt[b*x^(1/3) + a
*x])
```

---

**Rubi [A]** time = 0.693825, antiderivative size = 471, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2023, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{4807a^{11/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221b^7\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{24035a^3\sqrt{ax+b\sqrt[3]{x}}}{4641b^5x^{5/3}}-\frac{6555a^2\sqrt{ax+b\sqrt[3]{x}}}{1547b^4x^{7/3}}-\frac{4807a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+}{\left(\sqrt{a}\sqrt[3]{x}+\right.}}}{442b^{27/4}\sqrt{ax+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(10/3)*Sqrt[b*x^(1/3) + a*x]) - (4807*a^(11/2)*(b + a*x^(2/3))*x^(1/
3))/(221*b^7*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (23*Sqrt[
b*x^(1/3) + a*x])/(7*b^2*x^(11/3)) + (437*a*Sqrt[b*x^(1/3) + a*x])/(119*b^3
*x^3) - (6555*a^2*Sqrt[b*x^(1/3) + a*x])/(1547*b^4*x^(7/3)) + (24035*a^3*Sq
rt[b*x^(1/3) + a*x])/(4641*b^5*x^(5/3)) - (4807*a^4*Sqrt[b*x^(1/3) + a*x])/
(663*b^6*x) + (4807*a^5*Sqrt[b*x^(1/3) + a*x])/(221*b^7*x^(1/3)) + (4807*a^
(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*
x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2
21*b^(27/4)*Sqrt[b*x^(1/3) + a*x]) - (4807*a^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1
/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[
2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(442*b^(27/4)*Sqrt[b*x^(1/3) + a
*x])
```

### Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

### Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

### Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

### Rubi steps

$$\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{1}{x^{10}\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{69\,\text{Subst}\left(\int \frac{1}{x^{11}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} - \frac{(437a)\,\text{Subst}\left(\int \frac{1}{x^9\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{14b^2}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} + \frac{\left(6555a^2\right)\text{Subst}\left(\int \frac{1}{x^7\sqrt{bx+ax^3}}\,dx, x,\right.}{238b^3}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} - \left(72105a^3\right)\text{Sub}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} + \frac{24035a^3\sqrt{b}}{4641b^5x^{5/}}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} + \frac{24035a^3\sqrt{b}}{4641b^5x^{5/}}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} + \frac{24035a^3\sqrt{b}}{4641b^5x^{5/}}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} + \frac{24035a^3\sqrt{b}}{4641b^5x^{5/}}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} + \frac{24035a^3\sqrt{b}}{4641b^5x^{5/}}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} + \frac{24035a^3\sqrt{b}}{4641b^5x^{5/}}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{4807a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221b^7\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - 6$$

**Mathematica [C]** time = 0.0547918, size = 64, normalized size = 0.14

$$-\frac{2\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(-\frac{21}{4},\frac{3}{2};-\frac{17}{4};-\frac{ax^{2/3}}{b}\right)}{7bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-21/4, 3/2, -17/4, -((a*x^(2/
3))/b)])/(7*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])
```

---

**Maple [A]** time = 0.013, size = 413, normalized size = 0.9

$$\frac{1}{9282\,x^{7}b^{7}}\left(-201894\,a^{5}b\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{a\sqrt[3]{x}}{\sqrt{-ab}}}\,x^{\frac{20}{3}}\sqrt{\sqrt[3]{x}\left(b+ax^{2/3}\right)}\,\mathrm{EllipticE}\left(\sqrt{\frac{a\sqrt[3]{x}+\sqrt{-ab}}{\sqrt{-ab}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^(1/3)+a*x)^(3/2),x)
```

```
[Out] 1/9282*(-201894*a^5*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)
*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x^(1/3)*a/(-a*b)^(1/2))^(
1/2)*x^(20/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+100947*a^5*b*((a*x^(1/3)+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*(-x^(1/3)*a/(-a*b)^(1/2))^(1/2)*x^(20/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*El
lipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+201894*x
^(22/3)*(b*x^(1/3)+a*x)^(1/2)*a^6-19228*x^6*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a
^4*b^2-39452*x^(20/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^5*b+174048*x^(20/3)*(
b*x^(1/3)+a*x)^(1/2)*a^5*b+3588*x^4*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a*b^5-524
4*x^(14/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*a^2*b^4+8740*x^(16/3)*(x^(1/3)*(b+
a*x^(2/3)))^(1/2)*a^3*b^3-2652*x^(10/3)*(x^(1/3)*(b+a*x^(2/3)))^(1/2)*b^6)/
x^7/(b+a*x^(2/3))/b^7
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^4), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^{4}x^{3}+3\,a^{2}b^{2}x^{\frac{5}{3}}-2\,ab^{3}x-\left(2\,a^{3}bx^{2}-b^{4}\right)x^{\frac{1}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^{6}x^{9}+2\,a^{3}b^{3}x^{7}+b^{6}x^{5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^9 + 2*a^3*b^3*x^7 + b^6*x^5), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^4), x)
```

## 3.167 $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

**Optimal**. Leaf size=371

$$\frac{8388608b^{12}\left(ax+bx^{2/3}\right)^{3/2}}{152108775a^{13}x}-\frac{4194304b^{11}\left(ax+bx^{2/3}\right)^{3/2}}{50702925a^{12}x^{2/3}}+\frac{1048576b^{10}\left(ax+bx^{2/3}\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}}-\frac{524288b^{9}\left(ax+bx^{2/3}\right)^{3/2}}{4345965a^{10}}+$$

```
[Out] (-524288*b^9*(b*x^(2/3) + a*x)^(3/2))/(4345965*a^10) + (8388608*b^12*(b*x^(
2/3) + a*x)^(3/2))/(152108775*a^13*x) - (4194304*b^11*(b*x^(2/3) + a*x)^(3/
2))/(50702925*a^12*x^(2/3)) + (1048576*b^10*(b*x^(2/3) + a*x)^(3/2))/(10140
585*a^11*x^(1/3)) + (65536*b^8*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(482885*a^9
) - (360448*b^7*x^(2/3)*(b*x^(2/3) + a*x)^(3/2))/(2414425*a^8) + (90112*b^6
*x*(b*x^(2/3) + a*x)^(3/2))/(557175*a^7) - (45056*b^5*x^(4/3)*(b*x^(2/3) +
a*x)^(3/2))/(260015*a^6) + (2816*b^4*x^(5/3)*(b*x^(2/3) + a*x)^(3/2))/(1529
5*a^5) - (1408*b^3*x^2*(b*x^(2/3) + a*x)^(3/2))/(7245*a^4) + (352*b^2*x^(7/
3)*(b*x^(2/3) + a*x)^(3/2))/(1725*a^3) - (16*b*x^(8/3)*(b*x^(2/3) + a*x)^(3
/2))/(75*a^2) + (2*x^3*(b*x^(2/3) + a*x)^(3/2))/(9*a)
```

---

**Rubi [A]** time = 0.627282, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$\frac{8388608b^{12}\left(ax+bx^{2/3}\right)^{3/2}}{152108775a^{13}x}-\frac{4194304b^{11}\left(ax+bx^{2/3}\right)^{3/2}}{50702925a^{12}x^{2/3}}+\frac{1048576b^{10}\left(ax+bx^{2/3}\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}}-\frac{524288b^{9}\left(ax+bx^{2/3}\right)^{3/2}}{4345965a^{10}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-524288*b^9*(b*x^(2/3) + a*x)^(3/2))/(4345965*a^10) + (8388608*b^12*(b*x^(
2/3) + a*x)^(3/2))/(152108775*a^13*x) - (4194304*b^11*(b*x^(2/3) + a*x)^(3/
2))/(50702925*a^12*x^(2/3)) + (1048576*b^10*(b*x^(2/3) + a*x)^(3/2))/(10140
585*a^11*x^(1/3)) + (65536*b^8*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(482885*a^9
) - (360448*b^7*x^(2/3)*(b*x^(2/3) + a*x)^(3/2))/(2414425*a^8) + (90112*b^6
*x*(b*x^(2/3) + a*x)^(3/2))/(557175*a^7) - (45056*b^5*x^(4/3)*(b*x^(2/3) +
a*x)^(3/2))/(260015*a^6) + (2816*b^4*x^(5/3)*(b*x^(2/3) + a*x)^(3/2))/(1529
5*a^5) - (1408*b^3*x^2*(b*x^(2/3) + a*x)^(3/2))/(7245*a^4) + (352*b^2*x^(7/
3)*(b*x^(2/3) + a*x)^(3/2))/(1725*a^3) - (16*b*x^(8/3)*(b*x^(2/3) + a*x)^(3
/2))/(75*a^2) + (2*x^3*(b*x^(2/3) + a*x)^(3/2))/(9*a)
```

### Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

### Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
```

```
j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{bx^{2/3}+ax}\,dx &= \frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a} - \frac{(8b)\int x^{8/3}\sqrt{bx^{2/3}+ax}\,dx}{9a} \\
&= -\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2} + \frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a} + \frac{\left(176b^2\right)\int x^{7/3}\sqrt{bx^{2/3}+ax}\,dx}{225a^2} \\
&= \frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3} - \frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2} + \frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a} - \frac{\left(704b^3\right)\int x^2\sqrt{bx^{2/3}+a}}{1035a^3} \\
&= -\frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4} + \frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3} - \frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2} + \frac{2x^3\left(bx^{2/3}+ax\right.}{9a} \\
&= \frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5} - \frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4} + \frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3} - \frac{16bx^{8/3}\left(bx^{2/3}\right.}{75a^2} \\
&= -\frac{45056b^5x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{260015a^6} + \frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5} - \frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4} + \frac{352b^2x^{7/3}}{1} \\
&= \frac{90112b^6x\left(bx^{2/3}+ax\right)^{3/2}}{557175a^7} - \frac{45056b^5x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{260015a^6} + \frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5} - \frac{1408b^3x^2\left(\right.}{72} \\
&= -\frac{360448b^7x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{2414425a^8} + \frac{90112b^6x\left(bx^{2/3}+ax\right)^{3/2}}{557175a^7} - \frac{45056b^5x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{260015a^6} + \frac{2816b^4}{} \\
&= \frac{65536b^8\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{482885a^9} - \frac{360448b^7x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{2414425a^8} + \frac{90112b^6x\left(bx^{2/3}+ax\right)^{3/2}}{557175a^7} - \frac{45056b^5}{} \\
&= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}} + \frac{65536b^8\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{482885a^9} - \frac{360448b^7x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{2414425a^8} + \frac{90112b}{} \\
&= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}} + \frac{1048576b^{10}\left(bx^{2/3}+ax\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}} + \frac{65536b^8\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{482885a^9} - \frac{360448b}{} \\
&= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}} - \frac{4194304b^{11}\left(bx^{2/3}+ax\right)^{3/2}}{50702925a^{12}x^{2/3}} + \frac{1048576b^{10}\left(bx^{2/3}+ax\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}} + \frac{65536b^8}{} \\
&= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}} + \frac{8388608b^{12}\left(bx^{2/3}+ax\right)^{3/2}}{152108775a^{13}x} - \frac{4194304b^{11}\left(bx^{2/3}+ax\right)^{3/2}}{50702925a^{12}x^{2/3}} + \frac{1048576}{101}
\end{aligned}$$

**Mathematica [A]** time = 0.135226, size = 181, normalized size = 0.49

$$2\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\left(15519504a^{10}b^2x^{10/3}-14780480a^9b^3x^3+14002560a^8b^4x^{8/3}-13178880a^7b^5x^{7/3}+12300288a^6\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*(4194304*b^12 - 6291456*a*b^11*x^(
1/3) + 7864320*a^2*b^10*x^(2/3) - 9175040*a^3*b^9*x + 10321920*a^4*b^8*x^(4
/3) - 11354112*a^5*b^7*x^(5/3) + 12300288*a^6*b^6*x^2 - 13178880*a^7*b^5*x^
(7/3) + 14002560*a^8*b^4*x^(8/3) - 14780480*a^9*b^3*x^3 + 15519504*a^10*b^2
*x^(10/3) - 16224936*a^11*b*x^(11/3) + 16900975*a^12*x^4))/(152108775*a^13*
x^(1/3))
```

---

**Maple [A]** time = 0.01, size = 156, normalized size = 0.4

$$-\frac{2}{152108775\,a^{13}}\sqrt{bx^{\frac{2}{3}}+ax}\left(b+a\sqrt[3]{x}\right)\left(16224936\,x^{11/3}a^{11}b-15519504\,x^{10/3}a^{10}b^{2}-14002560\,x^{8/3}a^{8}b^{4}+13178880\,x^{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^(2/3)+a*x)^(1/2),x)
```

```
[Out] -2/152108775*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(16224936*x^(11/3)*a^11*b-
15519504*x^(10/3)*a^10*b^2-14002560*x^(8/3)*a^8*b^4+13178880*x^(7/3)*a^7*b^
5+11354112*x^(5/3)*a^5*b^7-10321920*x^(4/3)*a^4*b^8-16900975*x^4*a^12+14780
480*x^3*a^9*b^3-7864320*x^(2/3)*a^2*b^10-12300288*x^2*a^6*b^6+6291456*x^(1/
3)*a*b^11+9175040*x*a^3*b^9-4194304*b^12)/x^(1/3)/a^13
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax+bx^{\frac{2}{3}}}x^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))*x^3, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{3}\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*x + b*x**(2/3)), x)
```

---

**Giac [A]** time = 1.13299, size = 259, normalized size = 0.7

$$-\frac{8388608\,b^{\frac{27}{2}}}{152108775\,a^{13}}+\frac{2\left(16900975\left(ax^{\frac{1}{3}}+b\right)^{\frac{27}{2}}-219036636\left(ax^{\frac{1}{3}}+b\right)^{\frac{25}{2}}b+1309458150\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}b^{2}-4780561500\left(ax^{\frac{1}{3}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -8388608/152108775*b^(27/2)/a^13 + 2/152108775*(16900975*(a*x^(1/3) + b)^(2
7/2) - 219036636*(a*x^(1/3) + b)^(25/2)*b + 1309458150*(a*x^(1/3) + b)^(23/
2)*b^2 - 4780561500*(a*x^(1/3) + b)^(21/2)*b^3 + 11888501625*(a*x^(1/3) + b
)^(19/2)*b^4 - 21259438200*(a*x^(1/3) + b)^(17/2)*b^5 + 28109701620*(a*x^(1
/3) + b)^(15/2)*b^6 - 27800803800*(a*x^(1/3) + b)^(13/2)*b^7 + 20534684625*
(a*x^(1/3) + b)^(11/2)*b^8 - 11154643500*(a*x^(1/3) + b)^(9/2)*b^9 + 430250
5350*(a*x^(1/3) + b)^(7/2)*b^10 - 1095183180*(a*x^(1/3) + b)^(5/2)*b^11 + 1
52108775*(a*x^(1/3) + b)^(3/2)*b^12)/a^13
```

## 3.168 $\int x^2\sqrt{bx^{2/3}+ax}\,dx$

**Optimal**. Leaf size=283

$$-\frac{131072b^9\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^{10}x}+\frac{196608b^8\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^9x^{2/3}}-\frac{49152b^7\left(ax+bx^{2/3}\right)^{3/2}}{323323a^8\sqrt[3]{x}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{3/2}}{46189a^7}-\frac{9216b^5\sqrt[3]{}}{4}$$

```
[Out] (8192*b^6*(b*x^(2/3) + a*x)^(3/2))/(46189*a^7) - (131072*b^9*(b*x^(2/3) + a
*x)^(3/2))/(1616615*a^10*x) + (196608*b^8*(b*x^(2/3) + a*x)^(3/2))/(1616615
*a^9*x^(2/3)) - (49152*b^7*(b*x^(2/3) + a*x)^(3/2))/(323323*a^8*x^(1/3)) -
(9216*b^5*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(46189*a^6) + (4608*b^4*x^(2/3)*
(b*x^(2/3) + a*x)^(3/2))/(20995*a^5) - (384*b^3*x*(b*x^(2/3) + a*x)^(3/2))/
(1615*a^4) + (576*b^2*x^(4/3)*(b*x^(2/3) + a*x)^(3/2))/(2261*a^3) - (36*b*x
^(5/3)*(b*x^(2/3) + a*x)^(3/2))/(133*a^2) + (2*x^2*(b*x^(2/3) + a*x)^(3/2))
/(7*a)
```

---

**Rubi [A]** time = 0.441085, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{131072b^9\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^{10}x}+\frac{196608b^8\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^9x^{2/3}}-\frac{49152b^7\left(ax+bx^{2/3}\right)^{3/2}}{323323a^8\sqrt[3]{x}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{3/2}}{46189a^7}-\frac{9216b^5\sqrt[3]{}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (8192*b^6*(b*x^(2/3) + a*x)^(3/2))/(46189*a^7) - (131072*b^9*(b*x^(2/3) + a
*x)^(3/2))/(1616615*a^10*x) + (196608*b^8*(b*x^(2/3) + a*x)^(3/2))/(1616615
*a^9*x^(2/3)) - (49152*b^7*(b*x^(2/3) + a*x)^(3/2))/(323323*a^8*x^(1/3)) -
(9216*b^5*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(46189*a^6) + (4608*b^4*x^(2/3)*
(b*x^(2/3) + a*x)^(3/2))/(20995*a^5) - (384*b^3*x*(b*x^(2/3) + a*x)^(3/2))/
(1615*a^4) + (576*b^2*x^(4/3)*(b*x^(2/3) + a*x)^(3/2))/(2261*a^3) - (36*b*x
^(5/3)*(b*x^(2/3) + a*x)^(3/2))/(133*a^2) + (2*x^2*(b*x^(2/3) + a*x)^(3/2))
/(7*a)
```

### Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

### Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

### Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2\sqrt{bx^{2/3}+ax}\,dx &= \frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a}-\frac{(6b)\int x^{5/3}\sqrt{bx^{2/3}+ax}\,dx}{7a}\\
&= -\frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a}+\frac{\left(96b^2\right)\int x^{4/3}\sqrt{bx^{2/3}+ax}\,dx}{133a^2}\\
&= \frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3}-\frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a}-\frac{\left(192b^3\right)\int x\sqrt{bx^{2/3}+a}}{323a^3}\\
&= -\frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4}+\frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3}-\frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{3/}}{7a}\\
&= \frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5}-\frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4}+\frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3}-\frac{36bx^{5/3}(bx^{2/3}+}{133a^2}\\
&= -\frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6}+\frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5}-\frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4}+\frac{576b^2x^{4/3}(bx}{2261}\\
&= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7}-\frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6}+\frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5}-\frac{384b^3x(bx^{2/3}+}{1615a^4}\\
&= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7}-\frac{49152b^7\left(bx^{2/3}+ax\right)^{3/2}}{323323a^8\sqrt[3]{x}}-\frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6}+\frac{4608b^4x^{2/3}(bx^{2/3}}{20995a^5}\\
&= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7}+\frac{196608b^8\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^9x^{2/3}}-\frac{49152b^7\left(bx^{2/3}+ax\right)^{3/2}}{323323a^8\sqrt[3]{x}}-\frac{9216b^5\sqrt[3]{x}(bx^{2/3}+}{46189a^6}\\
&= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7}-\frac{131072b^9\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^{10}x}+\frac{196608b^8\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^9x^{2/3}}-\frac{49152b^7(bx^{2/3}+}{323323a^8}
\end{aligned}
$$

**Mathematica [A]** time = 0.0951391, size = 144, normalized size = 0.51

$$\frac{2\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\,(205920a^7b^2x^{7/3}-192192a^6b^3x^2+177408a^5b^4x^{5/3}-161280a^4b^5x^{4/3}-122880a^2b^7x^{2/3}+14336}{1616615a^{10}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*(-65536*b^9 + 98304*a*b^8*x^(1/3)
- 122880*a^2*b^7*x^(2/3) + 143360*a^3*b^6*x - 161280*a^4*b^5*x^(4/3) + 1774
08*a^5*b^4*x^(5/3) - 192192*a^6*b^3*x^2 + 205920*a^7*b^2*x^(7/3) - 218790*a
^8*b*x^(8/3) + 230945*a^9*x^3))/(1616615*a^10*x^(1/3))
```

---

**Maple [A]** time = 0.004, size = 123, normalized size = 0.4

$$-\frac{2}{1616615\,a^{10}}\sqrt{bx^{\frac{2}{3}}+ax}\left(b+a\sqrt[3]{x}\right)(218790\,x^{8/3}a^8b-205920\,x^{7/3}a^7b^2-177408\,x^{5/3}a^5b^4+161280\,x^{4/3}a^4b^5-230945\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^(2/3)+a*x)^(1/2),x)
```

```
[Out] -2/1616615*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(218790*x^(8/3)*a^8*b-205920
*x^(7/3)*a^7*b^2-177408*x^(5/3)*a^5*b^4+161280*x^(4/3)*a^4*b^5-230945*x^3*a
^9+122880*x^(2/3)*a^2*b^7+192192*x^2*a^6*b^3-98304*x^(1/3)*a*b^8-143360*x*a
^3*b^6+65536*b^9)/x^(1/3)/a^10
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{2}{3}}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))*x^2, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{ax + bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*x + b*x**(2/3)), x)
```

---

**Giac [A]** time = 1.12632, size = 203, normalized size = 0.72

$$\frac{131072\,b^{\frac{21}{2}}}{1616615\,a^{10}} + \frac{2\left(230945\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}} - 2297295\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b + 10270260\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^2 - 27159132\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 131072/1616615*b^(21/2)/a^10 + 2/1616615*(230945*(a*x^(1/3) + b)^(21/2) - 2
297295*(a*x^(1/3) + b)^(19/2)*b + 10270260*(a*x^(1/3) + b)^(17/2)*b^2 - 271
59132*(a*x^(1/3) + b)^(15/2)*b^3 + 47006190*(a*x^(1/3) + b)^(13/2)*b^4 - 55
552770*(a*x^(1/3) + b)^(11/2)*b^5 + 45265220*(a*x^(1/3) + b)^(9/2)*b^6 - 24
942060*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 1616
615*(a*x^(1/3) + b)^(3/2)*b^9)/a^10
```

## 3.169 $\int x\sqrt{bx^{2/3}+ax}\,dx$

**Optimal**. Leaf size=195

$$\frac{2048b^6\left(ax+bx^{2/3}\right)^{3/2}}{15015a^7x}-\frac{1024b^5\left(ax+bx^{2/3}\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(ax+bx^{2/3}\right)^{3/2}}{1001a^5\sqrt[3]{x}}-\frac{128b^3\left(ax+bx^{2/3}\right)^{3/2}}{429a^4}+\frac{48b^2\sqrt[3]{x}\left(ax+bx^{2/3}\right.}{143a^3}$$

```
[Out] (-128*b^3*(b*x^(2/3) + a*x)^(3/2))/(429*a^4) + (2048*b^6*(b*x^(2/3) + a*x)^
(3/2))/(15015*a^7*x) - (1024*b^5*(b*x^(2/3) + a*x)^(3/2))/(5005*a^6*x^(2/3)
) + (256*b^4*(b*x^(2/3) + a*x)^(3/2))/(1001*a^5*x^(1/3)) + (48*b^2*x^(1/3)*
(b*x^(2/3) + a*x)^(3/2))/(143*a^3) - (24*b*x^(2/3)*(b*x^(2/3) + a*x)^(3/2))
/(65*a^2) + (2*x*(b*x^(2/3) + a*x)^(3/2))/(5*a)
```

---

**Rubi [A]** time = 0.273272, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2016, 2002, 2014}

$$\frac{2048b^6\left(ax+bx^{2/3}\right)^{3/2}}{15015a^7x}-\frac{1024b^5\left(ax+bx^{2/3}\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(ax+bx^{2/3}\right)^{3/2}}{1001a^5\sqrt[3]{x}}-\frac{128b^3\left(ax+bx^{2/3}\right)^{3/2}}{429a^4}+\frac{48b^2\sqrt[3]{x}\left(ax+bx^{2/3}\right.}{143a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-128*b^3*(b*x^(2/3) + a*x)^(3/2))/(429*a^4) + (2048*b^6*(b*x^(2/3) + a*x)^
(3/2))/(15015*a^7*x) - (1024*b^5*(b*x^(2/3) + a*x)^(3/2))/(5005*a^6*x^(2/3)
) + (256*b^4*(b*x^(2/3) + a*x)^(3/2))/(1001*a^5*x^(1/3)) + (48*b^2*x^(1/3)*
(b*x^(2/3) + a*x)^(3/2))/(143*a^3) - (24*b*x^(2/3)*(b*x^(2/3) + a*x)^(3/2))
/(65*a^2) + (2*x*(b*x^(2/3) + a*x)^(3/2))/(5*a)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int x\sqrt{bx^{2/3}+ax}\,dx &= \frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}-\frac{(4b)\int x^{2/3}\sqrt{bx^{2/3}+ax}\,dx}{5a}\\
&=-\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}+\frac{\left(8b^2\right)\int \sqrt[3]{x}\sqrt{bx^{2/3}+ax}\,dx}{13a^2}\\
&=\frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3}-\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}-\frac{\left(64b^3\right)\int \sqrt{bx^{2/3}+ax}\,dx}{143a^3}\\
&=-\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}+\frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3}-\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a}+\\
&=-\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}+\frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}}+\frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3}-\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2}\\
&=-\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}-\frac{1024b^5\left(bx^{2/3}+ax\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}}+\frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3}\\
&=-\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4}+\frac{2048b^6\left(bx^{2/3}+ax\right)^{3/2}}{15015a^7x}-\frac{1024b^5\left(bx^{2/3}+ax\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}}+
\end{aligned}
$$

**Mathematica [A]** time = 0.0687215, size = 107, normalized size = 0.55

$$\frac{2\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\left(2520a^4b^2x^{4/3}+1920a^2b^4x^{2/3}-2240a^3b^3x-2772a^5bx^{5/3}+3003a^6x^2-1536ab^5\sqrt[3]{x}+1024b^6\right)}{15015a^7\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*(1024*b^6 - 1536*a*b^5*x^(1/3) + 1
920*a^2*b^4*x^(2/3) - 2240*a^3*b^3*x + 2520*a^4*b^2*x^(4/3) - 2772*a^5*b*x^
(5/3) + 3003*a^6*x^2))/(15015*a^7*x^(1/3))
```

---

**Maple [A]** time = 0.002, size = 90, normalized size = 0.5

$$-\frac{2}{15015\,a^7}\sqrt{bx^{\frac{2}{3}}+ax}\left(b+a\sqrt[3]{x}\right)\left(2772\,x^{5/3}a^5b-2520\,x^{4/3}a^4b^2-1920\,x^{2/3}a^2b^4-3003\,x^2a^6+1536\,\sqrt[3]{x}ab^5+2240\,xa^3b^3-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^(2/3)+a*x)^(1/2),x)
```

```
[Out] -2/15015*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(2772*x^(5/3)*a^5*b-2520*x^(4/
3)*a^4*b^2-1920*x^(2/3)*a^2*b^4-3003*x^2*a^6+1536*x^(1/3)*a*b^5+2240*x*a^3*
b^3-1024*b^6)/x^(1/3)/a^7
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax+bx^{\frac{2}{3}}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))*x, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*x + b*x**(2/3)), x)
```

---

**Giac [A]** time = 1.13058, size = 146, normalized size = 0.75

$$-\frac{2048\,b^{\frac{15}{2}}}{15015\,a^{7}}+\frac{2\left(3003\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}-20790\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b+61425\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{2}-100100\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{3}+96525\left(ax^{\frac{1}{3}}\right.\right.}{15015\,a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2048/15015*b^(15/2)/a^7 + 2/15015*(3003*(a*x^(1/3) + b)^(15/2) - 20790*(a*
x^(1/3) + b)^(13/2)*b + 61425*(a*x^(1/3) + b)^(11/2)*b^2 - 100100*(a*x^(1/3
) + b)^(9/2)*b^3 + 96525*(a*x^(1/3) + b)^(7/2)*b^4 - 54054*(a*x^(1/3) + b)^
(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6)/a^7
```

## 3.170 $\int \sqrt{bx^{2/3}+ax}\,dx$

**Optimal**. Leaf size=109

$$-\frac{32b^3\left(ax+bx^{2/3}\right)^{3/2}}{105a^4x}+\frac{16b^2\left(ax+bx^{2/3}\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{3/2}}{7a^2\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}$$

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(3*a) - (32*b^3*(b*x^(2/3) + a*x)^(3/2))/(105*a
^4*x) + (16*b^2*(b*x^(2/3) + a*x)^(3/2))/(35*a^3*x^(2/3)) - (4*b*(b*x^(2/3)
 + a*x)^(3/2))/(7*a^2*x^(1/3))
```

---

**Rubi [A]** time = 0.136737, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2002, 2016, 2014}

$$-\frac{32b^3\left(ax+bx^{2/3}\right)^{3/2}}{105a^4x}+\frac{16b^2\left(ax+bx^{2/3}\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{3/2}}{7a^2\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(3*a) - (32*b^3*(b*x^(2/3) + a*x)^(3/2))/(105*a
^4*x) + (16*b^2*(b*x^(2/3) + a*x)^(3/2))/(35*a^3*x^(2/3)) - (4*b*(b*x^(2/3)
 + a*x)^(3/2))/(7*a^2*x^(1/3))
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{bx^{2/3}+ax}\,dx &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a} - \frac{(2b)\int \frac{\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}\,dx}{3a} \\ &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a} - \frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}} + \frac{\left(8b^2\right)\int \frac{\sqrt{bx^{2/3}+ax}}{x^{2/3}}\,dx}{21a^2} \\ &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a} + \frac{16b^2\left(bx^{2/3}+ax\right)^{3/2}}{35a^3x^{2/3}} - \frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}} - \frac{\left(16b^3\right)\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx}{105a^3} \\ &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a} - \frac{32b^3\left(bx^{2/3}+ax\right)^{3/2}}{105a^4x} + \frac{16b^2\left(bx^{2/3}+ax\right)^{3/2}}{35a^3x^{2/3}} - \frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}}\end{aligned}$$

**Mathematica [A]** time = 0.0395579, size = 70, normalized size = 0.64

$$\frac{2\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\left(-30a^2bx^{2/3}+35a^3x+24ab^2\sqrt[3]{x}-16b^3\right)}{105a^4\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*(-16*b^3 + 24*a*b^2*x^(1/3) - 30*a
^2*b*x^(2/3) + 35*a^3*x))/(105*a^4*x^(1/3))
```

---

**Maple [A]** time = 0.003, size = 57, normalized size = 0.5

$$-\frac{2}{105\,a^4}\sqrt{bx^{\frac{2}{3}}+ax}\left(b+a\sqrt[3]{x}\right)\left(30\,x^{2/3}a^2b-24\,\sqrt[3]{x}ab^2-35\,xa^3+16\,b^3\right)\frac{1}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(1/2),x)
```

```
[Out] -2/105*(b*x^(2/3)+a*x)^(1/2)*(b+a*x^(1/3))*(30*x^(2/3)*a^2*b-24*x^(1/3)*a*b
^2-35*x*a^3+16*b^3)/x^(1/3)/a^4
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax + bx^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3)), x)
```

---

**Giac [A]** time = 1.11866, size = 89, normalized size = 0.82

$$\frac{32\, b^{\frac{9}{2}}}{105\, a^4} + \frac{2\left(35\left(ax^{\frac{1}{3}} + b\right)^{\frac{9}{2}} - 135\left(ax^{\frac{1}{3}} + b\right)^{\frac{7}{2}} b + 189\left(ax^{\frac{1}{3}} + b\right)^{\frac{5}{2}} b^2 - 105\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}} b^3\right)}{105\, a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 32/105*b^(9/2)/a^4 + 2/105*(35*(a*x^(1/3) + b)^(9/2) - 135*(a*x^(1/3) + b)^
(7/2)*b + 189*(a*x^(1/3) + b)^(5/2)*b^2 - 105*(a*x^(1/3) + b)^(3/2)*b^3)/a^
4
```

## 3.171 $\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx$

**Optimal**. Leaf size=23

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{ax}$$

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(a*x)
```

---

**Rubi [A]** time = 0.0404968, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x,x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(a*x)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx=\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{ax}$$

**Mathematica [A]** time = 0.0108147, size = 23, normalized size = 1.

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x,x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(a*x)
```

---

**Maple [A]** time = 0.003, size = 27, normalized size = 1.2

$$2\,\frac{\sqrt{bx^{2/3}+ax}\left(b+a\sqrt[3]{x}\right)}{a\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(1/2)/x,x)
```

```
[Out] 2*(b*x^(2/3)+a*x)^(1/2)/x^(1/3)*(b+a*x^(1/3))/a
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x, x)
```

---

**Giac [A]** time = 1.126, size = 31, normalized size = 1.35

$$\frac{2\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}}}{a} - \frac{2\,b^{\frac{3}{2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*(a*x^(1/3) + b)^(3/2)/a - 2*b^(3/2)/a
```

## 3.172 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx$

**Optimal**. Leaf size=90

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{4b^{3/2}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4bx^{2/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(2*x) - (3*a*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(2/3)
) + (3*a^2*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(4*b^(3/2))
```

---

**Rubi [A]** time = 0.138582, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2029, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{4b^{3/2}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4bx^{2/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^2,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(2*x) - (3*a*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(2/3)
) + (3*a^2*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(4*b^(3/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}+\frac{1}{4}a\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}-\frac{a^2\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{8b}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}+\frac{\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b^{3/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0415882, size = 57, normalized size = 0.63

$$
-\frac{2a^2\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^3\sqrt[3]{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^2,x]
```

```
[Out] (-2*a^2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 3, 5/2
, 1 + (a*x^(1/3))/b])/(b^3*x^(1/3))
```

---

**Maple [A]** time = 0.01, size = 80, normalized size = 0.9

$$
\frac{3}{4x}\sqrt{bx^{\frac{2}{3}}+ax}\left(\mathrm{Artanh}\left(\sqrt{b+a\sqrt[3]{x}}\frac{1}{\sqrt{b}}\right)ba^2x^{\frac{2}{3}}-b^{\frac{3}{2}}\left(b+a\sqrt[3]{x}\right)^{\frac{3}{2}}-b^{\frac{5}{2}}\sqrt{b+a\sqrt[3]{x}}\right)b^{-\frac{5}{2}}\frac{1}{\sqrt{b+a\sqrt[3]{x}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(1/2)/x^2,x)
```

```
[Out] 3/4*(b*x^(2/3)+a*x)^(1/2)*(arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b*a^2*x^(2/
3)-b^(3/2)*(b+a*x^(1/3))^(3/2)-b^(5/2)*(b+a*x^(1/3))^(1/2))/x/(b+a*x^(1/3))
^(1/2)/b^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^2, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**2, x)
```

---

**Giac [A]** time = 1.21738, size = 97, normalized size = 1.08

$$-\frac{3\left(\frac{a^3\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^3+\sqrt{ax^{\frac{1}{3}}+b}\,ba^3b}{a^2bx^{\frac{2}{3}}}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -3/4*(a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + ((a*x^(1/3) +
 b)^(3/2)*a^3 + sqrt(a*x^(1/3) + b)*a^3*b)/(a^2*b*x^(2/3)))/a
```

## 3.173 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx$

**Optimal**. Leaf size=178

$$\frac{21a^4\sqrt{ax+bx^{2/3}}}{128b^4x^{2/3}} - \frac{7a^3\sqrt{ax+bx^{2/3}}}{64b^3x} + \frac{7a^2\sqrt{ax+bx^{2/3}}}{80b^2x^{4/3}} - \frac{21a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{128b^{9/2}} - \frac{3a\sqrt{ax+bx^{2/3}}}{40bx^{5/3}} - \frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(5*x^2) - (3*a*Sqrt[b*x^(2/3) + a*x])/(40*b*x^(5
/3)) + (7*a^2*Sqrt[b*x^(2/3) + a*x])/(80*b^2*x^(4/3)) - (7*a^3*Sqrt[b*x^(2/
3) + a*x])/(64*b^3*x) + (21*a^4*Sqrt[b*x^(2/3) + a*x])/(128*b^4*x^(2/3)) -
(21*a^5*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(128*b^(9/2))
```

---

**Rubi [A]** time = 0.295909, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2029, 206}

$$\frac{21a^4\sqrt{ax+bx^{2/3}}}{128b^4x^{2/3}} - \frac{7a^3\sqrt{ax+bx^{2/3}}}{64b^3x} + \frac{7a^2\sqrt{ax+bx^{2/3}}}{80b^2x^{4/3}} - \frac{21a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{128b^{9/2}} - \frac{3a\sqrt{ax+bx^{2/3}}}{40bx^{5/3}} - \frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^3,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(5*x^2) - (3*a*Sqrt[b*x^(2/3) + a*x])/(40*b*x^(5
/3)) + (7*a^2*Sqrt[b*x^(2/3) + a*x])/(80*b^2*x^(4/3)) - (7*a^3*Sqrt[b*x^(2/
3) + a*x])/(64*b^3*x) + (21*a^4*Sqrt[b*x^(2/3) + a*x])/(128*b^4*x^(2/3)) -
(21*a^5*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(128*b^(9/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} + \frac{1}{10}a\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} - \frac{\left(7a^2\right)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{80b} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} + \frac{\left(7a^3\right)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{96b^2} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} - \frac{\left(7a^4\right)\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{128b^3} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} + \frac{21a^4\sqrt{bx^{2/3}+ax}}{128b^4x^{2/3}} + (7a \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} + \frac{21a^4\sqrt{bx^{2/3}+ax}}{128b^4x^{2/3}} - (21 \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} + \frac{21a^4\sqrt{bx^{2/3}+ax}}{128b^4x^{2/3}} - 21a
\end{aligned}$$

**Mathematica [C]** time = 0.0408602, size = 57, normalized size = 0.32

$$\frac{2a^5\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{3}{2},6;\frac{5}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^6\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^3,x]
```

```
[Out] (2*a^5*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 6, 5/2,
 1 + (a*x^(1/3))/b])/(b^6*x^(1/3))
```

---

**Maple [A]** time = 0.011, size = 125, normalized size = 0.7

$$-\frac{1}{640\,x^2}\sqrt{bx^{\frac{2}{3}}+ax}\left(105\,b^{17/2}\sqrt{b+a\sqrt[3]{x}}+790\,b^{15/2}\left(b+a\sqrt[3]{x}\right)^{3/2}-896\,b^{13/2}\left(b+a\sqrt[3]{x}\right)^{5/2}+490\,b^{11/2}\left(b+a\sqrt[3]{x}\right)^{7/2}-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(1/2)/x^3,x)
```

```
[Out] -1/640*(b*x^(2/3)+a*x)^(1/2)*(105*b^(17/2)*(b+a*x^(1/3))^(1/2)+790*b^(15/2)
*(b+a*x^(1/3))^(3/2)-896*b^(13/2)*(b+a*x^(1/3))^(5/2)+490*b^(11/2)*(b+a*x^(
1/3))^(7/2)-105*b^(9/2)*(b+a*x^(1/3))^(9/2)+105*arctanh((b+a*x^(1/3))^(1/2)
/b^(1/2))*b^4*a^5*x^(5/3))/x^2/(b+a*x^(1/3))^(1/2)/b^(17/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^3, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**3, x)
```

---

**Giac [A]** time = 1.20498, size = 170, normalized size = 0.96

$$\frac{\frac{105\,a^6 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4} + \frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^6-490\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^6b+896\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^6b^2-790\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^6b^3-105\sqrt{ax^{\frac{1}{3}}+b}\,a^6b^4}{a^5b^4x^{\frac{5}{3}}}}{640\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/640*(105*a^6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*(
a*x^(1/3) + b)^(9/2)*a^6 - 490*(a*x^(1/3) + b)^(7/2)*a^6*b + 896*(a*x^(1/3)
 + b)^(5/2)*a^6*b^2 - 790*(a*x^(1/3) + b)^(3/2)*a^6*b^3 - 105*sqrt(a*x^(1/3
) + b)*a^6*b^4)/(a^5*b^4*x^(5/3)))/a
```

## 3.174 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$

**Optimal**. Leaf size=266

$$-\frac{1287a^7\sqrt{ax+bx^{2/3}}}{16384b^7x^{2/3}}+\frac{429a^6\sqrt{ax+bx^{2/3}}}{8192b^6x}-\frac{429a^5\sqrt{ax+bx^{2/3}}}{10240b^5x^{4/3}}+\frac{1287a^4\sqrt{ax+bx^{2/3}}}{35840b^4x^{5/3}}-\frac{143a^3\sqrt{ax+bx^{2/3}}}{4480b^3x^2}+\frac{13a^2\sqrt{a}}{448b}$$

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(8*x^3) - (3*a*Sqrt[b*x^(2/3) + a*x])/(112*b*x^(
8/3)) + (13*a^2*Sqrt[b*x^(2/3) + a*x])/(448*b^2*x^(7/3)) - (143*a^3*Sqrt[b*
x^(2/3) + a*x])/(4480*b^3*x^2) + (1287*a^4*Sqrt[b*x^(2/3) + a*x])/(35840*b^
4*x^(5/3)) - (429*a^5*Sqrt[b*x^(2/3) + a*x])/(10240*b^5*x^(4/3)) + (429*a^6
*Sqrt[b*x^(2/3) + a*x])/(8192*b^6*x) - (1287*a^7*Sqrt[b*x^(2/3) + a*x])/(16
384*b^7*x^(2/3)) + (1287*a^8*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x
]])/(16384*b^(15/2))
```

---

**Rubi [A]** time = 0.475003, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2029, 206}

$$-\frac{1287a^7\sqrt{ax+bx^{2/3}}}{16384b^7x^{2/3}}+\frac{429a^6\sqrt{ax+bx^{2/3}}}{8192b^6x}-\frac{429a^5\sqrt{ax+bx^{2/3}}}{10240b^5x^{4/3}}+\frac{1287a^4\sqrt{ax+bx^{2/3}}}{35840b^4x^{5/3}}-\frac{143a^3\sqrt{ax+bx^{2/3}}}{4480b^3x^2}+\frac{13a^2\sqrt{a}}{448b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^4,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(8*x^3) - (3*a*Sqrt[b*x^(2/3) + a*x])/(112*b*x^(
8/3)) + (13*a^2*Sqrt[b*x^(2/3) + a*x])/(448*b^2*x^(7/3)) - (143*a^3*Sqrt[b*
x^(2/3) + a*x])/(4480*b^3*x^2) + (1287*a^4*Sqrt[b*x^(2/3) + a*x])/(35840*b^
4*x^(5/3)) - (429*a^5*Sqrt[b*x^(2/3) + a*x])/(10240*b^5*x^(4/3)) + (429*a^6
*Sqrt[b*x^(2/3) + a*x])/(8192*b^6*x) - (1287*a^7*Sqrt[b*x^(2/3) + a*x])/(16
384*b^7*x^(2/3)) + (1287*a^8*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x
]])/(16384*b^(15/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}+\frac{1}{16}a\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}-\frac{\left(13a^2\right)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{224b}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}}+\frac{\left(143a^3\right)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{2688b^2}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}}-\frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2}-\frac{\left(429a^4\right)\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx}{8960b^3}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}}-\frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2}+\frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}}+\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}}-\frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2}+\frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}}-4\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}}-\frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2}+\frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}}-4\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}}-\frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2}+\frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}}-4\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}}-\frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2}+\frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}}-4\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3}-\frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}}+\frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}}-\frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2}+\frac{1287a^4\sqrt{bx^{2/3}+ax}}{35840b^4x^{5/3}}-4
\end{aligned}$$

**Mathematica [C]** time = 0.0456889, size = 57, normalized size = 0.21

$$-\frac{2a^8\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{3}{2},9;\frac{5}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^9\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^4,x]
```

```
[Out] (-2*a^8*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 9, 5/2
, 1 + (a*x^(1/3))/b])/(b^9*x^(1/3))
```

---

**Maple [A]** time = 0.013, size = 167, normalized size = 0.6

$$-\frac{1}{573440\,x^3}\sqrt{bx^{\frac{2}{3}}+ax}\left(45045\,b^{15/2}\left(b+a\sqrt[3]{x}\right)^{15/2}-345345\,b^{17/2}\left(b+a\sqrt[3]{x}\right)^{13/2}+1150149\,b^{19/2}\left(b+a\sqrt[3]{x}\right)^{11/2}-2167737\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(1/2)/x^4,x)
```

```
[Out] -1/573440*(b*x^(2/3)+a*x)^(1/2)*(45045*b^(15/2)*(b+a*x^(1/3))^(15/2)-345345
*b^(17/2)*(b+a*x^(1/3))^(13/2)+1150149*b^(19/2)*(b+a*x^(1/3))^(11/2)-216773
7*b^(21/2)*(b+a*x^(1/3))^(9/2)+2518087*b^(23/2)*(b+a*x^(1/3))^(7/2)-1831739
*b^(25/2)*(b+a*x^(1/3))^(5/2)+801535*b^(27/2)*(b+a*x^(1/3))^(3/2)-45045*arc
tanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^7*a^8*x^(8/3)+45045*b^(29/2)*(b+a*x^(1/
3))^(1/2))/x^3/(b+a*x^(1/3))^(1/2)/b^(29/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^4, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**4, x)
```

---

**Giac [A]** time = 1.2688, size = 239, normalized size = 0.9

$$-\frac{\frac{45045\,a^{9}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{7}}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{9}-345345\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{9}b+1150149\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{9}b^{2}-2167737\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{9}b^{3}+2518087\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{9}b}{a^{8}b^{7}x^{\frac{8}{3}}}}{573440\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/573440*(45045*a^9*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) +
(45045*(a*x^(1/3) + b)^(15/2)*a^9 - 345345*(a*x^(1/3) + b)^(13/2)*a^9*b + 1
150149*(a*x^(1/3) + b)^(11/2)*a^9*b^2 - 2167737*(a*x^(1/3) + b)^(9/2)*a^9*b
^3 + 2518087*(a*x^(1/3) + b)^(7/2)*a^9*b^4 - 1831739*(a*x^(1/3) + b)^(5/2)*
a^9*b^5 + 801535*(a*x^(1/3) + b)^(3/2)*a^9*b^6 + 45045*sqrt(a*x^(1/3) + b)*
a^9*b^7)/(a^8*b^7*x^(8/3)))/a
```

# 3.175 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

**Optimal**. Leaf size=354

$$\frac{12597a^{10}\sqrt{ax+bx^{2/3}}}{262144b^{10}x^{2/3}}-\frac{4199a^9\sqrt{ax+bx^{2/3}}}{131072b^9x}+\frac{4199a^8\sqrt{ax+bx^{2/3}}}{163840b^8x^{4/3}}-\frac{12597a^7\sqrt{ax+bx^{2/3}}}{573440b^7x^{5/3}}+\frac{4199a^6\sqrt{ax+bx^{2/3}}}{215040b^6x^2}-$$

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(11*x^4) - (3*a*Sqrt[b*x^(2/3) + a*x])/(220*b*x^
(11/3)) + (19*a^2*Sqrt[b*x^(2/3) + a*x])/(1320*b^2*x^(10/3)) - (323*a^3*Sqr
t[b*x^(2/3) + a*x])/(21120*b^3*x^3) + (323*a^4*Sqrt[b*x^(2/3) + a*x])/(1971
2*b^4*x^(8/3)) - (4199*a^5*Sqrt[b*x^(2/3) + a*x])/(236544*b^5*x^(7/3)) + (4
199*a^6*Sqrt[b*x^(2/3) + a*x])/(215040*b^6*x^2) - (12597*a^7*Sqrt[b*x^(2/3)
 + a*x])/(573440*b^7*x^(5/3)) + (4199*a^8*Sqrt[b*x^(2/3) + a*x])/(163840*b^
8*x^(4/3)) - (4199*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^9*x) + (12597*a^10*
Sqrt[b*x^(2/3) + a*x])/(262144*b^10*x^(2/3)) - (12597*a^11*ArcTanh[(Sqrt[b]
*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(262144*b^(21/2))
```

---

**Rubi [A]** time = 0.660036, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2029, 206}

$$\frac{12597a^{10}\sqrt{ax+bx^{2/3}}}{262144b^{10}x^{2/3}}-\frac{4199a^9\sqrt{ax+bx^{2/3}}}{131072b^9x}+\frac{4199a^8\sqrt{ax+bx^{2/3}}}{163840b^8x^{4/3}}-\frac{12597a^7\sqrt{ax+bx^{2/3}}}{573440b^7x^{5/3}}+\frac{4199a^6\sqrt{ax+bx^{2/3}}}{215040b^6x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^5,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(11*x^4) - (3*a*Sqrt[b*x^(2/3) + a*x])/(220*b*x^
(11/3)) + (19*a^2*Sqrt[b*x^(2/3) + a*x])/(1320*b^2*x^(10/3)) - (323*a^3*Sqr
t[b*x^(2/3) + a*x])/(21120*b^3*x^3) + (323*a^4*Sqrt[b*x^(2/3) + a*x])/(1971
2*b^4*x^(8/3)) - (4199*a^5*Sqrt[b*x^(2/3) + a*x])/(236544*b^5*x^(7/3)) + (4
199*a^6*Sqrt[b*x^(2/3) + a*x])/(215040*b^6*x^2) - (12597*a^7*Sqrt[b*x^(2/3)
 + a*x])/(573440*b^7*x^(5/3)) + (4199*a^8*Sqrt[b*x^(2/3) + a*x])/(163840*b^
8*x^(4/3)) - (4199*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^9*x) + (12597*a^10*
Sqrt[b*x^(2/3) + a*x])/(262144*b^10*x^(2/3)) - (12597*a^11*ArcTanh[(Sqrt[b]
*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(262144*b^(21/2))
```

### Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

### Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}+\frac{1}{22}a\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}-\frac{\left(19a^2\right)\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{440b}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}+\frac{\left(323a^3\right)\int \frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{7920b^2}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}-\frac{\left(323a^4\right)\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,d}{8448b^3}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}+(4\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-41\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-41\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-41\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-41\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-41\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-41\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-41\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}+ax}}{19712b^4x^{8/3}}-41
\end{aligned}
$$

**Mathematica [C]** time = 0.0490546, size = 57, normalized size = 0.16

$$
\frac{2a^{11}\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{3}{2},12;\frac{5}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^{12}\sqrt[3]{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^5,x]
```

```
[Out] (2*a^11*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 12, 5/
2, 1 + (a*x^(1/3))/b])/(b^12*x^(1/3))
```

---

**Maple [A]** time = 0.015, size = 209, normalized size = 0.6

$$-\frac{1}{302776320\,x^{4}}\sqrt{b x^{\frac{2}{3}}+a x}\,\left(-14549535\left(b+a\sqrt[3]{x}\right)^{21/2} b^{21/2}+155195040\left(b+a\sqrt[3]{x}\right)^{19/2} b^{23/2}-749786037\left(b+a\sqrt[3]{x}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(1/2)/x^5,x)
```

```
[Out] -1/302776320*(b*x^(2/3)+a*x)^(1/2)*(-14549535*(b+a*x^(1/3))^(21/2)*b^(21/2)
+155195040*(b+a*x^(1/3))^(19/2)*b^(23/2)-749786037*(b+a*x^(1/3))^(17/2)*b^(
25/2)+2163862272*(b+a*x^(1/3))^(15/2)*b^(27/2)-4139920070*(b+a*x^(1/3))^(13
/2)*b^(29/2)+5503713280*(b+a*x^(1/3))^(11/2)*b^(31/2)-5174056250*(b+a*x^(1/
3))^(9/2)*b^(33/2)+3424523520*(b+a*x^(1/3))^(7/2)*b^(35/2)-1551313995*(b+a*
x^(1/3))^(5/2)*b^(37/2)+14549535*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^10*
a^11*x^(11/3)+450357600*(b+a*x^(1/3))^(3/2)*b^(39/2)+14549535*(b+a*x^(1/3))
^(1/2)*b^(41/2))/x^4/(b+a*x^(1/3))^(1/2)/b^(41/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a x + b x^{\frac{2}{3}}}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^5, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**5,x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.39868, size = 308, normalized size = 0.87

$$\frac{\frac{14549535\,a^{12}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{10}}+\frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{12}-155195040\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{12}b+749786037\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{12}b^{2}-2163862272\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{12}b^{3}+413992}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/302776320*(14549535*a^12*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b
^10) + (14549535*(a*x^(1/3) + b)^(21/2)*a^12 - 155195040*(a*x^(1/3) + b)^(1
9/2)*a^12*b + 749786037*(a*x^(1/3) + b)^(17/2)*a^12*b^2 - 2163862272*(a*x^(
1/3) + b)^(15/2)*a^12*b^3 + 4139920070*(a*x^(1/3) + b)^(13/2)*a^12*b^4 - 55
03713280*(a*x^(1/3) + b)^(11/2)*a^12*b^5 + 5174056250*(a*x^(1/3) + b)^(9/2)
*a^12*b^6 - 3424523520*(a*x^(1/3) + b)^(7/2)*a^12*b^7 + 1551313995*(a*x^(1/
3) + b)^(5/2)*a^12*b^8 - 450357600*(a*x^(1/3) + b)^(3/2)*a^12*b^9 - 1454953
5*sqrt(a*x^(1/3) + b)*a^12*b^10)/(a^11*b^10*x^(11/3)))/a
```

## 3.176 $\int x^2 \left(bx^{2/3}+ax\right)^{3/2} \, dx$

**Optimal**. Leaf size=343

$$-\frac{1048576b^{11}\left(ax+bx^{2/3}\right)^{5/2}}{152108775a^{12}x^{5/3}}+\frac{524288b^{10}\left(ax+bx^{2/3}\right)^{5/2}}{30421755a^{11}x^{4/3}}-\frac{131072b^{9}\left(ax+bx^{2/3}\right)^{5/2}}{4345965a^{10}x}+\frac{65536b^{8}\left(ax+bx^{2/3}\right)^{5/2}}{1448655a^{9}x^{2/3}}-901$$

```
[Out] (45056*b^6*(b*x^(2/3) + a*x)^(5/2))/(557175*a^7) - (1048576*b^11*(b*x^(2/3)
 + a*x)^(5/2))/(152108775*a^12*x^(5/3)) + (524288*b^10*(b*x^(2/3) + a*x)^(5
/2))/(30421755*a^11*x^(4/3)) - (131072*b^9*(b*x^(2/3) + a*x)^(5/2))/(434596
5*a^10*x) + (65536*b^8*(b*x^(2/3) + a*x)^(5/2))/(1448655*a^9*x^(2/3)) - (90
112*b^7*(b*x^(2/3) + a*x)^(5/2))/(1448655*a^8*x^(1/3)) - (11264*b^5*x^(1/3)
*(b*x^(2/3) + a*x)^(5/2))/(111435*a^6) + (5632*b^4*x^(2/3)*(b*x^(2/3) + a*x
)^(5/2))/(45885*a^5) - (352*b^3*x*(b*x^(2/3) + a*x)^(5/2))/(2415*a^4) + (17
6*b^2*x^(4/3)*(b*x^(2/3) + a*x)^(5/2))/(1035*a^3) - (44*b*x^(5/3)*(b*x^(2/3
) + a*x)^(5/2))/(225*a^2) + (2*x^2*(b*x^(2/3) + a*x)^(5/2))/(9*a)
```

---

**Rubi [A]** time = 0.616286, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{1048576b^{11}\left(ax+bx^{2/3}\right)^{5/2}}{152108775a^{12}x^{5/3}}+\frac{524288b^{10}\left(ax+bx^{2/3}\right)^{5/2}}{30421755a^{11}x^{4/3}}-\frac{131072b^{9}\left(ax+bx^{2/3}\right)^{5/2}}{4345965a^{10}x}+\frac{65536b^{8}\left(ax+bx^{2/3}\right)^{5/2}}{1448655a^{9}x^{2/3}}-901$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (45056*b^6*(b*x^(2/3) + a*x)^(5/2))/(557175*a^7) - (1048576*b^11*(b*x^(2/3)
 + a*x)^(5/2))/(152108775*a^12*x^(5/3)) + (524288*b^10*(b*x^(2/3) + a*x)^(5
/2))/(30421755*a^11*x^(4/3)) - (131072*b^9*(b*x^(2/3) + a*x)^(5/2))/(434596
5*a^10*x) + (65536*b^8*(b*x^(2/3) + a*x)^(5/2))/(1448655*a^9*x^(2/3)) - (90
112*b^7*(b*x^(2/3) + a*x)^(5/2))/(1448655*a^8*x^(1/3)) - (11264*b^5*x^(1/3)
*(b*x^(2/3) + a*x)^(5/2))/(111435*a^6) + (5632*b^4*x^(2/3)*(b*x^(2/3) + a*x
)^(5/2))/(45885*a^5) - (352*b^3*x*(b*x^(2/3) + a*x)^(5/2))/(2415*a^4) + (17
6*b^2*x^(4/3)*(b*x^(2/3) + a*x)^(5/2))/(1035*a^3) - (44*b*x^(5/3)*(b*x^(2/3
) + a*x)^(5/2))/(225*a^2) + (2*x^2*(b*x^(2/3) + a*x)^(5/2))/(9*a)
```

### Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

### Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\left(bx^{2/3}+ax\right)^{3/2}dx &= \frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}-\frac{(22b)\int x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}dx}{27a}\\
&= -\frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}+\frac{\left(88b^2\right)\int x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}dx}{135a^2}\\
&= \frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3}-\frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}-\frac{\left(176b^3\right)\int x\left(bx^{2/3}+\right.}{345a^3}\\
&= -\frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4}+\frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3}-\frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2}+\frac{2x^2\left(bx^{2/3}+a\right.}{9a}\\
&= \frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5}-\frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4}+\frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3}-\frac{44bx^{5/3}\left(bx^{2/}\right.}{225a}\\
&= -\frac{11264b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6}+\frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5}-\frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4}+\frac{176b^2x^{4/3}}{1}\\
&= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7}-\frac{11264b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6}+\frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5}-\frac{352b^3x\left(b\right.}{24}\\
&= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7}-\frac{90112b^7\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^8\sqrt[3]{x}}-\frac{11264b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6}+\frac{5632b^4x^{2/3}\left(\right.}{458}\\
&= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7}+\frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^9x^{2/3}}-\frac{90112b^7\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^8\sqrt[3]{x}}-\frac{11264b^5\sqrt[3]{x}\left(bx\right.}{111435}\\
&= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7}-\frac{131072b^9\left(bx^{2/3}+ax\right)^{5/2}}{4345965a^{10}x}+\frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^9x^{2/3}}-\frac{90112b^7\left(bx^{2/3}\right.}{1448655a}\\
&= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7}+\frac{524288b^{10}\left(bx^{2/3}+ax\right)^{5/2}}{30421755a^{11}x^{4/3}}-\frac{131072b^9\left(bx^{2/3}+ax\right)^{5/2}}{4345965a^{10}x}+\frac{65536b^8\left(b\right.}{144865}\\
&= \frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7}-\frac{1048576b^{11}\left(bx^{2/3}+ax\right)^{5/2}}{152108775a^{12}x^{5/3}}+\frac{524288b^{10}\left(bx^{2/3}+ax\right)^{5/2}}{30421755a^{11}x^{4/3}}-\frac{131072b^9}{434}
\end{aligned}$$

**Mathematica [A]** time = 0.131135, size = 172, normalized size = 0.5

$$\frac{2\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\left(12932920a^9b^2x^3-11085360a^8b^3x^{8/3}+9335040a^7b^4x^{7/3}-7687680a^6b^5x^2+6150144a^5b^6x^{5/3}\right.}{152}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*(-524288*b^11 + 1310720*a*b^10*x
^(1/3) - 2293760*a^2*b^9*x^(2/3) + 3440640*a^3*b^8*x - 4730880*a^4*b^7*x^(4
/3) + 6150144*a^5*b^6*x^(5/3) - 7687680*a^6*b^5*x^2 + 9335040*a^7*b^4*x^(7/
3) - 11085360*a^8*b^3*x^(8/3) + 12932920*a^9*b^2*x^3 - 14872858*a^10*b*x^(1
```

```
0/3) + 16900975*a^11*x^(11/3)))/(152108775*a^12*x^(1/3))
```

---

**Maple [A]** time = 0.004, size = 145, normalized size = 0.4

$$\frac{2}{152108775\,xa^{12}}\Big(bx^{\frac{2}{3}}+ax\Big)^{\frac{3}{2}}\big(b+a\sqrt[3]{x}\big)\big(16900975\,x^{11/3}a^{11}-14872858\,x^{10/3}a^{10}b+12932920\,x^{3}a^{9}b^{2}-11085360\,x^{8/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^(2/3)+a*x)^(3/2),x)
```

```
[Out] 2/152108775*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(16900975*x^(11/3)*a^11-148
72858*x^(10/3)*a^10*b+12932920*x^3*a^9*b^2-11085360*x^(8/3)*a^8*b^3+9335040
*x^(7/3)*a^7*b^4-7687680*x^2*a^6*b^5+6150144*x^(5/3)*a^5*b^6-4730880*x^(4/3
)*a^4*b^7+3440640*x*a^3*b^8-2293760*x^(2/3)*a^2*b^9+1310720*x^(1/3)*a*b^10-
524288*b^11)/x/a^12
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\Big(ax+bx^{\frac{2}{3}}\Big)^{\frac{3}{2}}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)*x^2, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.18264, size = 508, normalized size = 1.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/16900975*b*(524288*b^(25/2)/a^12 + (2028117*(a*x^(1/3) + b)^(25/2) - 2424
9225*(a*x^(1/3) + b)^(23/2)*b + 132793375*(a*x^(1/3) + b)^(21/2)*b^2 - 4403
14875*(a*x^(1/3) + b)^(19/2)*b^3 + 984233250*(a*x^(1/3) + b)^(17/2)*b^4 - 1
561650090*(a*x^(1/3) + b)^(15/2)*b^5 + 1801903950*(a*x^(1/3) + b)^(13/2)*b^
6 - 1521087750*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a*x^(1/3) + b)^(9/2)
*b^8 - 398380125*(a*x^(1/3) + b)^(7/2)*b^9 + 111546435*(a*x^(1/3) + b)^(5/2
)*b^10 - 16900975*(a*x^(1/3) + b)^(3/2)*b^11)/a^12) - 2/152108775*a*(419430
4*b^(27/2)/a^13 - (16900975*(a*x^(1/3) + b)^(27/2) - 219036636*(a*x^(1/3) +
 b)^(25/2)*b + 1309458150*(a*x^(1/3) + b)^(23/2)*b^2 - 4780561500*(a*x^(1/3
) + b)^(21/2)*b^3 + 11888501625*(a*x^(1/3) + b)^(19/2)*b^4 - 21259438200*(a
*x^(1/3) + b)^(17/2)*b^5 + 28109701620*(a*x^(1/3) + b)^(15/2)*b^6 - 2780080
3800*(a*x^(1/3) + b)^(13/2)*b^7 + 20534684625*(a*x^(1/3) + b)^(11/2)*b^8 -
11154643500*(a*x^(1/3) + b)^(9/2)*b^9 + 4302505350*(a*x^(1/3) + b)^(7/2)*b^
10 - 1095183180*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x^(1/3) + b)^(3/2
)*b^12)/a^13)
```

## 3.177 $\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx$

**Optimal**. Leaf size=255

$$\frac{65536b^8\left(ax+bx^{2/3}\right)^{5/2}}{4849845a^9x^{5/3}}-\frac{32768b^7\left(ax+bx^{2/3}\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(ax+bx^{2/3}\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(ax+b\right.}{4199a^5\sqrt[3]{}}$$

```
[Out] (-256*b^3*(b*x^(2/3) + a*x)^(5/2))/(1615*a^4) + (65536*b^8*(b*x^(2/3) + a*x
)^(5/2))/(4849845*a^9*x^(5/3)) - (32768*b^7*(b*x^(2/3) + a*x)^(5/2))/(96996
9*a^8*x^(4/3)) + (8192*b^6*(b*x^(2/3) + a*x)^(5/2))/(138567*a^7*x) - (4096*
b^5*(b*x^(2/3) + a*x)^(5/2))/(46189*a^6*x^(2/3)) + (512*b^4*(b*x^(2/3) + a*
x)^(5/2))/(4199*a^5*x^(1/3)) + (64*b^2*x^(1/3)*(b*x^(2/3) + a*x)^(5/2))/(32
3*a^3) - (32*b*x^(2/3)*(b*x^(2/3) + a*x)^(5/2))/(133*a^2) + (2*x*(b*x^(2/3)
 + a*x)^(5/2))/(7*a)
```

---

**Rubi [A]** time = 0.42205, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2016, 2002, 2014}

$$\frac{65536b^8\left(ax+bx^{2/3}\right)^{5/2}}{4849845a^9x^{5/3}}-\frac{32768b^7\left(ax+bx^{2/3}\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(ax+bx^{2/3}\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(ax+b\right.}{4199a^5\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-256*b^3*(b*x^(2/3) + a*x)^(5/2))/(1615*a^4) + (65536*b^8*(b*x^(2/3) + a*x
)^(5/2))/(4849845*a^9*x^(5/3)) - (32768*b^7*(b*x^(2/3) + a*x)^(5/2))/(96996
9*a^8*x^(4/3)) + (8192*b^6*(b*x^(2/3) + a*x)^(5/2))/(138567*a^7*x) - (4096*
b^5*(b*x^(2/3) + a*x)^(5/2))/(46189*a^6*x^(2/3)) + (512*b^4*(b*x^(2/3) + a*
x)^(5/2))/(4199*a^5*x^(1/3)) + (64*b^2*x^(1/3)*(b*x^(2/3) + a*x)^(5/2))/(32
3*a^3) - (32*b*x^(2/3)*(b*x^(2/3) + a*x)^(5/2))/(133*a^2) + (2*x*(b*x^(2/3)
 + a*x)^(5/2))/(7*a)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
```

```
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx &= \frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}-\frac{(16b)\int x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}\,dx}{21a}\\
&= -\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}+\frac{\left(32b^2\right)\int\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}\,dx}{57a^2}\\
&= \frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}-\frac{\left(128b^3\right)\int\left(bx^{2/3}+ax\right)^{3}}{323a^3}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)}{133a^2}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}-\frac{4096b^5\left(bx^{2/3}+ax\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)}{323a^3}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{8192b^6\left(bx^{2/3}+ax\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(bx^{2/3}+ax\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(bx^{2/3}+ax\right)^{5}}{4199a^5\sqrt[3]{x}}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}-\frac{32768b^7\left(bx^{2/3}+ax\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(bx^{2/3}+ax\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(bx^{2/3}+a\right.}{46189a^6x^{2/3}}\\
&= -\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{4849845a^9x^{5/3}}-\frac{32768b^7\left(bx^{2/3}+ax\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(bx^{2/3}+\right.}{138567a^7x}
\end{aligned}
$$

**Mathematica [A]** time = 0.0855035, size = 135, normalized size = 0.53

$$
\frac{2\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\left(480480a^6b^2x^2-384384a^5b^3x^{5/3}+295680a^4b^4x^{4/3}+143360a^2b^6x^{2/3}-215040a^3b^5x-58344\right.}{4849845a^9\sqrt[3]{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*(32768*b^8 - 81920*a*b^7*x^(1/3)
 + 143360*a^2*b^6*x^(2/3) - 215040*a^3*b^5*x + 295680*a^4*b^4*x^(4/3) - 384
384*a^5*b^3*x^(5/3) + 480480*a^6*b^2*x^2 - 583440*a^7*b*x^(7/3) + 692835*a^
8*x^(8/3)))/(4849845*a^9*x^(1/3))
```

---

**Maple [A]** time = 0.004, size = 112, normalized size = 0.4

$$
\frac{2}{4849845\,xa^9}\left(bx^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\sqrt[3]{x}\right)\left(692835\,x^{8/3}a^8-583440\,x^{7/3}a^7b+480480\,x^2a^6b^2-384384\,x^{5/3}a^5b^3+295680\,x^{4/3}a\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^(2/3)+a*x)^(3/2),x)
```

```
[Out] 2/4849845*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(692835*x^(8/3)*a^8-583440*x^
(7/3)*a^7*b+480480*x^2*a^6*b^2-384384*x^(5/3)*a^5*b^3+295680*x^(4/3)*a^4*b^
4-215040*x*a^3*b^5+143360*x^(2/3)*a^2*b^6-81920*x^(1/3)*a*b^7+32768*b^8)/x/
a^9
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)*x, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.16902, size = 394, normalized size = 1.55

$$-\frac{2}{692835}\, b \left( \frac{32768\, b^{\frac{19}{2}}}{a^9} - \frac{109395 \left(ax^{\frac{1}{3}} + b\right)^{\frac{19}{2}} - 978120 \left(ax^{\frac{1}{3}} + b\right)^{\frac{17}{2}} b + 3879876 \left(ax^{\frac{1}{3}} + b\right)^{\frac{15}{2}} b^2 - 8953560 \left(ax^{\frac{1}{3}} + b\right)}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/692835*b*(32768*b^(19/2)/a^9 - (109395*(a*x^(1/3) + b)^(19/2) - 978120*(
a*x^(1/3) + b)^(17/2)*b + 3879876*(a*x^(1/3) + b)^(15/2)*b^2 - 8953560*(a*x
^(1/3) + b)^(13/2)*b^3 + 13226850*(a*x^(1/3) + b)^(11/2)*b^4 - 12932920*(a*
x^(1/3) + b)^(9/2)*b^5 + 8314020*(a*x^(1/3) + b)^(7/2)*b^6 - 3325608*(a*x^(
1/3) + b)^(5/2)*b^7 + 692835*(a*x^(1/3) + b)^(3/2)*b^8)/a^9) + 2/1616615*a*
(65536*b^(21/2)/a^10 + (230945*(a*x^(1/3) + b)^(21/2) - 2297295*(a*x^(1/3)
+ b)^(19/2)*b + 10270260*(a*x^(1/3) + b)^(17/2)*b^2 - 27159132*(a*x^(1/3) +
 b)^(15/2)*b^3 + 47006190*(a*x^(1/3) + b)^(13/2)*b^4 - 55552770*(a*x^(1/3)
+ b)^(11/2)*b^5 + 45265220*(a*x^(1/3) + b)^(9/2)*b^6 - 24942060*(a*x^(1/3)
+ b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 1616615*(a*x^(1/3) + b
)^(3/2)*b^9)/a^10)
```

## 3.178 $\int \left(bx^{2/3}+ax\right)^{3/2} \, dx$

**Optimal**. Leaf size=169

$$-\frac{512b^5\left(ax+bx^{2/3}\right)^{5/2}}{15015a^6x^{5/3}}+\frac{256b^4\left(ax+bx^{2/3}\right)^{5/2}}{3003a^5x^{4/3}}-\frac{64b^3\left(ax+bx^{2/3}\right)^{5/2}}{429a^4x}+\frac{32b^2\left(ax+bx^{2/3}\right)^{5/2}}{143a^3x^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{5/2}}{13a^2\sqrt[3]{x}}+\frac{2}{}$$

```
[Out] (2*(b*x^(2/3) + a*x)^(5/2))/(5*a) - (512*b^5*(b*x^(2/3) + a*x)^(5/2))/(1501
5*a^6*x^(5/3)) + (256*b^4*(b*x^(2/3) + a*x)^(5/2))/(3003*a^5*x^(4/3)) - (64
*b^3*(b*x^(2/3) + a*x)^(5/2))/(429*a^4*x) + (32*b^2*(b*x^(2/3) + a*x)^(5/2)
)/(143*a^3*x^(2/3)) - (4*b*(b*x^(2/3) + a*x)^(5/2))/(13*a^2*x^(1/3))
```

---

**Rubi [A]** time = 0.249476, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2002, 2016, 2014}

$$-\frac{512b^5\left(ax+bx^{2/3}\right)^{5/2}}{15015a^6x^{5/3}}+\frac{256b^4\left(ax+bx^{2/3}\right)^{5/2}}{3003a^5x^{4/3}}-\frac{64b^3\left(ax+bx^{2/3}\right)^{5/2}}{429a^4x}+\frac{32b^2\left(ax+bx^{2/3}\right)^{5/2}}{143a^3x^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{5/2}}{13a^2\sqrt[3]{x}}+\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(5/2))/(5*a) - (512*b^5*(b*x^(2/3) + a*x)^(5/2))/(1501
5*a^6*x^(5/3)) + (256*b^4*(b*x^(2/3) + a*x)^(5/2))/(3003*a^5*x^(4/3)) - (64
*b^3*(b*x^(2/3) + a*x)^(5/2))/(429*a^4*x) + (32*b^2*(b*x^(2/3) + a*x)^(5/2)
)/(143*a^3*x^(2/3)) - (4*b*(b*x^(2/3) + a*x)^(5/2))/(13*a^2*x^(1/3))
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \left(bx^{2/3}+ax\right)^{3/2}dx &= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a}-\frac{(2b)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{\sqrt[3]{x}}dx}{3a} \\
&= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a}-\frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}}+\frac{\left(16b^2\right)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{2/3}}dx}{39a^2} \\
&= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a}+\frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}}-\frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}}-\frac{\left(32b^3\right)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}dx}{143a^3} \\
&= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a}-\frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x}+\frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}}-\frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}}+ \\
&= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a}+\frac{256b^4\left(bx^{2/3}+ax\right)^{5/2}}{3003a^5x^{4/3}}-\frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x}+\frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}}- \\
&= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a}-\frac{512b^5\left(bx^{2/3}+ax\right)^{5/2}}{15015a^6x^{5/3}}+\frac{256b^4\left(bx^{2/3}+ax\right)^{5/2}}{3003a^5x^{4/3}}-\frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x}+
\end{aligned}
$$

**Mathematica [A]** time = 0.0578055, size = 98, normalized size = 0.58

$$\frac{2\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\left(-1120a^2b^3x^{2/3}+1680a^3b^2x-2310a^4bx^{4/3}+3003a^5x^{5/3}+640ab^4\sqrt[3]{x}-256b^5\right)}{15015a^6\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*(-256*b^5 + 640*a*b^4*x^(1/3) -
1120*a^2*b^3*x^(2/3) + 1680*a^3*b^2*x - 2310*a^4*b*x^(4/3) + 3003*a^5*x^(5/
3)))/(15015*a^6*x^(1/3))
```

---

**Maple [A]** time = 0.003, size = 79, normalized size = 0.5

$$\frac{2}{15015\,xa^6}\left(bx^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\sqrt[3]{x}\right)\left(3003\,x^{5/3}a^5-2310\,x^{4/3}a^4b+1680\,xa^3b^2-1120\,x^{2/3}a^2b^3+640\,\sqrt[3]{x}ab^4-256\,b^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(3/2),x)
```

```
[Out] 2/15015*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(3003*x^(5/3)*a^5-2310*x^(4/3)*
a^4*b+1680*x*a^3*b^2-1120*x^(2/3)*a^2*b^3+640*x^(1/3)*a*b^4-256*b^5)/x/a^6
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2), x)
```

---

**Giac [A]** time = 1.22769, size = 281, normalized size = 1.66

$$\frac{2}{3003}b\left(\frac{256\,b^{\frac{13}{2}}}{a^6} + \frac{693\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}} - 4095\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b + 10010\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^2 - 12870\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^3 + 9009\left(ax^{\frac{1}{3}}+\right.}{a^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3003*b*(256*b^(13/2)/a^6 + (693*(a*x^(1/3) + b)^(13/2) - 4095*(a*x^(1/3)
+ b)^(11/2)*b + 10010*(a*x^(1/3) + b)^(9/2)*b^2 - 12870*(a*x^(1/3) + b)^(7/
2)*b^3 + 9009*(a*x^(1/3) + b)^(5/2)*b^4 - 3003*(a*x^(1/3) + b)^(3/2)*b^5)/a
^6) - 2/15015*a*(1024*b^(15/2)/a^7 - (3003*(a*x^(1/3) + b)^(15/2) - 20790*(
a*x^(1/3) + b)^(13/2)*b + 61425*(a*x^(1/3) + b)^(11/2)*b^2 - 100100*(a*x^(1
/3) + b)^(9/2)*b^3 + 96525*(a*x^(1/3) + b)^(7/2)*b^4 - 54054*(a*x^(1/3) + b
)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6)/a^7)
```

## 3.179 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx$

**Optimal**. Leaf size=84

$$\frac{16b^2\left(ax+bx^{2/3}\right)^{5/2}}{105a^3x^{5/3}}-\frac{8b\left(ax+bx^{2/3}\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}$$

```
[Out] (16*b^2*(b*x^(2/3) + a*x)^(5/2))/(105*a^3*x^(5/3)) - (8*b*(b*x^(2/3) + a*x)
^(5/2))/(21*a^2*x^(4/3)) + (2*(b*x^(2/3) + a*x)^(5/2))/(3*a*x)
```

---

**Rubi [A]** time = 0.138779, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{16b^2\left(ax+bx^{2/3}\right)^{5/2}}{105a^3x^{5/3}}-\frac{8b\left(ax+bx^{2/3}\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x,x]
```

```
[Out] (16*b^2*(b*x^(2/3) + a*x)^(5/2))/(105*a^3*x^(5/3)) - (8*b*(b*x^(2/3) + a*x)
^(5/2))/(21*a^2*x^(4/3)) + (2*(b*x^(2/3) + a*x)^(5/2))/(3*a*x)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx&=\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}-\frac{(4b)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{4/3}}\,dx}{9a}\\&=-\frac{8b\left(bx^{2/3}+ax\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}+\frac{\left(8b^2\right)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{5/3}}\,dx}{63a^2}\\&=\frac{16b^2\left(bx^{2/3}+ax\right)^{5/2}}{105a^3x^{5/3}}-\frac{8b\left(bx^{2/3}+ax\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}\end{aligned}$$

**Mathematica [A]** time = 0.0579474, size = 63, normalized size = 0.75

$$\frac{2\left(a\sqrt[3]{x}+b\right)^2\left(35a^2x^{2/3}-20ab\sqrt[3]{x}+8b^2\right)\sqrt{ax+bx^{2/3}}}{105a^3\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x,x]
```

```
[Out] (2*(b + a*x^(1/3))^2*(8*b^2 - 20*a*b*x^(1/3) + 35*a^2*x^(2/3))*Sqrt[b*x^(2/
3) + a*x]])/(105*a^3*x^(1/3))
```

---

**Maple [A]** time = 0.004, size = 48, normalized size = 0.6

$$\frac{2}{105\,xa^3}\left(bx^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(b+a\sqrt[3]{x}\right)\left(35\,x^{2/3}a^2-20\,\sqrt[3]{x}ab+8\,b^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x,x)
```

```
[Out] 2/105*(b*x^(2/3)+a*x)^(3/2)*(b+a*x^(1/3))*(35*x^(2/3)*a^2-20*x^(1/3)*a*b+8*
b^2)/x/a^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x, x)
```

---

**Giac [A]** time = 1.17726, size = 155, normalized size = 1.85

$$-\frac{16\,b^{\frac{9}{2}}}{105\,a^{3}}+\frac{2\left(\frac{3\left(15\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-42\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{2}\right)b}{a^{2}}+\frac{35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}-135\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b+189\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{2}-105\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{3}}{a^{2}}\right)}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -16/105*b^(9/2)/a^3 + 2/105*(3*(15*(a*x^(1/3) + b)^(7/2) - 42*(a*x^(1/3) +
b)^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2)*b/a^2 + (35*(a*x^(1/3) + b)^(9/2
) - 135*(a*x^(1/3) + b)^(7/2)*b + 189*(a*x^(1/3) + b)^(5/2)*b^2 - 105*(a*x^
(1/3) + b)^(3/2)*b^3)/a^2)/a
```

## 3.180 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx$

**Optimal**. Leaf size=78

$$-6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)+\frac{6b\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

```
[Out] (6*b*Sqrt[b*x^(2/3) + a*x])/x^(1/3) + (2*(b*x^(2/3) + a*x)^(3/2))/x - 6*b^(
3/2)*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]]
```

---

**Rubi [A]** time = 0.136998, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2021, 2029, 206}

$$-6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)+\frac{6b\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] (6*b*Sqrt[b*x^(2/3) + a*x])/x^(1/3) + (2*(b*x^(2/3) + a*x)^(3/2))/x - 6*b^(
3/2)*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}+b\int \frac{\sqrt{bx^{2/3}+ax}}{x^{4/3}}\,dx\\ &= \frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}+b^2\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx\\ &= \frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}-\left(6b^2\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)\\ &= \frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}-6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)\end{aligned}$$

**Mathematica [A]** time = 0.0689394, size = 88, normalized size = 1.13

$$\frac{2\sqrt{ax+bx^{2/3}}\left(\sqrt{a\sqrt[3]{x}+b}\left(a\sqrt[3]{x}+4b\right)-3b^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sqrt[3]{x}+b}}{\sqrt{b}}\right)\right)}{\sqrt[3]{x}\sqrt{a\sqrt[3]{x}+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(Sqrt[b + a*x^(1/3)]*(4*b + a*x^(1/3)) - 3*b^(3/2)
*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]]))/(Sqrt[b + a*x^(1/3)]*x^(1/3))
```

---

**Maple [A]** time = 0.004, size = 69, normalized size = 0.9

$$-2\,\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x\left(b+a\sqrt[3]{x}\right)^{3/2}}\left(3\,b^{3/2}\mathrm{Artanh}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)-\left(b+a\sqrt[3]{x}\right)^{3/2}-3\,b\sqrt{b+a\sqrt[3]{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x^2,x)
```

```
[Out] -2*(b*x^(2/3)+a*x)^(3/2)*(3*b^(3/2)*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))-(b
+a*x^(1/3))^(3/2)-3*b*(b+a*x^(1/3))^(1/2))/x/(b+a*x^(1/3))^(3/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^2, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**2,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**2, x)
```

---

**Giac [A]** time = 1.21538, size = 112, normalized size = 1.44

$$\frac{6\,b^2 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + 2\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}} + 6\sqrt{ax^{\frac{1}{3}} + b}\,b - \frac{2\left(3\,b^2 \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + 4\sqrt{-b}\,b^{\frac{3}{2}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 6*b^2*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/sqrt(-b) + 2*(a*x^(1/3) + b)^(3/
2) + 6*sqrt(a*x^(1/3) + b)*b - 2*(3*b^2*arctan(sqrt(b)/sqrt(-b)) + 4*sqrt(-
b)*b^(3/2))/sqrt(-b)
```

## 3.181 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx$

**Optimal**. Leaf size=113

$$\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{3/2}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{8bx^{2/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4x}-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(4*x) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(8*b*x^(
2/3)) - (b*x^(2/3) + a*x)^(3/2)/x^2 + (3*a^3*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt
[b*x^(2/3) + a*x]])/(8*b^(3/2))
```

---

**Rubi [A]** time = 0.183797, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2029, 206}

$$\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{3/2}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{8bx^{2/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4x}-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(4*x) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(8*b*x^(
2/3)) - (b*x^(2/3) + a*x)^(3/2)/x^2 + (3*a^3*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt
[b*x^(2/3) + a*x]])/(8*b^(3/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx &= -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{1}{2}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{1}{8}a^2\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}-\frac{a^3\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{16b}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{\left(3a^3\right)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{8b}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{8b^{3/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0525169, size = 61, normalized size = 0.54

$$
\frac{6a^3\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{5b^4\sqrt[3]{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (6*a^3*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 4, 7/
2, 1 + (a*x^(1/3))/b])/(5*b^4*x^(1/3))
```

---

**Maple [A]** time = 0.01, size = 93, normalized size = 0.8

$$
\frac{1}{8x^2}\left(bx^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(3\,b^{7/2}\sqrt{b+a\sqrt[3]{x}}-8\,b^{5/2}\left(b+a\sqrt[3]{x}\right)^{3/2}-3\,b^{3/2}\left(b+a\sqrt[3]{x}\right)^{5/2}+3\operatorname{Artanh}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)ba^3x\right)b^{-\frac{5}{2}}\left(b+\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x^3,x)
```

```
[Out] 1/8*(b*x^(2/3)+a*x)^(3/2)*(3*b^(7/2)*(b+a*x^(1/3))^(1/2)-8*b^(5/2)*(b+a*x^(
1/3))^(3/2)-3*b^(3/2)*(b+a*x^(1/3))^(5/2)+3*arctanh((b+a*x^(1/3))^(1/2)/b^(
1/2))*b*a^3*x)/x^2/(b+a*x^(1/3))^(3/2)/b^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^3, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**3,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**3, x)
```

---

**Giac [A]** time = 1.24507, size = 124, normalized size = 1.1

$$-\frac{\frac{3\,a^4\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{3\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^4+8\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^4b-3\sqrt{ax^{\frac{1}{3}}+b}\,a^4b^2}{a^3bx}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*a^4*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + (3*(a*x^(1/
3) + b)^(5/2)*a^4 + 8*(a*x^(1/3) + b)^(3/2)*a^4*b - 3*sqrt(a*x^(1/3) + b)*a
^4*b^2)/(a^3*b*x))/a
```

## 3.182 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx$

**Optimal**. Leaf size=203

$$\frac{21a^5\sqrt{ax+bx^{2/3}}}{512b^4x^{2/3}}-\frac{7a^4\sqrt{ax+bx^{2/3}}}{256b^3x}+\frac{7a^3\sqrt{ax+bx^{2/3}}}{320b^2x^{4/3}}-\frac{21a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{9/2}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{160bx^{5/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{20x^2}$$

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(20*x^2) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(160*
b*x^(5/3)) + (7*a^3*Sqrt[b*x^(2/3) + a*x])/(320*b^2*x^(4/3)) - (7*a^4*Sqrt[
b*x^(2/3) + a*x])/(256*b^3*x) + (21*a^5*Sqrt[b*x^(2/3) + a*x])/(512*b^4*x^(
2/3)) - (b*x^(2/3) + a*x)^(3/2)/(2*x^3) - (21*a^6*ArcTanh[(Sqrt[b]*x^(1/3))
/Sqrt[b*x^(2/3) + a*x]])/(512*b^(9/2))
```

---

**Rubi [A]** time = 0.34037, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2029, 206}

$$\frac{21a^5\sqrt{ax+bx^{2/3}}}{512b^4x^{2/3}}-\frac{7a^4\sqrt{ax+bx^{2/3}}}{256b^3x}+\frac{7a^3\sqrt{ax+bx^{2/3}}}{320b^2x^{4/3}}-\frac{21a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{9/2}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{160bx^{5/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{20x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(20*x^2) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(160*
b*x^(5/3)) + (7*a^3*Sqrt[b*x^(2/3) + a*x])/(320*b^2*x^(4/3)) - (7*a^4*Sqrt[
b*x^(2/3) + a*x])/(256*b^3*x) + (21*a^5*Sqrt[b*x^(2/3) + a*x])/(512*b^4*x^(
2/3)) - (b*x^(2/3) + a*x)^(3/2)/(2*x^3) - (21*a^6*ArcTanh[(Sqrt[b]*x^(1/3))
/Sqrt[b*x^(2/3) + a*x]])/(512*b^(9/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx &= -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}+\frac{1}{4}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}+\frac{1}{40}a^2\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}-\frac{\left(7a^3\right)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{320b}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}+\frac{\left(7a^4\right)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{384b^2}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}-(7a^5\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x}+\frac{21a^5\sqrt{bx^{2/3}+ax}}{512b^4x^{2/3}}-(b\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x}+\frac{21a^5\sqrt{bx^{2/3}+ax}}{512b^4x^{2/3}}-(b\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x}+\frac{21a^5\sqrt{bx^{2/3}+ax}}{512b^4x^{2/3}}-(b
\end{aligned}$$

**Mathematica [C]** time = 0.0463573, size = 61, normalized size = 0.3

$$-\frac{6a^6\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{5}{2},7;\frac{7}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{5b^7\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-6*a^6*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 7, 7
/2, 1 + (a*x^(1/3))/b])/(5*b^7*x^(1/3))
```

---

**Maple [A]** time = 0.012, size = 139, normalized size = 0.7

$$\frac{1}{2560\,x^3}\left(bx^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(105\,b^{9/2}\left(b+a\sqrt[3]{x}\right)^{11/2}-595\,b^{11/2}\left(b+a\sqrt[3]{x}\right)^{9/2}+1386\,b^{13/2}\left(b+a\sqrt[3]{x}\right)^{7/2}-1686\,b^{15/2}\left(b+a\sqrt[3]{x}\right)^{5/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x^4,x)
```

```
[Out] 1/2560*(b*x^(2/3)+a*x)^(3/2)*(105*b^(9/2)*(b+a*x^(1/3))^(11/2)-595*b^(11/2)
*(b+a*x^(1/3))^(9/2)+1386*b^(13/2)*(b+a*x^(1/3))^(7/2)-1686*b^(15/2)*(b+a*x
```

```
^(1/3))^(5/2)-595*b^(17/2)*(b+a*x^(1/3))^(3/2)+105*b^(19/2)*(b+a*x^(1/3))^(
1/2)-105*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^4*a^6*x^2)/x^3/(b+a*x^(1/3)
)^(3/2)/b^(17/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^4, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**4,x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.25095, size = 193, normalized size = 0.95

$$\frac{\frac{105\,a^7\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4}+\frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^7-595\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^7b+1386\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^7b^2-1686\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^7b^3-595\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^7b^4+105\sqrt{ax^{\frac{1}{3}}+b}\,a^7b^5}{a^6b^4x^2}}{2560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2560*(105*a^7*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*
(a*x^(1/3) + b)^(11/2)*a^7 - 595*(a*x^(1/3) + b)^(9/2)*a^7*b + 1386*(a*x^(1
/3) + b)^(7/2)*a^7*b^2 - 1686*(a*x^(1/3) + b)^(5/2)*a^7*b^3 - 595*(a*x^(1/3
) + b)^(3/2)*a^7*b^4 + 105*sqrt(a*x^(1/3) + b)*a^7*b^5)/(a^6*b^4*x^2))/a
```

## 3.183 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx$

**Optimal**. Leaf size=291

$$-\frac{429a^8\sqrt{ax+bx^{2/3}}}{32768b^7x^{2/3}}+\frac{143a^7\sqrt{ax+bx^{2/3}}}{16384b^6x}-\frac{143a^6\sqrt{ax+bx^{2/3}}}{20480b^5x^{4/3}}+\frac{429a^5\sqrt{ax+bx^{2/3}}}{71680b^4x^{5/3}}-\frac{143a^4\sqrt{ax+bx^{2/3}}}{26880b^3x^2}+\frac{13a^3\sqrt{ax+bx}}{2688b^2x^{7/3}}$$

```
[Out] -(a*Sqrt[b*x^(2/3) + a*x])/(16*x^3) - (a^2*Sqrt[b*x^(2/3) + a*x])/(224*b*x^
(8/3)) + (13*a^3*Sqrt[b*x^(2/3) + a*x])/(2688*b^2*x^(7/3)) - (143*a^4*Sqrt[
b*x^(2/3) + a*x])/(26880*b^3*x^2) + (429*a^5*Sqrt[b*x^(2/3) + a*x])/(71680*
b^4*x^(5/3)) - (143*a^6*Sqrt[b*x^(2/3) + a*x])/(20480*b^5*x^(4/3)) + (143*a
^7*Sqrt[b*x^(2/3) + a*x])/(16384*b^6*x) - (429*a^8*Sqrt[b*x^(2/3) + a*x])/(
32768*b^7*x^(2/3)) - (b*x^(2/3) + a*x)^(3/2)/(3*x^4) + (429*a^9*ArcTanh[(Sq
rt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(32768*b^(15/2))
```

---

**Rubi [A]** time = 0.521562, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2029, 206}

$$-\frac{429a^8\sqrt{ax+bx^{2/3}}}{32768b^7x^{2/3}}+\frac{143a^7\sqrt{ax+bx^{2/3}}}{16384b^6x}-\frac{143a^6\sqrt{ax+bx^{2/3}}}{20480b^5x^{4/3}}+\frac{429a^5\sqrt{ax+bx^{2/3}}}{71680b^4x^{5/3}}-\frac{143a^4\sqrt{ax+bx^{2/3}}}{26880b^3x^2}+\frac{13a^3\sqrt{ax+bx}}{2688b^2x^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] -(a*Sqrt[b*x^(2/3) + a*x])/(16*x^3) - (a^2*Sqrt[b*x^(2/3) + a*x])/(224*b*x^
(8/3)) + (13*a^3*Sqrt[b*x^(2/3) + a*x])/(2688*b^2*x^(7/3)) - (143*a^4*Sqrt[
b*x^(2/3) + a*x])/(26880*b^3*x^2) + (429*a^5*Sqrt[b*x^(2/3) + a*x])/(71680*
b^4*x^(5/3)) - (143*a^6*Sqrt[b*x^(2/3) + a*x])/(20480*b^5*x^(4/3)) + (143*a
^7*Sqrt[b*x^(2/3) + a*x])/(16384*b^6*x) - (429*a^8*Sqrt[b*x^(2/3) + a*x])/(
32768*b^7*x^(2/3)) - (b*x^(2/3) + a*x)^(3/2)/(3*x^4) + (429*a^9*ArcTanh[(Sq
rt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(32768*b^(15/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx &= -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{1}{6}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{1}{96}a^2\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}-\frac{\left(13a^3\right)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{1344b} \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{\left(143a^4\right)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+a}}}{16128b^2} \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}- \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}} \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}} \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}} \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}} \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}} \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{bx^{2/3}+ax}}{71680b^4x^{5/3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0501321, size = 61, normalized size = 0.21

$$
\frac{6a^9\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{5}{2},10;\frac{7}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{5b^{10}\sqrt[3]{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] (6*a^9*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 10, 7
/2, 1 + (a*x^(1/3))/b])/(5*b^10*x^(1/3))
```

**Maple [A]** time = 0.014, size = 181, normalized size = 0.6

$$\frac{1}{3440640\,x^{4}}\left(bx^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(45045\,b^{\frac{31}{2}}\sqrt{b+a\sqrt[3]{x}}-390390\,b^{\frac{29}{2}}\left(b+a\sqrt[3]{x}\right)^{3/2}-2633274\,b^{\frac{27}{2}}\left(b+a\sqrt[3]{x}\right)^{5/2}+4349826\,b^{\frac{25}{2}}\left(b-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x^5,x)
```

```
[Out] 1/3440640*(b*x^(2/3)+a*x)^(3/2)*(45045*b^(31/2)*(b+a*x^(1/3))^(1/2)-390390*
b^(29/2)*(b+a*x^(1/3))^(3/2)-2633274*b^(27/2)*(b+a*x^(1/3))^(5/2)+4349826*b
^(25/2)*(b+a*x^(1/3))^(7/2)-4685824*b^(23/2)*(b+a*x^(1/3))^(9/2)+3317886*b^
(21/2)*(b+a*x^(1/3))^(11/2)-1495494*b^(19/2)*(b+a*x^(1/3))^(13/2)+390390*b^
(17/2)*(b+a*x^(1/3))^(15/2)-45045*b^(15/2)*(b+a*x^(1/3))^(17/2)+45045*arcta
nh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^7*a^9*x^3)/x^4/(b+a*x^(1/3))^(3/2)/b^(29/
2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^5, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**5,x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.372, size = 262, normalized size = 0.9

$$-\frac{\frac{45045\,a^{10}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{7}}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{10}-390390\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{10}b+1495494\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{10}b^{2}-3317886\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{10}b^{3}+4685824\left(ax^{\frac{1}{3}}+b\right)}{a^{9}b^{7}x^{3}}}{3440640\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/3440640*(45045*a^10*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7)
+ (45045*(a*x^(1/3) + b)^(17/2)*a^10 - 390390*(a*x^(1/3) + b)^(15/2)*a^10*b
 + 1495494*(a*x^(1/3) + b)^(13/2)*a^10*b^2 - 3317886*(a*x^(1/3) + b)^(11/2)
*a^10*b^3 + 4685824*(a*x^(1/3) + b)^(9/2)*a^10*b^4 - 4349826*(a*x^(1/3) + b
)^(7/2)*a^10*b^5 + 2633274*(a*x^(1/3) + b)^(5/2)*a^10*b^6 + 390390*(a*x^(1/
3) + b)^(3/2)*a^10*b^7 - 45045*sqrt(a*x^(1/3) + b)*a^10*b^8)/(a^9*b^7*x^3))
/a
```

## 3.184 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

**Optimal**. Leaf size=379

$$\frac{12597a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{10}x^{2/3}}-\frac{4199a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^9x}+\frac{4199a^9\sqrt{ax+bx^{2/3}}}{1310720b^8x^{4/3}}-\frac{12597a^8\sqrt{ax+bx^{2/3}}}{4587520b^7x^{5/3}}+\frac{4199a^7\sqrt{ax+bx^{2/3}}}{1720320b^6x^2}-$$

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(88*x^4) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(1760
*b*x^(11/3)) + (19*a^3*Sqrt[b*x^(2/3) + a*x])/(10560*b^2*x^(10/3)) - (323*a
^4*Sqrt[b*x^(2/3) + a*x])/(168960*b^3*x^3) + (323*a^5*Sqrt[b*x^(2/3) + a*x]
)/(157696*b^4*x^(8/3)) - (4199*a^6*Sqrt[b*x^(2/3) + a*x])/(1892352*b^5*x^(7
/3)) + (4199*a^7*Sqrt[b*x^(2/3) + a*x])/(1720320*b^6*x^2) - (12597*a^8*Sqrt
[b*x^(2/3) + a*x])/(4587520*b^7*x^(5/3)) + (4199*a^9*Sqrt[b*x^(2/3) + a*x])
/(1310720*b^8*x^(4/3)) - (4199*a^10*Sqrt[b*x^(2/3) + a*x])/(1048576*b^9*x)
+ (12597*a^11*Sqrt[b*x^(2/3) + a*x])/(2097152*b^10*x^(2/3)) - (b*x^(2/3) +
a*x)^(3/2)/(4*x^5) - (12597*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) +
 a*x]])/(2097152*b^(21/2))
```

---

**Rubi [A]** time = 0.718415, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2029, 206}

$$\frac{12597a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{10}x^{2/3}}-\frac{4199a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^9x}+\frac{4199a^9\sqrt{ax+bx^{2/3}}}{1310720b^8x^{4/3}}-\frac{12597a^8\sqrt{ax+bx^{2/3}}}{4587520b^7x^{5/3}}+\frac{4199a^7\sqrt{ax+bx^{2/3}}}{1720320b^6x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(88*x^4) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(1760
*b*x^(11/3)) + (19*a^3*Sqrt[b*x^(2/3) + a*x])/(10560*b^2*x^(10/3)) - (323*a
^4*Sqrt[b*x^(2/3) + a*x])/(168960*b^3*x^3) + (323*a^5*Sqrt[b*x^(2/3) + a*x]
)/(157696*b^4*x^(8/3)) - (4199*a^6*Sqrt[b*x^(2/3) + a*x])/(1892352*b^5*x^(7
/3)) + (4199*a^7*Sqrt[b*x^(2/3) + a*x])/(1720320*b^6*x^2) - (12597*a^8*Sqrt
[b*x^(2/3) + a*x])/(4587520*b^7*x^(5/3)) + (4199*a^9*Sqrt[b*x^(2/3) + a*x])
/(1310720*b^8*x^(4/3)) - (4199*a^10*Sqrt[b*x^(2/3) + a*x])/(1048576*b^9*x)
+ (12597*a^11*Sqrt[b*x^(2/3) + a*x])/(2097152*b^10*x^(2/3)) - (b*x^(2/3) +
a*x)^(3/2)/(4*x^5) - (12597*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) +
 a*x]])/(2097152*b^(21/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
```

```
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx &= -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}+\frac{1}{8}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}+\frac{1}{176}a^2\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}-\frac{\left(19a^3\right)\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{3520b}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}+\frac{\left(323a^4\right)\int \frac{1}{x^{10/3}\sqrt{bx^2}}}{63360b^2}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{bx^{2/3}+}}{157696b^4x^{8/}}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{bx^{2/3}+}}{157696b^4x^{8/}}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{bx^{2/3}+}}{157696b^4x^{8/}}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{bx^{2/3}+}}{157696b^4x^{8/}}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{bx^{2/3}+}}{157696b^4x^{8/}}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{bx^{2/3}+}}{157696b^4x^{8/}}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{bx^{2/3}+}}{157696b^4x^{8/}}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{bx^{2/3}+}}{157696b^4x^{8/}}\\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\sqrt{bx^{2/3}+}}{157696b^4x^{8/}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0609208, size = 61, normalized size = 0.16

$$-\frac{6a^{12}\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{5}{2},13;\frac{7}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{5b^{13}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-6*a^12*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 13,
 7/2, 1 + (a*x^(1/3))/b])/(5*b^13*x^(1/3))
```

---

**Maple [A]** time = 0.016, size = 223, normalized size = 0.6

$$\frac{1}{2422210560\,x^5}\left(bx^{\frac{2}{3}}+ax\right)^{\frac{3}{2}}\left(14549535\left(b+a\sqrt[3]{x}\right)^{23/2}b^{21/2}-169744575\left(b+a\sqrt[3]{x}\right)^{21/2}b^{23/2}+904981077\left(b+a\sqrt[3]{x}\right)^{19/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(2/3)+a*x)^(3/2)/x^6,x)
```

```
[Out] 1/2422210560*(b*x^(2/3)+a*x)^(3/2)*(14549535*(b+a*x^(1/3))^(23/2)*b^(21/2)-
169744575*(b+a*x^(1/3))^(21/2)*b^(23/2)+904981077*(b+a*x^(1/3))^(19/2)*b^(2
5/2)-2913648309*(b+a*x^(1/3))^(17/2)*b^(27/2)+6303782342*(b+a*x^(1/3))^(15/
2)*b^(29/2)-9643633350*(b+a*x^(1/3))^(13/2)*b^(31/2)+10677769530*(b+a*x^(1/
3))^(11/2)*b^(33/2)-8598579770*(b+a*x^(1/3))^(9/2)*b^(35/2)+4975837515*(b+a
*x^(1/3))^(7/2)*b^(37/2)-2001671595*(b+a*x^(1/3))^(5/2)*b^(39/2)-169744575*
(b+a*x^(1/3))^(3/2)*b^(41/2)+14549535*(b+a*x^(1/3))^(1/2)*b^(43/2)-14549535
*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b^10*a^12*x^4)/x^5/(b+a*x^(1/3))^(3/2
)/b^(41/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^6, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**6,x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.44672, size = 331, normalized size = 0.87

$$\frac{\frac{14549535\,a^{13}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{10}}+\frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}a^{13}-169744575\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{13}b+904981077\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{13}b^{2}-2913648309\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{13}b^{3}+63}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/2422210560*(14549535*a^13*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*
b^10) + (14549535*(a*x^(1/3) + b)^(23/2)*a^13 - 169744575*(a*x^(1/3) + b)^(
21/2)*a^13*b + 904981077*(a*x^(1/3) + b)^(19/2)*a^13*b^2 - 2913648309*(a*x^
(1/3) + b)^(17/2)*a^13*b^3 + 6303782342*(a*x^(1/3) + b)^(15/2)*a^13*b^4 - 9
643633350*(a*x^(1/3) + b)^(13/2)*a^13*b^5 + 10677769530*(a*x^(1/3) + b)^(11
/2)*a^13*b^6 - 8598579770*(a*x^(1/3) + b)^(9/2)*a^13*b^7 + 4975837515*(a*x^
(1/3) + b)^(7/2)*a^13*b^8 - 2001671595*(a*x^(1/3) + b)^(5/2)*a^13*b^9 - 169
744575*(a*x^(1/3) + b)^(3/2)*a^13*b^10 + 14549535*sqrt(a*x^(1/3) + b)*a^13*
b^11)/(a^12*b^10*x^4))/a
```

## 3.185 $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

**Optimal**. Leaf size=401

$$-\frac{16777216b^{13}\sqrt{ax+bx^{2/3}}}{11700675a^{14}\sqrt[3]{x}}+\frac{8388608b^{12}\sqrt{ax+bx^{2/3}}}{11700675a^{13}}-\frac{2097152b^{11}\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{3900225a^{12}}+\frac{1048576b^{10}x^{2/3}\sqrt{ax+bx^{2/3}}}{2340135a^{11}}-131$$

```
[Out] (8388608*b^12*Sqrt[b*x^(2/3) + a*x])/(11700675*a^13) - (16777216*b^13*Sqrt[
b*x^(2/3) + a*x])/(11700675*a^14*x^(1/3)) - (2097152*b^11*x^(1/3)*Sqrt[b*x^
(2/3) + a*x])/(3900225*a^12) + (1048576*b^10*x^(2/3)*Sqrt[b*x^(2/3) + a*x])
/(2340135*a^11) - (131072*b^9*x*Sqrt[b*x^(2/3) + a*x])/(334305*a^10) + (655
36*b^8*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(185725*a^9) - (180224*b^7*x^(5/3)*Sq
rt[b*x^(2/3) + a*x])/(557175*a^8) + (1171456*b^6*x^2*Sqrt[b*x^(2/3) + a*x])
/(3900225*a^7) - (73216*b^5*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(260015*a^6) + (
36608*b^4*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(137655*a^5) - (9152*b^3*x^3*Sqrt[
b*x^(2/3) + a*x])/(36225*a^4) + (416*b^2*x^(10/3)*Sqrt[b*x^(2/3) + a*x])/(1
725*a^3) - (52*b*x^(11/3)*Sqrt[b*x^(2/3) + a*x])/(225*a^2) + (2*x^4*Sqrt[b*
x^(2/3) + a*x])/(9*a)
```

---

**Rubi [A]** time = 0.728113, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{16777216b^{13}\sqrt{ax+bx^{2/3}}}{11700675a^{14}\sqrt[3]{x}}+\frac{8388608b^{12}\sqrt{ax+bx^{2/3}}}{11700675a^{13}}-\frac{2097152b^{11}\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{3900225a^{12}}+\frac{1048576b^{10}x^{2/3}\sqrt{ax+bx^{2/3}}}{2340135a^{11}}-131$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (8388608*b^12*Sqrt[b*x^(2/3) + a*x])/(11700675*a^13) - (16777216*b^13*Sqrt[
b*x^(2/3) + a*x])/(11700675*a^14*x^(1/3)) - (2097152*b^11*x^(1/3)*Sqrt[b*x^
(2/3) + a*x])/(3900225*a^12) + (1048576*b^10*x^(2/3)*Sqrt[b*x^(2/3) + a*x])
/(2340135*a^11) - (131072*b^9*x*Sqrt[b*x^(2/3) + a*x])/(334305*a^10) + (655
36*b^8*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(185725*a^9) - (180224*b^7*x^(5/3)*Sq
rt[b*x^(2/3) + a*x])/(557175*a^8) + (1171456*b^6*x^2*Sqrt[b*x^(2/3) + a*x])
/(3900225*a^7) - (73216*b^5*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(260015*a^6) + (
36608*b^4*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(137655*a^5) - (9152*b^3*x^3*Sqrt[
b*x^(2/3) + a*x])/(36225*a^4) + (416*b^2*x^(10/3)*Sqrt[b*x^(2/3) + a*x])/(1
725*a^3) - (52*b*x^(11/3)*Sqrt[b*x^(2/3) + a*x])/(225*a^2) + (2*x^4*Sqrt[b*
x^(2/3) + a*x])/(9*a)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
```

```
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

## Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

## Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx &= \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a}-\frac{(26b)\int \frac{x^{11/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{27a}\\
&= -\frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2}+\frac{2x^4\sqrt{bx^{2/3}+ax}}{9a}+\frac{\left(208b^2\right)\int \frac{x^{10/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{225a^2}\\
&= \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3}-\frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2}+\frac{2x^4\sqrt{bx^{2/3}+ax}}{9a}-\frac{\left(4576b^3\right)\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx}{5175a^3}\\
&= -\frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4}+\frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3}-\frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2}+\frac{2x^4\sqrt{bx^{2/3}+ax}}{9a}+\\
&= \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5}-\frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4}+\frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3}-\\
&= -\frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6}+\frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5}-\frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4}+\\
&= \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7}-\frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6}+\frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5}-\\
&= -\frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8}+\frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7}-\frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6}+\\
&= \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9}-\frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8}+\frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7}-\\
&= -\frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}}+\frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9}-\frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8}+\\
&= \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}}-\frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}}+\frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9}-\\
&= -\frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{3900225a^{12}}+\frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}}-\frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}}+\\
&= \frac{8388608b^{12}\sqrt{bx^{2/3}+ax}}{11700675a^{13}}-\frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{3900225a^{12}}+\frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}}-\\
&= \frac{8388608b^{12}\sqrt{bx^{2/3}+ax}}{11700675a^{13}}-\frac{16777216b^{13}\sqrt{bx^{2/3}+ax}}{11700675a^{14}\sqrt[3]{x}}-\frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{3900225a^{12}}+
\end{aligned}
$$

**Mathematica [A]** time = 0.153033, size = 185, normalized size = 0.46

$$2\sqrt{ax+bx^{2/3}}\Big(1410864a^{11}b^2x^{11/3}-1478048a^{10}b^3x^{10/3}+1555840a^9b^4x^3-1647360a^8b^5x^{8/3}+1757184a^7b^6x^{7/3}-$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-8388608*b^13 + 4194304*a*b^12*x^(1/3) - 3145728*
a^2*b^11*x^(2/3) + 2621440*a^3*b^10*x - 2293760*a^4*b^9*x^(4/3) + 2064384*a
^5*b^8*x^(5/3) - 1892352*a^6*b^7*x^2 + 1757184*a^7*b^6*x^(7/3) - 1647360*a^
8*b^5*x^(8/3) + 1555840*a^9*b^4*x^3 - 1478048*a^10*b^3*x^(10/3) + 1410864*a
^11*b^2*x^(11/3) - 1352078*a^12*b*x^4 + 1300075*a^13*x^(13/3)))/(11700675*a
^14*x^(1/3))
```

---

**Maple [A]** time = 0.004, size = 167, normalized size = 0.4

$$\frac{2}{11700675\,a^{14}}\sqrt[3]{x}\left(b+a\sqrt[3]{x}\right)\big(1300075\,x^{13/3}a^{13}-1352078\,x^{4}a^{12}b+1410864\,x^{11/3}a^{11}b^{2}-1478048\,x^{10/3}a^{10}b^{3}+1555840\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^(2/3)+a*x)^(1/2),x)
```

```
[Out] 2/11700675*x^(1/3)*(b+a*x^(1/3))*(1300075*x^(13/3)*a^13-1352078*x^4*a^12*b+
1410864*x^(11/3)*a^11*b^2-1478048*x^(10/3)*a^10*b^3+1555840*x^3*a^9*b^4-164
7360*x^(8/3)*a^8*b^5+1757184*x^(7/3)*a^7*b^6-1892352*x^2*a^6*b^7+2064384*x^
(5/3)*a^5*b^8-2293760*x^(4/3)*a^4*b^9+2621440*x*a^3*b^10-3145728*x^(2/3)*a^
2*b^11+4194304*x^(1/3)*a*b^12-8388608*b^13)/(b*x^(2/3)+a*x)^(1/2)/a^14
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(a*x + b*x^(2/3)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.14363, size = 278, normalized size = 0.69

$$\frac{16777216\, b^{\frac{27}{2}}}{11700675\, a^{14}} + 2\left(1300075\left(ax^{\frac{1}{3}}+b\right)^{\frac{27}{2}} - 18253053\left(ax^{\frac{1}{3}}+b\right)^{\frac{25}{2}} b + 119041650\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}} b^{2} - 478056150\left(ax^{\frac{1}{3}}+b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 16777216/11700675*b^(27/2)/a^14 + 2/11700675*(1300075*(a*x^(1/3) + b)^(27/2
) - 18253053*(a*x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^
2 - 478056150*(a*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2
)*b^4 - 2657429775*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^
(15/2)*b^6 - 4633467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3)
+ b)^(11/2)*b^8 - 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1
/3) + b)^(7/2)*b^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x
^(1/3) + b)^(3/2)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^14
```

## 3.186 $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

**Optimal**. Leaf size=313

$$\frac{524288b^{10}\sqrt{ax+bx^{2/3}}}{323323a^{11}\sqrt[3]{x}}-\frac{262144b^{9}\sqrt{ax+bx^{2/3}}}{323323a^{10}}+\frac{196608b^{8}\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{323323a^{9}}-\frac{163840b^{7}x^{2/3}\sqrt{ax+bx^{2/3}}}{323323a^{8}}+\frac{20480b^{6}x\sqrt{ax}}{46189a}$$

```
[Out] (-262144*b^9*Sqrt[b*x^(2/3) + a*x])/(323323*a^10) + (524288*b^10*Sqrt[b*x^(
2/3) + a*x])/(323323*a^11*x^(1/3)) + (196608*b^8*x^(1/3)*Sqrt[b*x^(2/3) + a
*x])/(323323*a^9) - (163840*b^7*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(323323*a^8)
 + (20480*b^6*x*Sqrt[b*x^(2/3) + a*x])/(46189*a^7) - (18432*b^5*x^(4/3)*Sqr
t[b*x^(2/3) + a*x])/(46189*a^6) + (1536*b^4*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/
(4199*a^5) - (768*b^3*x^2*Sqrt[b*x^(2/3) + a*x])/(2261*a^4) + (720*b^2*x^(7
/3)*Sqrt[b*x^(2/3) + a*x])/(2261*a^3) - (40*b*x^(8/3)*Sqrt[b*x^(2/3) + a*x]
)/(133*a^2) + (2*x^3*Sqrt[b*x^(2/3) + a*x])/(7*a)
```

---

**Rubi [A]** time = 0.53092, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$\frac{524288b^{10}\sqrt{ax+bx^{2/3}}}{323323a^{11}\sqrt[3]{x}}-\frac{262144b^{9}\sqrt{ax+bx^{2/3}}}{323323a^{10}}+\frac{196608b^{8}\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{323323a^{9}}-\frac{163840b^{7}x^{2/3}\sqrt{ax+bx^{2/3}}}{323323a^{8}}+\frac{20480b^{6}x\sqrt{ax}}{46189a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-262144*b^9*Sqrt[b*x^(2/3) + a*x])/(323323*a^10) + (524288*b^10*Sqrt[b*x^(
2/3) + a*x])/(323323*a^11*x^(1/3)) + (196608*b^8*x^(1/3)*Sqrt[b*x^(2/3) + a
*x])/(323323*a^9) - (163840*b^7*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(323323*a^8)
 + (20480*b^6*x*Sqrt[b*x^(2/3) + a*x])/(46189*a^7) - (18432*b^5*x^(4/3)*Sqr
t[b*x^(2/3) + a*x])/(46189*a^6) + (1536*b^4*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/
(4199*a^5) - (768*b^3*x^2*Sqrt[b*x^(2/3) + a*x])/(2261*a^4) + (720*b^2*x^(7
/3)*Sqrt[b*x^(2/3) + a*x])/(2261*a^3) - (40*b*x^(8/3)*Sqrt[b*x^(2/3) + a*x]
)/(133*a^2) + (2*x^3*Sqrt[b*x^(2/3) + a*x])/(7*a)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} - \frac{(20b)\int \frac{x^{8/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a}$$

$$= -\frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} + \frac{\left(120b^2\right)\int \frac{x^{7/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{133a^2}$$

$$= \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} - \frac{\left(1920b^3\right)\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx}{2261a^3}$$

$$= -\frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} + \Big(256$$

$$= \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} +$$

$$= -\frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} + \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+}}{2261a^3}$$

$$= \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} + \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3}+}}{2261a^4}$$

$$= -\frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} + \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} + \frac{1536b^4x^{5/3}\sqrt{b}}{4199a}$$

$$= \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} + \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{b}}{46189a}$$

$$= -\frac{262144b^9\sqrt{bx^{2/3}+ax}}{323323a^{10}} + \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} + \frac{20480b^6x\sqrt{b}}{46189a}$$

$$= -\frac{262144b^9\sqrt{bx^{2/3}+ax}}{323323a^{10}} + \frac{524288b^{10}\sqrt{bx^{2/3}+ax}}{323323a^{11}\sqrt[3]{x}} + \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{b}}{323323a}$$

**Mathematica [A]** time = 0.115059, size = 148, normalized size = 0.47

$$\frac{2\sqrt{ax+bx^{2/3}}\left(51480a^8b^2x^{8/3} - 54912a^7b^3x^{7/3} + 59136a^6b^4x^2 - 64512a^5b^5x^{5/3} + 71680a^4b^6x^{4/3} + 98304a^2b^8x^{2/3} - 81\right.}{323323a^{11}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(262144*b^10 - 131072*a*b^9*x^(1/3) + 98304*a^2*b^
8*x^(2/3) - 81920*a^3*b^7*x + 71680*a^4*b^6*x^(4/3) - 64512*a^5*b^5*x^(5/3)
 + 59136*a^6*b^4*x^2 - 54912*a^7*b^3*x^(7/3) + 51480*a^8*b^2*x^(8/3) - 4862
0*a^9*b*x^3 + 46189*a^10*x^(10/3)))/(323323*a^11*x^(1/3))
```

**Maple [A]** time = 0.003, size = 134, normalized size = 0.4

$$\frac{2}{323323\,a^{11}}\sqrt[3]{x}\left(b+a\sqrt[3]{x}\right)\left(46189\,x^{10/3}a^{10}-48620\,x^{3}a^{9}b+51480\,x^{8/3}a^{8}b^{2}-54912\,x^{7/3}a^{7}b^{3}+59136\,x^{2}a^{6}b^{4}-64512\,x^{5/3}a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(2/3)+a*x)^(1/2),x)
```

```
[Out] 2/323323*x^(1/3)*(b+a*x^(1/3))*(46189*x^(10/3)*a^10-48620*x^3*a^9*b+51480*x
^(8/3)*a^8*b^2-54912*x^(7/3)*a^7*b^3+59136*x^2*a^6*b^4-64512*x^(5/3)*a^5*b^
5+71680*x^(4/3)*a^4*b^6-81920*x*a^3*b^7+98304*x^(2/3)*a^2*b^8-131072*x^(1/3
)*a*b^9+262144*b^10)/(b*x^(2/3)+a*x)^(1/2)/a^11
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(a*x + b*x^(2/3)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a*x + b*x**(2/3)), x)
```

---

**Giac [A]** time = 1.15723, size = 221, normalized size = 0.71

$$-\frac{524288\,b^{\frac{21}{2}}}{323323\,a^{11}} + \frac{2\left(46189\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}} - 510510\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b + 2567565\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^{2} - 7759752\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^{3} + 15\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -524288/323323*b^(21/2)/a^11 + 2/323323*(46189*(a*x^(1/3) + b)^(21/2) - 510
510*(a*x^(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3) + b)^(17/2)*b^2 - 7759752
*(a*x^(1/3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3) + b)^(13/2)*b^4 - 2222110
8*(a*x^(1/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 1662804
0*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233230*(
a*x^(1/3) + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3) + b)*b^10)/a^11
```

## 3.187 $\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx$

**Optimal**. Leaf size=225

$$-\frac{4096b^7\sqrt{ax+bx^{2/3}}}{2145a^8\sqrt[3]{x}}+\frac{2048b^6\sqrt{ax+bx^{2/3}}}{2145a^7}-\frac{512b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^6}+\frac{256b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^5}-\frac{224b^3x\sqrt{ax+bx^{2/3}}}{429a^4}+$$

```
[Out] (2048*b^6*Sqrt[b*x^(2/3) + a*x])/(2145*a^7) - (4096*b^7*Sqrt[b*x^(2/3) + a*
x])/(2145*a^8*x^(1/3)) - (512*b^5*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^6)
+ (256*b^4*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(429*a^5) - (224*b^3*x*Sqrt[b*x^(
2/3) + a*x])/(429*a^4) + (336*b^2*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^3)
- (28*b*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(65*a^2) + (2*x^2*Sqrt[b*x^(2/3) + a
*x])/(5*a)
```

---

**Rubi [A]** time = 0.34578, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{4096b^7\sqrt{ax+bx^{2/3}}}{2145a^8\sqrt[3]{x}}+\frac{2048b^6\sqrt{ax+bx^{2/3}}}{2145a^7}-\frac{512b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^6}+\frac{256b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^5}-\frac{224b^3x\sqrt{ax+bx^{2/3}}}{429a^4}+$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2048*b^6*Sqrt[b*x^(2/3) + a*x])/(2145*a^7) - (4096*b^7*Sqrt[b*x^(2/3) + a*
x])/(2145*a^8*x^(1/3)) - (512*b^5*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^6)
+ (256*b^4*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(429*a^5) - (224*b^3*x*Sqrt[b*x^(
2/3) + a*x])/(429*a^4) + (336*b^2*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^3)
- (28*b*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(65*a^2) + (2*x^2*Sqrt[b*x^(2/3) + a
*x])/(5*a)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx &= \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} - \frac{(14b)\int \frac{x^{5/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{15a} \\
&= -\frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} + \frac{\left(56b^2\right)\int \frac{x^{4/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{65a^2} \\
&= \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} - \frac{\left(112b^3\right)\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx}{143a^3} \\
&= -\frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} + (896b \\
&= \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + 2 \\
&= -\frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \\
&= \frac{2048b^6\sqrt{bx^{2/3}+ax}}{2145a^7} - \frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + 33 \\
&= \frac{2048b^6\sqrt{bx^{2/3}+ax}}{2145a^7} - \frac{4096b^7\sqrt{bx^{2/3}+ax}}{2145a^8\sqrt[3]{x}} - \frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - 22
\end{aligned}
$$

**Mathematica [A]** time = 0.0879218, size = 111, normalized size = 0.49

$$
\frac{2\sqrt{ax+bx^{2/3}}\left(504a^5b^2x^{5/3} - 560a^4b^3x^{4/3} - 768a^2b^5x^{2/3} + 640a^3b^4x - 462a^6bx^2 + 429a^7x^{7/3} + 1024ab^6\sqrt[3]{x} - 2048b^7\right)}{2145a^8\sqrt[3]{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-2048*b^7 + 1024*a*b^6*x^(1/3) - 768*a^2*b^5*x^(2
/3) + 640*a^3*b^4*x - 560*a^4*b^3*x^(4/3) + 504*a^5*b^2*x^(5/3) - 462*a^6*b
*x^2 + 429*a^7*x^(7/3)))/(2145*a^8*x^(1/3))
```

---

**Maple [A]** time = 0.005, size = 101, normalized size = 0.5

$$
\frac{2}{2145\,a^8}\sqrt[3]{x}\left(b+a\sqrt[3]{x}\right)\left(429\,x^{7/3}a^7 - 462\,x^2a^6b + 504\,x^{5/3}a^5b^2 - 560\,x^{4/3}a^4b^3 + 640\,xa^3b^4 - 768\,x^{2/3}a^2b^5 + 1024\,\sqrt[3]{x}ab^6
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(2/3)+a*x)^(1/2),x)
```

```
[Out] 2/2145*x^(1/3)*(b+a*x^(1/3))*(429*x^(7/3)*a^7-462*x^2*a^6*b+504*x^(5/3)*a^5
*b^2-560*x^(4/3)*a^4*b^3+640*x*a^3*b^4-768*x^(2/3)*a^2*b^5+1024*x^(1/3)*a*b
^6-2048*b^7)/(b*x^(2/3)+a*x)^(1/2)/a^8
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a*x + b*x^(2/3)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*x + b*x**(2/3)), x)
```

---

**Giac [A]** time = 1.16623, size = 165, normalized size = 0.73

$$\frac{4096\,b^{\frac{15}{2}}}{2145\,a^{8}}+\frac{2\left(429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}-3465\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b+12285\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^{2}-25025\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^{3}+32175\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^{4}\right.}{2145\,a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4096/2145*b^(15/2)/a^8 + 2/2145*(429*(a*x^(1/3) + b)^(15/2) - 3465*(a*x^(1/
3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^2 - 25025*(a*x^(1/3) + b)
^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 27027*(a*x^(1/3) + b)^(5/2)*
b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(a*x^(1/3) + b)*b^7)/a^8
```

# 3.188 $\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx$

**Optimal**. Leaf size=137

$$\frac{256b^4\sqrt{ax+bx^{2/3}}}{105a^5\sqrt[3]{x}}-\frac{128b^3\sqrt{ax+bx^{2/3}}}{105a^4}+\frac{32b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{35a^3}-\frac{16bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^2}+\frac{2x\sqrt{ax+bx^{2/3}}}{3a}$$

```
[Out] (-128*b^3*Sqrt[b*x^(2/3) + a*x])/(105*a^4) + (256*b^4*Sqrt[b*x^(2/3) + a*x]
)/(105*a^5*x^(1/3)) + (32*b^2*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(35*a^3) - (16
*b*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(21*a^2) + (2*x*Sqrt[b*x^(2/3) + a*x])/(3
*a)
```

---

**Rubi [A]** time = 0.179859, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2016, 2002, 2014}

$$\frac{256b^4\sqrt{ax+bx^{2/3}}}{105a^5\sqrt[3]{x}}-\frac{128b^3\sqrt{ax+bx^{2/3}}}{105a^4}+\frac{32b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{35a^3}-\frac{16bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^2}+\frac{2x\sqrt{ax+bx^{2/3}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-128*b^3*Sqrt[b*x^(2/3) + a*x])/(105*a^4) + (256*b^4*Sqrt[b*x^(2/3) + a*x]
)/(105*a^5*x^(1/3)) + (32*b^2*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(35*a^3) - (16
*b*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(21*a^2) + (2*x*Sqrt[b*x^(2/3) + a*x])/(3
*a)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2x\sqrt{bx^{2/3}+ax}}{3a} - \frac{(8b)\int \frac{x^{2/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{9a}$$
$$= -\frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}+ax}}{3a} + \frac{\left(16b^2\right)\int \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a^2}$$
$$= \frac{32b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}+ax}}{3a} - \frac{\left(64b^3\right)\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx}{105a^3}$$
$$= -\frac{128b^3\sqrt{bx^{2/3}+ax}}{105a^4} + \frac{32b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}+ax}}{3a} + \frac{\left(128b^4\right)\int \frac{}{\sqrt[3]{x}}}{315}$$
$$= -\frac{128b^3\sqrt{bx^{2/3}+ax}}{105a^4} + \frac{256b^4\sqrt{bx^{2/3}+ax}}{105a^5\sqrt[3]{x}} + \frac{32b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}}}{3a}$$

**Mathematica [A]** time = 0.063737, size = 74, normalized size = 0.54

$$\frac{2\sqrt{ax+bx^{2/3}}\left(48a^2b^2x^{2/3}-40a^3bx+35a^4x^{4/3}-64ab^3\sqrt[3]{x}+128b^4\right)}{105a^5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(128*b^4 - 64*a*b^3*x^(1/3) + 48*a^2*b^2*x^(2/3) -
 40*a^3*b*x + 35*a^4*x^(4/3)))/(105*a^5*x^(1/3))
```

---

**Maple [A]** time = 0.003, size = 68, normalized size = 0.5

$$\frac{2}{105\,a^5}\sqrt[3]{x}\left(b+a\sqrt[3]{x}\right)\left(35\,x^{4/3}a^4-40\,xa^3b+48\,x^{2/3}a^2b^2-64\,\sqrt[3]{x}ab^3+128\,b^4\right)\frac{1}{\sqrt{bx^{\frac{2}{3}}+ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(2/3)+a*x)^(1/2),x)
```

```
[Out] 2/105*x^(1/3)*(b+a*x^(1/3))*(35*x^(4/3)*a^4-40*x*a^3*b+48*x^(2/3)*a^2*b^2-6
4*x^(1/3)*a*b^3+128*b^4)/(b*x^(2/3)+a*x)^(1/2)/a^5
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(a*x + b*x^(2/3)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{ax + bx^{\frac{2}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*x + b*x**(2/3)), x)
```

---

**Giac [A]** time = 1.13309, size = 108, normalized size = 0.79

$$-\frac{256\,b^{\frac{9}{2}}}{105\,a^5} + \frac{2\left(35\left(ax^{\frac{1}{3}} + b\right)^{\frac{9}{2}} - 180\left(ax^{\frac{1}{3}} + b\right)^{\frac{7}{2}}b + 378\left(ax^{\frac{1}{3}} + b\right)^{\frac{5}{2}}b^2 - 420\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}}b^3 + 315\sqrt{ax^{\frac{1}{3}} + b}\,b^4\right)}{105\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -256/105*b^(9/2)/a^5 + 2/105*(35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b
)^(7/2)*b + 378*(a*x^(1/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 +
 315*sqrt(a*x^(1/3) + b)*b^4)/a^5
```

## 3.189 $\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx$

**Optimal**. Leaf size=47

$$\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

```
[Out] (2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

---

**Rubi [A]** time = 0.0495725, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2002, 2014}

$$\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx&=\frac{2\sqrt{bx^{2/3}+ax}}{a}-\frac{(2b)\int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{3a}\\&=\frac{2\sqrt{bx^{2/3}+ax}}{a}-\frac{4b\sqrt{bx^{2/3}+ax}}{a^2\sqrt[3]{x}}\end{aligned}$$

**Mathematica [A]** time = 0.0274036, size = 36, normalized size = 0.77

$$\frac{2\left(a\sqrt[3]{x}-2b\right)\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(-2*b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

---

**Maple [A]** time = 0.003, size = 36, normalized size = 0.8

$$2\,{\frac{\sqrt[3]{x}\left(b+a\sqrt[3]{x}\right)\left(a\sqrt[3]{x}-2\,b\right)}{\sqrt{b{x}^{2/3}+ax}{a}^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(2/3)+a*x)^(1/2),x)
```

```
[Out] 2*x^(1/3)*(b+a*x^(1/3))*(a*x^(1/3)-2*b)/(b*x^(2/3)+a*x)^(1/2)/a^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x + b*x^(2/3)), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**(2/3)), x)
```

---

**Giac [A]** time = 1.1424, size = 49, normalized size = 1.04

$$\frac{4\,b^{\frac{3}{2}}}{a^2}+\frac{2\left(\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}-3\sqrt{ax^{\frac{1}{3}}+b}b\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4*b^(3/2)/a^2 + 2*((a*x^(1/3) + b)^(3/2) - 3*sqrt(a*x^(1/3) + b)*b)/a^2
```

## 3.190 $\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx$

**Optimal**. Leaf size=61

$$\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqr
t[b*x^(2/3) + a*x]])/b^(3/2)
```

---

**Rubi [A]** time = 0.0934507, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2029, 206}

$$\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqr
t[b*x^(2/3) + a*x]])/b^(3/2)
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}} - \frac{a\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{2b} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.0868025, size = 90, normalized size = 1.48

$$
\frac{6a\sqrt[3]{x}\left(a\sqrt[3]{x}+b\right)\left(\frac{\tanh^{-1}\left(\sqrt{\frac{a\sqrt[3]{x}}{b}+1}\right)}{2\sqrt{\frac{a\sqrt[3]{x}}{b}+1}}-\frac{b}{2a\sqrt[3]{x}}\right)}{b^2\sqrt{x^{2/3}\left(a\sqrt[3]{x}+b\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (6*a*(b + a*x^(1/3))*x^(1/3)*(-b/(2*a*x^(1/3)) + ArcTanh[Sqrt[1 + (a*x^(1/3
))/b]]/(2*Sqrt[1 + (a*x^(1/3))/b])))/(b^2*Sqrt[(b + a*x^(1/3))*x^(2/3)])
```

---

**Maple [A]** time = 0.007, size = 61, normalized size = 1.

$$
3\,\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{bx^{2/3}+ax}\,b^{5/2}}\left(\text{Artanh}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)ba\sqrt[3]{x}-\sqrt{b+a\sqrt[3]{x}}\,b^{3/2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(2/3)+a*x)^(1/2),x)
```

```
[Out] 3*(b+a*x^(1/3))^(1/2)*(arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b*a*x^(1/3)-(b+
a*x^(1/3))^(1/2)*b^(3/2))/(b*x^(2/3)+a*x)^(1/2)/b^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\,x}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{ax + bx^{\frac{2}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*x + b*x**(2/3))), x)
```

---

**Giac [A]** time = 1.15623, size = 69, normalized size = 1.13

$$-\frac{3\left(\frac{a^2\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} + \frac{\sqrt{ax^{\frac{1}{3}}+b}a}{bx^{\frac{1}{3}}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -3*(a^2*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(a*x^(1/3)
+ b)*a/(b*x^(1/3)))/a
```

## 3.191 $\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx$

**Optimal**. Leaf size=153

$$\frac{105a^3\sqrt{ax+bx^{2/3}}}{64b^4x^{2/3}}-\frac{35a^2\sqrt{ax+bx^{2/3}}}{32b^3x}-\frac{105a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{64b^{9/2}}+\frac{7a\sqrt{ax+bx^{2/3}}}{8b^2x^{4/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) + (7*a*Sqrt[b*x^(2/3) + a*x])/(8*b
^2*x^(4/3)) - (35*a^2*Sqrt[b*x^(2/3) + a*x])/(32*b^3*x) + (105*a^3*Sqrt[b*x
^(2/3) + a*x])/(64*b^4*x^(2/3)) - (105*a^4*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b
*x^(2/3) + a*x]])/(64*b^(9/2))
```

---

**Rubi [A]** time = 0.239309, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2029, 206}

$$\frac{105a^3\sqrt{ax+bx^{2/3}}}{64b^4x^{2/3}}-\frac{35a^2\sqrt{ax+bx^{2/3}}}{32b^3x}-\frac{105a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{64b^{9/2}}+\frac{7a\sqrt{ax+bx^{2/3}}}{8b^2x^{4/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) + (7*a*Sqrt[b*x^(2/3) + a*x])/(8*b
^2*x^(4/3)) - (35*a^2*Sqrt[b*x^(2/3) + a*x])/(32*b^3*x) + (105*a^3*Sqrt[b*x
^(2/3) + a*x])/(64*b^4*x^(2/3)) - (105*a^4*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b
*x^(2/3) + a*x]])/(64*b^(9/2))
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} - \frac{(7a)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{8b} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} + \frac{\left(35a^2\right)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{48b^2} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} - \frac{\left(35a^3\right)\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{64b^3} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}} + \frac{\left(35a^4\right)\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}}}}{128b^4} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}} - \frac{\left(105a^4\right)\text{Subst}\left(\int\right.}{} \\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}} - \frac{105a^4\tanh^{-1}\left(\frac{}{\sqrt{b}}\right.}{64b^{9/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0545172, size = 48, normalized size = 0.31

$$
-\frac{6a^4\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^5\sqrt[3]{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-6*a^4*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[1/2, 5, 3/2, 1 + (a*x^(1/3)
)/b])/(b^5*x^(1/3))
```

---

**Maple [A]** time = 0.007, size = 126, normalized size = 0.8

$$
-\frac{1}{64\,x^2}\sqrt{b+a\sqrt[3]{x}}\left(-56\,b^{7/2}x^{4/3}\sqrt{b+a\sqrt[3]{x}}a+70\,b^{5/2}x^{5/3}\sqrt{b+a\sqrt[3]{x}}a^2+105\,\mathrm{Artanh}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)x^{7/3}a^4b+48\,\sqrt{b+a}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(2/3)+a*x)^(1/2),x)
```

```
[Out] -1/64*(b+a*x^(1/3))^(1/2)*(-56*b^(7/2)*x^(4/3)*(b+a*x^(1/3))^(1/2)*a+70*b^(
5/2)*x^(5/3)*(b+a*x^(1/3))^(1/2)*a^2+105*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2
))*x^(7/3)*a^4*b+48*(b+a*x^(1/3))^(1/2)*b^(9/2)*x-105*b^(3/2)*x^2*(b+a*x^(1
/3))^(1/2)*a^3)/x^2/(b*x^(2/3)+a*x)^(1/2)/b^(11/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\,x^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}\sqrt{ax + bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*x + b*x**(2/3))), x)
```

---

**Giac [A]** time = 1.18958, size = 147, normalized size = 0.96

$$\frac{\frac{105\,a^{5}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{4}}+\frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{5}-385\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{5}b+511\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{5}b^{2}-279\sqrt{ax^{\frac{1}{3}}+b}a^{5}b^{3}}{a^{4}b^{4}x^{\frac{4}{3}}}}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(105*a^5*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*(a
*x^(1/3) + b)^(7/2)*a^5 - 385*(a*x^(1/3) + b)^(5/2)*a^5*b + 511*(a*x^(1/3)
+ b)^(3/2)*a^5*b^2 - 279*sqrt(a*x^(1/3) + b)*a^5*b^3)/(a^4*b^4*x^(4/3)))/a
```

## 3.192 $\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$

**Optimal**. Leaf size=241

$$-\frac{1287a^6\sqrt{ax+bx^{2/3}}}{1024b^7x^{2/3}}+\frac{429a^5\sqrt{ax+bx^{2/3}}}{512b^6x}-\frac{429a^4\sqrt{ax+bx^{2/3}}}{640b^5x^{4/3}}+\frac{1287a^3\sqrt{ax+bx^{2/3}}}{2240b^4x^{5/3}}-\frac{143a^2\sqrt{ax+bx^{2/3}}}{280b^3x^2}+\frac{1287a^7\ \mathrm{t}}{}$$

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x^(8/3)) + (13*a*Sqrt[b*x^(2/3) + a*x])/(28
*b^2*x^(7/3)) - (143*a^2*Sqrt[b*x^(2/3) + a*x])/(280*b^3*x^2) + (1287*a^3*S
qrt[b*x^(2/3) + a*x])/(2240*b^4*x^(5/3)) - (429*a^4*Sqrt[b*x^(2/3) + a*x])/
(640*b^5*x^(4/3)) + (429*a^5*Sqrt[b*x^(2/3) + a*x])/(512*b^6*x) - (1287*a^6
*Sqrt[b*x^(2/3) + a*x])/(1024*b^7*x^(2/3)) + (1287*a^7*ArcTanh[(Sqrt[b]*x^(
1/3))/Sqrt[b*x^(2/3) + a*x]])/(1024*b^(15/2))
```

---

**Rubi [A]** time = 0.407532, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2029, 206}

$$-\frac{1287a^6\sqrt{ax+bx^{2/3}}}{1024b^7x^{2/3}}+\frac{429a^5\sqrt{ax+bx^{2/3}}}{512b^6x}-\frac{429a^4\sqrt{ax+bx^{2/3}}}{640b^5x^{4/3}}+\frac{1287a^3\sqrt{ax+bx^{2/3}}}{2240b^4x^{5/3}}-\frac{143a^2\sqrt{ax+bx^{2/3}}}{280b^3x^2}+\frac{1287a^7\ \mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x^(8/3)) + (13*a*Sqrt[b*x^(2/3) + a*x])/(28
*b^2*x^(7/3)) - (143*a^2*Sqrt[b*x^(2/3) + a*x])/(280*b^3*x^2) + (1287*a^3*S
qrt[b*x^(2/3) + a*x])/(2240*b^4*x^(5/3)) - (429*a^4*Sqrt[b*x^(2/3) + a*x])/
(640*b^5*x^(4/3)) + (429*a^5*Sqrt[b*x^(2/3) + a*x])/(512*b^6*x) - (1287*a^6
*Sqrt[b*x^(2/3) + a*x])/(1024*b^7*x^(2/3)) + (1287*a^7*ArcTanh[(Sqrt[b]*x^(
1/3))/Sqrt[b*x^(2/3) + a*x]])/(1024*b^(15/2))
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} - \frac{(13a)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{14b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} + \frac{\left(143a^2\right)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{168b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} - \frac{\left(429a^3\right)\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx}{560b^3}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} + \frac{\left(429a^4\right)\int \frac{1}{x^{5/3}\sqrt{bx}}}{640b^4}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - \frac{429a^4\sqrt{bx^{2/3}+a}}{640b^5x^{4/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - \frac{429a^4\sqrt{bx^{2/3}+a}}{640b^5x^{4/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - \frac{429a^4\sqrt{bx^{2/3}+a}}{640b^5x^{4/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - \frac{429a^4\sqrt{bx^{2/3}+a}}{640b^5x^{4/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - \frac{429a^4\sqrt{bx^{2/3}+a}}{640b^5x^{4/3}}$$

**Mathematica [C]** time = 0.0558736, size = 48, normalized size = 0.2

$$\frac{6a^7\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{1}{2},8;\frac{3}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^8\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (6*a^7*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[1/2, 8, 3/2, 1 + (a*x^(1/3))
/b])/(b^8*x^(1/3))
```

---

**Maple [A]** time = 0.006, size = 188, normalized size = 0.8

$$-\frac{1}{35840\,x^4}\sqrt{b+a\sqrt[3]{x}}\left(24024\,b^{7/2}x^{10/3}\sqrt{b+a\sqrt[3]{x}}a^4+45045\,b^{3/2}x^4\sqrt{b+a\sqrt[3]{x}}a^6-45045\,\mathrm{Artanh}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)x^{13/3}a^7b-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(2/3)+a*x)^(1/2),x)
```

```
[Out] -1/35840/x^4*(b+a*x^(1/3))^(1/2)*(24024*b^(7/2)*x^(10/3)*(b+a*x^(1/3))^(1/2
)*a^4+45045*b^(3/2)*x^4*(b+a*x^(1/3))^(1/2)*a^6-45045*arctanh((b+a*x^(1/3))
^(1/2)/b^(1/2))*x^(13/3)*a^7*b-16640*b^(13/2)*x^(7/3)*(b+a*x^(1/3))^(1/2)*a
-30030*b^(5/2)*x^(11/3)*(b+a*x^(1/3))^(1/2)*a^5-20592*b^(9/2)*x^3*(b+a*x^(1
/3))^(1/2)*a^3+18304*b^(11/2)*x^(8/3)*(b+a*x^(1/3))^(1/2)*a^2+15360*(b+a*x^
```

```
(1/3))^(1/2)*b^(15/2)*x^2)/(b*x^(2/3)+a*x)^(1/2)/b^(17/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax + bx^{\frac{2}{3}}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^3), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{ax + bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a*x + b*x**(2/3))), x)
```

---

**Giac [A]** time = 1.26854, size = 216, normalized size = 0.9

$$-\frac{\frac{45045\,a^8\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^7}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^8-300300\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^8b+849849\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^8b^2-1317888\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^8b^3+1200199\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^8b^4-}{a^7b^7x^{\frac{7}{3}}}}{35840\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/35840*(45045*a^8*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) + (
45045*(a*x^(1/3) + b)^(13/2)*a^8 - 300300*(a*x^(1/3) + b)^(11/2)*a^8*b + 84
9849*(a*x^(1/3) + b)^(9/2)*a^8*b^2 - 1317888*(a*x^(1/3) + b)^(7/2)*a^8*b^3
+ 1200199*(a*x^(1/3) + b)^(5/2)*a^8*b^4 - 631540*(a*x^(1/3) + b)^(3/2)*a^8*
b^5 + 169995*sqrt(a*x^(1/3) + b)*a^8*b^6)/(a^7*b^7*x^(7/3)))/a
```

## 3.193 $\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$

**Optimal**. Leaf size=329

$$\frac{138567a^9\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{2/3}}-\frac{46189a^8\sqrt{ax+bx^{2/3}}}{65536b^9x}+\frac{46189a^7\sqrt{ax+bx^{2/3}}}{81920b^8x^{4/3}}-\frac{138567a^6\sqrt{ax+bx^{2/3}}}{286720b^7x^{5/3}}+\frac{46189a^5\sqrt{ax+bx^{2/3}}}{107520b^6x^2}-$$

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(10*b*x^(11/3)) + (19*a*Sqrt[b*x^(2/3) + a*x])/(
60*b^2*x^(10/3)) - (323*a^2*Sqrt[b*x^(2/3) + a*x])/(960*b^3*x^3) + (323*a^3
*Sqrt[b*x^(2/3) + a*x])/(896*b^4*x^(8/3)) - (4199*a^4*Sqrt[b*x^(2/3) + a*x]
)/(10752*b^5*x^(7/3)) + (46189*a^5*Sqrt[b*x^(2/3) + a*x])/(107520*b^6*x^2)
- (138567*a^6*Sqrt[b*x^(2/3) + a*x])/(286720*b^7*x^(5/3)) + (46189*a^7*Sqrt
[b*x^(2/3) + a*x])/(81920*b^8*x^(4/3)) - (46189*a^8*Sqrt[b*x^(2/3) + a*x])/
(65536*b^9*x) + (138567*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(2/3)) -
(138567*a^10*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(131072*b^(2
1/2))
```

---

**Rubi [A]** time = 0.577139, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2029, 206}

$$\frac{138567a^9\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{2/3}}-\frac{46189a^8\sqrt{ax+bx^{2/3}}}{65536b^9x}+\frac{46189a^7\sqrt{ax+bx^{2/3}}}{81920b^8x^{4/3}}-\frac{138567a^6\sqrt{ax+bx^{2/3}}}{286720b^7x^{5/3}}+\frac{46189a^5\sqrt{ax+bx^{2/3}}}{107520b^6x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(10*b*x^(11/3)) + (19*a*Sqrt[b*x^(2/3) + a*x])/(
60*b^2*x^(10/3)) - (323*a^2*Sqrt[b*x^(2/3) + a*x])/(960*b^3*x^3) + (323*a^3
*Sqrt[b*x^(2/3) + a*x])/(896*b^4*x^(8/3)) - (4199*a^4*Sqrt[b*x^(2/3) + a*x]
)/(10752*b^5*x^(7/3)) + (46189*a^5*Sqrt[b*x^(2/3) + a*x])/(107520*b^6*x^2)
- (138567*a^6*Sqrt[b*x^(2/3) + a*x])/(286720*b^7*x^(5/3)) + (46189*a^7*Sqrt
[b*x^(2/3) + a*x])/(81920*b^8*x^(4/3)) - (46189*a^8*Sqrt[b*x^(2/3) + a*x])/
(65536*b^9*x) + (138567*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(2/3)) -
(138567*a^10*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(131072*b^(2
1/2))
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}-\frac{(19a)\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{20b}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}+\frac{\left(323a^2\right)\int \frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{360b^2}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}-\frac{\left(323a^3\right)\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx}{384b^3}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}+\frac{\left(4199a^4\right)\int \frac{}{x^{8/3}}}{5376}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}-\frac{4199a^4\sqrt{bx^{2/3}}}{10752b^5x^{7/3}}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}-\frac{4199a^4\sqrt{bx^{2/3}}}{10752b^5x^{7/3}}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}-\frac{4199a^4\sqrt{bx^{2/3}}}{10752b^5x^{7/3}}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}-\frac{4199a^4\sqrt{bx^{2/3}}}{10752b^5x^{7/3}}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}-\frac{4199a^4\sqrt{bx^{2/3}}}{10752b^5x^{7/3}}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}-\frac{4199a^4\sqrt{bx^{2/3}}}{10752b^5x^{7/3}}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}-\frac{4199a^4\sqrt{bx^{2/3}}}{10752b^5x^{7/3}}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}}+\frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}}-\frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3}+\frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}}-\frac{4199a^4\sqrt{bx^{2/3}}}{10752b^5x^{7/3}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0547898, size = 48, normalized size = 0.15

$$
-\frac{6a^{10}\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{1}{2},11;\frac{3}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^{11}\sqrt[3]{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-6*a^10*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[1/2, 11, 3/2, 1 + (a*x^(1/
3))/b])/(b^11*x^(1/3))
```

**Maple [A]** time = 0.007, size = 248, normalized size = 0.8

$$-\frac{1}{13762560\,x^{6}}\sqrt{b+a\sqrt[3]{x}}\left(-4358144\,b^{19/2}x^{10/3}\sqrt{b+a\sqrt[3]{x}}a+4630528\,b^{17/2}x^{11/3}\sqrt{b+a\sqrt[3]{x}}a^{2}+5374720\,b^{13/2}x^{13/3}\sqrt{b+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^(2/3)+a*x)^(1/2),x)
```

```
[Out] -1/13762560*(b+a*x^(1/3))^(1/2)*(-4358144*b^(19/2)*x^(10/3)*(b+a*x^(1/3))^(
1/2)*a+4630528*b^(17/2)*x^(11/3)*(b+a*x^(1/3))^(1/2)*a^2+5374720*b^(13/2)*x
^(13/3)*(b+a*x^(1/3))^(1/2)*a^4-5912192*b^(11/2)*x^(14/3)*(b+a*x^(1/3))^(1/
2)*a^5-7759752*b^(7/2)*x^(16/3)*(b+a*x^(1/3))^(1/2)*a^7+9699690*b^(5/2)*x^(
17/3)*(b+a*x^(1/3))^(1/2)*a^8+14549535*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))
*x^(19/3)*a^10*b+4128768*(b+a*x^(1/3))^(1/2)*b^(21/2)*x^3-4961280*b^(15/2)*
x^4*(b+a*x^(1/3))^(1/2)*a^3+6651216*b^(9/2)*x^5*(b+a*x^(1/3))^(1/2)*a^6-145
49535*b^(3/2)*x^6*(b+a*x^(1/3))^(1/2)*a^9)/x^6/(b*x^(2/3)+a*x)^(1/2)/b^(23/
2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^4), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.35374, size = 285, normalized size = 0.87

$$\frac{\frac{14549535\,a^{11}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^{10}}+\frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{11}-140645505\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{11}b+609140532\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{11}b^{2}-1554721740\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{11}b^{3}+25}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/13762560*(14549535*a^11*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^
10) + (14549535*(a*x^(1/3) + b)^(19/2)*a^11 - 140645505*(a*x^(1/3) + b)^(17
/2)*a^11*b + 609140532*(a*x^(1/3) + b)^(15/2)*a^11*b^2 - 1554721740*(a*x^(1
/3) + b)^(13/2)*a^11*b^3 + 2585198330*(a*x^(1/3) + b)^(11/2)*a^11*b^4 - 291
8514950*(a*x^(1/3) + b)^(9/2)*a^11*b^5 + 2255541300*(a*x^(1/3) + b)^(7/2)*a
^11*b^6 - 1168982220*(a*x^(1/3) + b)^(5/2)*a^11*b^7 + 382331775*(a*x^(1/3)
+ b)^(3/2)*a^11*b^8 - 68025825*sqrt(a*x^(1/3) + b)*a^11*b^9)/(a^10*b^10*x^(
10/3)))/a
```

## 3.194 $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=336

$$\frac{1048576b^{10}\sqrt{ax+bx^{2/3}}}{29393a^{12}\sqrt[3]{x}}-\frac{524288b^9\sqrt{ax+bx^{2/3}}}{29393a^{11}}+\frac{393216b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{29393a^{10}}-\frac{327680b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{29393a^9}+\frac{40960b^6x\sqrt{a}}{4199}$$

```
[Out] (-6*x^4)/(a*Sqrt[b*x^(2/3) + a*x]) - (524288*b^9*Sqrt[b*x^(2/3) + a*x])/(29
393*a^11) + (1048576*b^10*Sqrt[b*x^(2/3) + a*x])/(29393*a^12*x^(1/3)) + (39
3216*b^8*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(29393*a^10) - (327680*b^7*x^(2/3)*
Sqrt[b*x^(2/3) + a*x])/(29393*a^9) + (40960*b^6*x*Sqrt[b*x^(2/3) + a*x])/(4
199*a^8) - (36864*b^5*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(4199*a^7) + (33792*b^
4*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(4199*a^6) - (16896*b^3*x^2*Sqrt[b*x^(2/3)
 + a*x])/(2261*a^5) + (15840*b^2*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(2261*a^4)
- (880*b*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(133*a^3) + (44*x^3*Sqrt[b*x^(2/3)
+ a*x])/(7*a^2)
```

---

**Rubi [A]** time = 0.599225, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2015, 2016, 2002, 2014}

$$\frac{1048576b^{10}\sqrt{ax+bx^{2/3}}}{29393a^{12}\sqrt[3]{x}}-\frac{524288b^9\sqrt{ax+bx^{2/3}}}{29393a^{11}}+\frac{393216b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{29393a^{10}}-\frac{327680b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{29393a^9}+\frac{40960b^6x\sqrt{a}}{4199}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x^4)/(a*Sqrt[b*x^(2/3) + a*x]) - (524288*b^9*Sqrt[b*x^(2/3) + a*x])/(29
393*a^11) + (1048576*b^10*Sqrt[b*x^(2/3) + a*x])/(29393*a^12*x^(1/3)) + (39
3216*b^8*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(29393*a^10) - (327680*b^7*x^(2/3)*
Sqrt[b*x^(2/3) + a*x])/(29393*a^9) + (40960*b^6*x*Sqrt[b*x^(2/3) + a*x])/(4
199*a^8) - (36864*b^5*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(4199*a^7) + (33792*b^
4*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(4199*a^6) - (16896*b^3*x^2*Sqrt[b*x^(2/3)
 + a*x])/(2261*a^5) + (15840*b^2*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(2261*a^4)
- (880*b*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(133*a^3) + (44*x^3*Sqrt[b*x^(2/3)
+ a*x])/(7*a^2)
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}+\frac{22\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx}{a}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}+\frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2}-\frac{(440b)\int \frac{x^{8/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a^2}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}-\frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3}+\frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2}+\frac{\left(2640b^2\right)\int \frac{x^{7/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{133a^3}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}+\frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4}-\frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3}+\frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2}-(422\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}-\frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5}+\frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4}-\frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3}+\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}+\frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6}-\frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5}+\frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+a}}{2261a^4}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}-\frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7}+\frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6}-\frac{16896b^3x^2\sqrt{bx^{2/3}+a}}{2261a^5}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}+\frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8}-\frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7}+\frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}-\frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{29393a^9}+\frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8}-\frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+a}}{4199a^7}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}+\frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{29393a^{10}}-\frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{29393a^9}+\frac{40960b^6x\sqrt{bx^{2/3}+}}{4199a^8}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}-\frac{524288b^9\sqrt{bx^{2/3}+ax}}{29393a^{11}}+\frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{29393a^{10}}-\frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+}}{29393a^9}\\
&= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}}-\frac{524288b^9\sqrt{bx^{2/3}+ax}}{29393a^{11}}+\frac{1048576b^{10}\sqrt{bx^{2/3}+ax}}{29393a^{12}\sqrt[3]{x}}+\frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3}+a}}{29393a^{10}}
\end{aligned}
$$

**Mathematica [A]** time = 0.132949, size = 161, normalized size = 0.48

$$\frac{2\sqrt[3]{x}\left(5720a^9b^2x^3-6864a^8b^3x^{8/3}+8448a^7b^4x^{7/3}-10752a^6b^5x^2+14336a^5b^6x^{5/3}-20480a^4b^7x^{4/3}-65536a^2b^9x^{2/3}+32\right.}{29393a^{12}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(1/3)*(524288*b^11 + 262144*a*b^10*x^(1/3) - 65536*a^2*b^9*x^(2/3) + 3
2768*a^3*b^8*x - 20480*a^4*b^7*x^(4/3) + 14336*a^5*b^6*x^(5/3) - 10752*a^6*
b^5*x^2 + 8448*a^7*b^4*x^(7/3) - 6864*a^8*b^3*x^(8/3) + 5720*a^9*b^2*x^3 -
4862*a^10*b*x^(10/3) + 4199*a^11*x^(11/3)))/(29393*a^12*Sqrt[b*x^(2/3) + a*
x])
```

---

**Maple [A]** time = 0.005, size = 143, normalized size = 0.4

$$\frac{2x}{29393a^{12}}\left(b+a\sqrt[3]{x}\right)\left(4199x^{11/3}a^{11}-4862x^{10/3}a^{10}b+5720x^3a^9b^2-6864x^{8/3}a^8b^3+8448x^{7/3}a^7b^4-10752x^2a^6b^5+143\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^(2/3)+a*x)^(3/2),x)
```

```
[Out] 2/29393*x*(b+a*x^(1/3))*(4199*x^(11/3)*a^11-4862*x^(10/3)*a^10*b+5720*x^3*a
^9*b^2-6864*x^(8/3)*a^8*b^3+8448*x^(7/3)*a^7*b^4-10752*x^2*a^6*b^5+14336*x^
(5/3)*a^5*b^6-20480*x^(4/3)*a^4*b^7+32768*x*a^3*b^8-65536*x^(2/3)*a^2*b^9+2
62144*x^(1/3)*a*b^10+524288*b^11)/(b*x^(2/3)+a*x)^(3/2)/a^12
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(a*x + b*x^(2/3))^(3/2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.23366, size = 289, normalized size = 0.86

$$-\frac{1048576\,b^{\frac{21}{2}}}{29393\,a^{12}}+\frac{6\,b^{11}}{\sqrt{ax^{\frac{1}{3}}+b}\,a^{12}}+\frac{2\left(4199\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{240}-51051\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{240}b+285285\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{240}b^{2}-96\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -1048576/29393*b^(21/2)/a^12 + 6*b^11/(sqrt(a*x^(1/3) + b)*a^12) + 2/29393*
(4199*(a*x^(1/3) + b)^(21/2)*a^240 - 51051*(a*x^(1/3) + b)^(19/2)*a^240*b +
 285285*(a*x^(1/3) + b)^(17/2)*a^240*b^2 - 969969*(a*x^(1/3) + b)^(15/2)*a^
240*b^3 + 2238390*(a*x^(1/3) + b)^(13/2)*a^240*b^4 - 3703518*(a*x^(1/3) + b
)^(11/2)*a^240*b^5 + 4526522*(a*x^(1/3) + b)^(9/2)*a^240*b^6 - 4157010*(a*x
^(1/3) + b)^(7/2)*a^240*b^7 + 2909907*(a*x^(1/3) + b)^(5/2)*a^240*b^8 - 161
6615*(a*x^(1/3) + b)^(3/2)*a^240*b^9 + 969969*sqrt(a*x^(1/3) + b)*a^240*b^1
0)/a^252
```

## 3.195 $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=248

$$-\frac{65536b^7\sqrt{ax+bx^{2/3}}}{2145a^9\sqrt[3]{x}}+\frac{32768b^6\sqrt{ax+bx^{2/3}}}{2145a^8}-\frac{8192b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^7}+\frac{4096b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^6}-\frac{3584b^3x\sqrt{ax+bx^{2/3}}}{429a^5}$$

```
[Out] (-6*x^3)/(a*Sqrt[b*x^(2/3) + a*x]) + (32768*b^6*Sqrt[b*x^(2/3) + a*x])/(214
5*a^8) - (65536*b^7*Sqrt[b*x^(2/3) + a*x])/(2145*a^9*x^(1/3)) - (8192*b^5*x
^(1/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^7) + (4096*b^4*x^(2/3)*Sqrt[b*x^(2/3)
+ a*x])/(429*a^6) - (3584*b^3*x*Sqrt[b*x^(2/3) + a*x])/(429*a^5) + (5376*b^
2*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^4) - (448*b*x^(5/3)*Sqrt[b*x^(2/3)
+ a*x])/(65*a^3) + (32*x^2*Sqrt[b*x^(2/3) + a*x])/(5*a^2)
```

---

**Rubi [A]** time = 0.413968, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2015, 2016, 2002, 2014}

$$-\frac{65536b^7\sqrt{ax+bx^{2/3}}}{2145a^9\sqrt[3]{x}}+\frac{32768b^6\sqrt{ax+bx^{2/3}}}{2145a^8}-\frac{8192b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^7}+\frac{4096b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^6}-\frac{3584b^3x\sqrt{ax+bx^{2/3}}}{429a^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x^3)/(a*Sqrt[b*x^(2/3) + a*x]) + (32768*b^6*Sqrt[b*x^(2/3) + a*x])/(214
5*a^8) - (65536*b^7*Sqrt[b*x^(2/3) + a*x])/(2145*a^9*x^(1/3)) - (8192*b^5*x
^(1/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^7) + (4096*b^4*x^(2/3)*Sqrt[b*x^(2/3)
+ a*x])/(429*a^6) - (3584*b^3*x*Sqrt[b*x^(2/3) + a*x])/(429*a^5) + (5376*b^
2*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^4) - (448*b*x^(5/3)*Sqrt[b*x^(2/3)
+ a*x])/(65*a^3) + (32*x^2*Sqrt[b*x^(2/3) + a*x])/(5*a^2)
```

### Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

### Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

### Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
```

```
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{16\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx}{a}\\
&= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2}-\frac{(224b)\int \frac{x^{5/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{15a^2}\\
&= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}-\frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3}+\frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2}+\frac{\left(896b^2\right)\int \frac{x^{4/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{65a^3}\\
&= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4}-\frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3}+\frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2}- (1792\\
&= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}-\frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5}+\frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4}-\frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3}+ 32\\
&= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6}-\frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5}+\frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4}-\\
&= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}-\frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^7}+\frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6}-\frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5}+\\
&= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{32768b^6\sqrt{bx^{2/3}+ax}}{2145a^8}-\frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^7}+\frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6}-\\
&= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{32768b^6\sqrt{bx^{2/3}+ax}}{2145a^8}-\frac{65536b^7\sqrt{bx^{2/3}+ax}}{2145a^9\sqrt[3]{x}}-\frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^7}+
\end{aligned}
$$

**Mathematica [A]** time = 0.0885618, size = 122, normalized size = 0.49

$$
\frac{2(672a^6b^2x^{7/3}-896a^5b^3x^2+1280a^4b^4x^{5/3}-2048a^3b^5x^{4/3}+4096a^2b^6x-528a^7bx^{8/3}+429a^8x^3-16384ab^7x^{2/3}-32}{2145a^9\sqrt{ax+bx^{2/3}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(-32768*b^8*x^(1/3) - 16384*a*b^7*x^(2/3) + 4096*a^2*b^6*x - 2048*a^3*b^
5*x^(4/3) + 1280*a^4*b^4*x^(5/3) - 896*a^5*b^3*x^2 + 672*a^6*b^2*x^(7/3) -
528*a^7*b*x^(8/3) + 429*a^8*x^3))/(2145*a^9*Sqrt[b*x^(2/3) + a*x])
```

**Maple [A]** time = 0.006, size = 110, normalized size = 0.4

$$\frac{2x}{2145a^9}\big(b+a\sqrt[3]{x}\big)\big(429x^{8/3}a^8-528x^{7/3}a^7b+672x^2a^6b^2-896x^{5/3}a^5b^3+1280x^{4/3}a^4b^4-2048xa^3b^5+4096x^{2/3}a^2b^6-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^(2/3)+a*x)^(3/2),x)
```

```
[Out] 2/2145*x*(b+a*x^(1/3))*(429*x^(8/3)*a^8-528*x^(7/3)*a^7*b+672*x^2*a^6*b^2-8
96*x^(5/3)*a^5*b^3+1280*x^(4/3)*a^4*b^4-2048*x*a^3*b^5+4096*x^(2/3)*a^2*b^6
-16384*x^(1/3)*a*b^7-32768*b^8)/(b*x^(2/3)+a*x)^(3/2)/a^9
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(a*x + b*x^(2/3))^(3/2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(a*x + b*x**(2/3))**(3/2), x)
```

---

**Giac [A]** time = 1.21474, size = 220, normalized size = 0.89

$$\frac{65536\,b^{\frac{15}{2}}}{2145\,a^{9}}-\frac{6\,b^{8}}{\sqrt{ax^{\frac{1}{3}}+b}\,a^{9}}+\frac{2\left(429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{126}-3960\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{126}b+16380\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{126}b^{2}-40040\left(ax\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 65536/2145*b^(15/2)/a^9 - 6*b^8/(sqrt(a*x^(1/3) + b)*a^9) + 2/2145*(429*(a*
x^(1/3) + b)^(15/2)*a^126 - 3960*(a*x^(1/3) + b)^(13/2)*a^126*b + 16380*(a*
x^(1/3) + b)^(11/2)*a^126*b^2 - 40040*(a*x^(1/3) + b)^(9/2)*a^126*b^3 + 643
50*(a*x^(1/3) + b)^(7/2)*a^126*b^4 - 72072*(a*x^(1/3) + b)^(5/2)*a^126*b^5
+ 60060*(a*x^(1/3) + b)^(3/2)*a^126*b^6 - 51480*sqrt(a*x^(1/3) + b)*a^126*b
^7)/a^135
```

## 3.196 $\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=160

$$\frac{512b^4\sqrt{ax+bx^{2/3}}}{21a^6\sqrt[3]{x}}-\frac{256b^3\sqrt{ax+bx^{2/3}}}{21a^5}+\frac{64b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{7a^4}-\frac{160bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^3}+\frac{20x\sqrt{ax+bx^{2/3}}}{3a^2}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

```
[Out] (-6*x^2)/(a*Sqrt[b*x^(2/3) + a*x]) - (256*b^3*Sqrt[b*x^(2/3) + a*x])/(21*a^
5) + (512*b^4*Sqrt[b*x^(2/3) + a*x])/(21*a^6*x^(1/3)) + (64*b^2*x^(1/3)*Sqr
t[b*x^(2/3) + a*x])/(7*a^4) - (160*b*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(21*a^3
) + (20*x*Sqrt[b*x^(2/3) + a*x])/(3*a^2)
```

---

**Rubi [A]** time = 0.242094, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2015, 2016, 2002, 2014}

$$\frac{512b^4\sqrt{ax+bx^{2/3}}}{21a^6\sqrt[3]{x}}-\frac{256b^3\sqrt{ax+bx^{2/3}}}{21a^5}+\frac{64b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{7a^4}-\frac{160bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^3}+\frac{20x\sqrt{ax+bx^{2/3}}}{3a^2}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x^2)/(a*Sqrt[b*x^(2/3) + a*x]) - (256*b^3*Sqrt[b*x^(2/3) + a*x])/(21*a^
5) + (512*b^4*Sqrt[b*x^(2/3) + a*x])/(21*a^6*x^(1/3)) + (64*b^2*x^(1/3)*Sqr
t[b*x^(2/3) + a*x])/(7*a^4) - (160*b*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(21*a^3
) + (20*x*Sqrt[b*x^(2/3) + a*x])/(3*a^2)
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}}+\frac{10\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx}{a}\\
&= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}}+\frac{20x\sqrt{bx^{2/3}+ax}}{3a^2}-\frac{(80b)\int \frac{x^{2/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{9a^2}\\
&= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}}-\frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3}+\frac{20x\sqrt{bx^{2/3}+ax}}{3a^2}+\frac{\left(160b^2\right)\int \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a^3}\\
&= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}}+\frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{7a^4}-\frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3}+\frac{20x\sqrt{bx^{2/3}+ax}}{3a^2}-\frac{\left(128b^3\right)\int}{2}\\
&= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}}-\frac{256b^3\sqrt{bx^{2/3}+ax}}{21a^5}+\frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{7a^4}-\frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3}+\frac{20x\sqrt{bx}}{3}\\
&= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}}-\frac{256b^3\sqrt{bx^{2/3}+ax}}{21a^5}+\frac{512b^4\sqrt{bx^{2/3}+ax}}{21a^6\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{7a^4}-\frac{160bx^{2/3}\sqrt{}}{21}
\end{aligned}$$

**Mathematica [A]** time = 0.0714595, size = 85, normalized size = 0.53

$$\frac{32a^3b^2x^{4/3}-64a^2b^3x-20a^4bx^{5/3}+14a^5x^2+256ab^4x^{2/3}+512b^5\sqrt[3]{x}}{21a^6\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (512*b^5*x^(1/3) + 256*a*b^4*x^(2/3) - 64*a^2*b^3*x + 32*a^3*b^2*x^(4/3) -
20*a^4*b*x^(5/3) + 14*a^5*x^2)/(21*a^6*Sqrt[b*x^(2/3) + a*x])
```

---

**Maple [A]** time = 0.006, size = 77, normalized size = 0.5

$$\frac{2\,x}{21\,a^6}\left(b+a\sqrt[3]{x}\right)\left(7\,x^{5/3}a^5-10\,x^{4/3}a^4b+16\,xa^3b^2-32\,x^{2/3}a^2b^3+128\,\sqrt[3]{x}ab^4+256\,b^5\right)\left(bx^{\frac{2}{3}}+ax\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(2/3)+a*x)^(3/2),x)
```

```
[Out] 2/21*x*(b+a*x^(1/3))*(7*x^(5/3)*a^5-10*x^(4/3)*a^4*b+16*x*a^3*b^2-32*x^(2/3
)*a^2*b^3+128*x^(1/3)*a*b^4+256*b^5)/(b*x^(2/3)+a*x)^(3/2)/a^6
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x + b*x^(2/3))^(3/2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(a*x + b*x**(2/3))**(3/2), x)
```

---

**Giac [A]** time = 1.14521, size = 151, normalized size = 0.94

$$-\frac{512\,b^{\frac{9}{2}}}{21\,a^{6}}+\frac{6\,b^{5}}{\sqrt{ax^{\frac{1}{3}}+b}\,a^{6}}+\frac{2\left(7\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{48}-45\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{48}b+126\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{48}b^{2}-210\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{48}b^{3}+315\right.}{21\,a^{54}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -512/21*b^(9/2)/a^6 + 6*b^5/(sqrt(a*x^(1/3) + b)*a^6) + 2/21*(7*(a*x^(1/3)
+ b)^(9/2)*a^48 - 45*(a*x^(1/3) + b)^(7/2)*a^48*b + 126*(a*x^(1/3) + b)^(5/
2)*a^48*b^2 - 210*(a*x^(1/3) + b)^(3/2)*a^48*b^3 + 315*sqrt(a*x^(1/3) + b)*
a^48*b^4)/a^54
```

## 3.197 $\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=68

$$-\frac{16b\sqrt{ax+bx^{2/3}}}{a^3\sqrt[3]{x}}+\frac{8\sqrt{ax+bx^{2/3}}}{a^2}-\frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

```
[Out] (-6*x)/(a*Sqrt[b*x^(2/3) + a*x]) + (8*Sqrt[b*x^(2/3) + a*x])/a^2 - (16*b*Sq
rt[b*x^(2/3) + a*x])/(a^3*x^(1/3))
```

---

**Rubi [A]** time = 0.0838576, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2015, 2002, 2014}

$$-\frac{16b\sqrt{ax+bx^{2/3}}}{a^3\sqrt[3]{x}}+\frac{8\sqrt{ax+bx^{2/3}}}{a^2}-\frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x)/(a*Sqrt[b*x^(2/3) + a*x]) + (8*Sqrt[b*x^(2/3) + a*x])/a^2 - (16*b*Sq
rt[b*x^(2/3) + a*x])/(a^3*x^(1/3))
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= -\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{4\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx}{a}\\
&= -\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{8\sqrt{bx^{2/3}+ax}}{a^2}-\frac{(8b)\int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{3a^2}\\
&= -\frac{6x}{a\sqrt{bx^{2/3}+ax}}+\frac{8\sqrt{bx^{2/3}+ax}}{a^2}-\frac{16b\sqrt{bx^{2/3}+ax}}{a^3\sqrt[3]{x}}
\end{aligned}
$$

**Mathematica [A]** time = 0.0503076, size = 60, normalized size = 0.88

$$
\frac{2\left(a^2x^{2/3}-4ab\sqrt[3]{x}-8b^2\right)\sqrt{ax+bx^{2/3}}}{a^3\sqrt[3]{x}\left(a\sqrt[3]{x}+b\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(-8*b^2 - 4*a*b*x^(1/3) + a^2*x^(2/3))*Sqrt[b*x^(2/3) + a*x])/(a^3*(b +
a*x^(1/3))*x^(1/3))
```

---

**Maple [A]** time = 0.005, size = 45, normalized size = 0.7

$$
2\,{\frac{x\left(b+a\sqrt[3]{x}\right)\left(x^{2/3}a^2-4\,\sqrt[3]{x}ab-8\,b^2\right)}{\left(bx^{2/3}+ax\right)^{3/2}a^3}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^(2/3)+a*x)^(3/2),x)
```

```
[Out] 2*x*(b+a*x^(1/3))*(x^(2/3)*a^2-4*x^(1/3)*a*b-8*b^2)/(b*x^(2/3)+a*x)^(3/2)/a
^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{x}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(a*x + b*x^(2/3))^(3/2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x/(a*x + b*x**(2/3))**(3/2), x)
```

---

**Giac [A]** time = 1.17889, size = 81, normalized size = 1.19

$$\frac{16\, b^{\frac{3}{2}}}{a^3} - \frac{6\, b^2}{\sqrt{ax^{\frac{1}{3}} + b}a^3} + \frac{2\left(\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}} a^6 - 6\sqrt{ax^{\frac{1}{3}} + b}a^6 b\right)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 16*b^(3/2)/a^3 - 6*b^2/(sqrt(a*x^(1/3) + b)*a^3) + 2*((a*x^(1/3) + b)^(3/2)
*a^6 - 6*sqrt(a*x^(1/3) + b)*a^6*b)/a^9
```

## 3.198 $\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=60

$$\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}}-\frac{6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}$$

```
[Out] (6*x^(1/3))/(b*Sqrt[b*x^(2/3) + a*x]) - (6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b
*x^(2/3) + a*x]])/b^(3/2)
```

---

**Rubi [A]** time = 0.0560228, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2006, 2029, 206}

$$\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}}-\frac{6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(-3/2),x]
```

```
[Out] (6*x^(1/3))/(b*Sqrt[b*x^(2/3) + a*x]) - (6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b
*x^(2/3) + a*x]])/b^(3/2)
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}+\frac{\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}\\
&= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}-\frac{6\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b}\\
&= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}-\frac{6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0259092, size = 45, normalized size = 0.75

$$
\frac{6\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b\sqrt{ax+bx^{2/3}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(-3/2),x]
```

```
[Out] (6*x^(1/3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (a*x^(1/3))/b])/(b*Sqrt[b*x^
(2/3) + a*x])
```

---

**Maple [A]** time = 0.006, size = 55, normalized size = 0.9

$$
6\,\frac{x\left(b+a\sqrt[3]{x}\right)}{\left(bx^{2/3}+ax\right)^{3/2}b^{5/2}}\left(b^{3/2}-\text{Artanh}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)b\sqrt{b+a\sqrt[3]{x}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(2/3)+a*x)^(3/2),x)
```

```
[Out] 6*x*(b+a*x^(1/3))*(b^(3/2)-arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*b*(b+a*x^(1
/3))^(1/2))/(b*x^(2/3)+a*x)^(3/2)/b^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(-3/2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(-3/2), x)
```

---

**Giac [A]** time = 1.16026, size = 96, normalized size = 1.6

$$\frac{6 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} - \frac{6\left(\sqrt{b}\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) + \sqrt{-b}\right)}{\sqrt{-b}b^{\frac{3}{2}}} + \frac{6}{\sqrt{ax^{\frac{1}{3}} + b}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) - 6*(sqrt(b)*arctan(sqr
t(b)/sqrt(-b)) + sqrt(-b))/(sqrt(-b)*b^(3/2)) + 6/(sqrt(a*x^(1/3) + b)*b)
```

## 3.199 $\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=146

$$-\frac{105a^2\sqrt{ax+bx^{2/3}}}{8b^4x^{2/3}}+\frac{105a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{9/2}}+\frac{35a\sqrt{ax+bx^{2/3}}}{4b^3x}-\frac{7\sqrt{ax+bx^{2/3}}}{b^2x^{4/3}}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

```
[Out] 6/(b*x^(2/3)*Sqrt[b*x^(2/3) + a*x]) - (7*Sqrt[b*x^(2/3) + a*x])/(b^2*x^(4/3
)) + (35*a*Sqrt[b*x^(2/3) + a*x])/(4*b^3*x) - (105*a^2*Sqrt[b*x^(2/3) + a*x
])/(8*b^4*x^(2/3)) + (105*a^3*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*
x]])/(8*b^(9/2))
```

---

**Rubi [A]** time = 0.240522, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2023, 2025, 2029, 206}

$$-\frac{105a^2\sqrt{ax+bx^{2/3}}}{8b^4x^{2/3}}+\frac{105a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{9/2}}+\frac{35a\sqrt{ax+bx^{2/3}}}{4b^3x}-\frac{7\sqrt{ax+bx^{2/3}}}{b^2x^{4/3}}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(2/3)*Sqrt[b*x^(2/3) + a*x]) - (7*Sqrt[b*x^(2/3) + a*x])/(b^2*x^(4/3
)) + (35*a*Sqrt[b*x^(2/3) + a*x])/(4*b^3*x) - (105*a^2*Sqrt[b*x^(2/3) + a*x
])/(8*b^4*x^(2/3)) + (105*a^3*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*
x]])/(8*b^(9/2))
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}+\frac{7\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}\\
&= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}-\frac{(35a)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{6b^2}\\
&= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}+\frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x}+\frac{\left(35a^2\right)\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{8b^3}\\
&= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}+\frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x}-\frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}}-\frac{\left(35a^3\right)\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+}}}{16b^4}\\
&= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}+\frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x}-\frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}}+\frac{\left(105a^3\right)\text{Subst}\left(\int\right.}{8}\\
&= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}}-\frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}}+\frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x}-\frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}}+\frac{105a^3\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{bx}}\right.}{8b^{9/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0565704, size = 48, normalized size = 0.33

$$
-\frac{6a^3\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},4;\frac{1}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^4\sqrt{ax+bx^{2/3}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (-6*a^3*x^(1/3)*Hypergeometric2F1[-1/2, 4, 1/2, 1 + (a*x^(1/3))/b])/(b^4*Sq
rt[b*x^(2/3) + a*x])
```

---

**Maple [A]** time = 0.013, size = 88, normalized size = 0.6

$$
-\frac{1}{8}\left(b+a\sqrt[3]{x}\right)\left(105\sqrt{b}xa^3+35x^{2/3}b^{3/2}a^2-14\sqrt[3]{x}b^{5/2}a+8b^{7/2}-105\,\text{Artanh}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)\sqrt{b+a\sqrt[3]{x}}xa^3\right)\left(bx^{\frac{2}{3}}+ax\right)^{-\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^(2/3)+a*x)^(3/2),x)
```

```
[Out] -1/8*(b+a*x^(1/3))*(105*b^(1/2)*x*a^3+35*x^(2/3)*b^(3/2)*a^2-14*x^(1/3)*b^(
5/2)*a+8*b^(7/2)-105*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*(b+a*x^(1/3))^(1/
2)*x*a^3)/(b*x^(2/3)+a*x)^(3/2)/b^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(a*x + b*x**(2/3))**(3/2)), x)
```

---

**Giac [A]** time = 1.23205, size = 142, normalized size = 0.97

$$-\frac{105\,a^{3}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{8\,\sqrt{-b}b^{4}} - \frac{6\,a^{3}}{\sqrt{ax^{\frac{1}{3}} + b}\,b^{4}} - \frac{57\left(ax^{\frac{1}{3}} + b\right)^{\frac{5}{2}}a^{3} - 136\left(ax^{\frac{1}{3}} + b\right)^{\frac{3}{2}}a^{3}b + 87\sqrt{ax^{\frac{1}{3}} + b}\,a^{3}b^{2}}{8\,a^{3}b^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -105/8*a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) - 6*a^3/(sqr
t(a*x^(1/3) + b)*b^4) - 1/8*(57*(a*x^(1/3) + b)^(5/2)*a^3 - 136*(a*x^(1/3)
+ b)^(3/2)*a^3*b + 87*sqrt(a*x^(1/3) + b)*a^3*b^2)/(a^3*b^4*x)
```

## 3.200 $\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=236

$$\frac{9009a^5\sqrt{ax+bx^{2/3}}}{512b^7x^{2/3}}-\frac{3003a^4\sqrt{ax+bx^{2/3}}}{256b^6x}+\frac{3003a^3\sqrt{ax+bx^{2/3}}}{320b^5x^{4/3}}-\frac{1287a^2\sqrt{ax+bx^{2/3}}}{160b^4x^{5/3}}-\frac{9009a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{15/2}}+14$$

```
[Out] 6/(b*x^(5/3)*Sqrt[b*x^(2/3) + a*x]) - (13*Sqrt[b*x^(2/3) + a*x])/(2*b^2*x^(
7/3)) + (143*a*Sqrt[b*x^(2/3) + a*x])/(20*b^3*x^2) - (1287*a^2*Sqrt[b*x^(2/
3) + a*x])/(160*b^4*x^(5/3)) + (3003*a^3*Sqrt[b*x^(2/3) + a*x])/(320*b^5*x^
(4/3)) - (3003*a^4*Sqrt[b*x^(2/3) + a*x])/(256*b^6*x) + (9009*a^5*Sqrt[b*x^
(2/3) + a*x])/(512*b^7*x^(2/3)) - (9009*a^6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[
b*x^(2/3) + a*x]])/(512*b^(15/2))
```

---

**Rubi [A]** time = 0.410442, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2023, 2025, 2029, 206}

$$\frac{9009a^5\sqrt{ax+bx^{2/3}}}{512b^7x^{2/3}}-\frac{3003a^4\sqrt{ax+bx^{2/3}}}{256b^6x}+\frac{3003a^3\sqrt{ax+bx^{2/3}}}{320b^5x^{4/3}}-\frac{1287a^2\sqrt{ax+bx^{2/3}}}{160b^4x^{5/3}}-\frac{9009a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{15/2}}+14$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(5/3)*Sqrt[b*x^(2/3) + a*x]) - (13*Sqrt[b*x^(2/3) + a*x])/(2*b^2*x^(
7/3)) + (143*a*Sqrt[b*x^(2/3) + a*x])/(20*b^3*x^2) - (1287*a^2*Sqrt[b*x^(2/
3) + a*x])/(160*b^4*x^(5/3)) + (3003*a^3*Sqrt[b*x^(2/3) + a*x])/(320*b^5*x^
(4/3)) - (3003*a^4*Sqrt[b*x^(2/3) + a*x])/(256*b^6*x) + (9009*a^5*Sqrt[b*x^
(2/3) + a*x])/(512*b^7*x^(2/3)) - (9009*a^6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[
b*x^(2/3) + a*x]])/(512*b^(15/2))
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}+\frac{13\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}\\
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}-\frac{(143a)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{12b^2}\\
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}+\frac{\left(429a^2\right)\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx}{40b^3}\\
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}-\frac{\left(3003a^3\right)\int}{3}\\
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003a^3\sqrt{bx}}{320b^5}\\
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003a^3\sqrt{bx}}{320b^5}\\
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003a^3\sqrt{bx}}{320b^5}\\
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003a^3\sqrt{bx}}{320b^5}\\
&= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003a^3\sqrt{bx}}{320b^5}
\end{aligned}
$$

**Mathematica [C]** time = 0.0578095, size = 48, normalized size = 0.2

$$\frac{6a^6\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},7;\frac{1}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^7\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (6*a^6*x^(1/3)*Hypergeometric2F1[-1/2, 7, 1/2, 1 + (a*x^(1/3))/b])/(b^7*Sqr
t[b*x^(2/3) + a*x])
```

---

**Maple [A]** time = 0.017, size = 126, normalized size = 0.5

$$-\frac{1}{2560\,x}\left(b+a\sqrt[3]{x}\right)\left(45045\sqrt{b+a\sqrt[3]{x}}\,\mathrm{Artanh}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)x^2a^6-1664\,b^{11/2}\sqrt[3]{x}a+2288\,b^{9/2}x^{2/3}a^2-3432\,b^{7/2}xa^3+6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^(2/3)+a*x)^(3/2),x)
```

```
[Out] -1/2560*(b+a*x^(1/3))*(45045*(b+a*x^(1/3))^(1/2)*arctanh((b+a*x^(1/3))^(1/2
)/b^(1/2))*x^2*a^6-1664*b^(11/2)*x^(1/3)*a+2288*b^(9/2)*x^(2/3)*a^2-3432*b^
(7/2)*x*a^3+6006*b^(5/2)*x^(4/3)*a^4-15015*b^(3/2)*x^(5/3)*a^5-45045*x^2*a^
6*b^(1/2)+1280*b^(13/2))/x/(b*x^(2/3)+a*x)^(3/2)/b^(15/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}} x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a*x + b*x**(2/3))**(3/2)), x)
```

---

**Giac [A]** time = 1.29799, size = 211, normalized size = 0.89

$$\frac{9009\,a^6 \arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{512\,\sqrt{-b}b^7} + \frac{6\,a^6}{\sqrt{ax^{\frac{1}{3}} + b}b^7} + \frac{29685\left(ax^{\frac{1}{3}} + b\right)^{\frac{11}{2}} a^6 - 163095\left(ax^{\frac{1}{3}} + b\right)^{\frac{9}{2}} a^6 b + 364194\left(ax^{\frac{1}{3}} + b\right)^{\frac{7}{2}} a^6 b^2 - 41}{2560\,a^6 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 9009/512*a^6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) + 6*a^6/(s
qrt(a*x^(1/3) + b)*b^7) + 1/2560*(29685*(a*x^(1/3) + b)^(11/2)*a^6 - 163095
*(a*x^(1/3) + b)^(9/2)*a^6*b + 364194*(a*x^(1/3) + b)^(7/2)*a^6*b^2 - 41609
4*(a*x^(1/3) + b)^(5/2)*a^6*b^3 + 246505*(a*x^(1/3) + b)^(3/2)*a^6*b^4 - 62
475*sqrt(a*x^(1/3) + b)*a^6*b^5)/(a^6*b^7*x^2)
```

## 3.201 $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=324

$$-\frac{692835a^8\sqrt{ax+bx^{2/3}}}{32768b^{10}x^{2/3}}+\frac{230945a^7\sqrt{ax+bx^{2/3}}}{16384b^9x}-\frac{46189a^6\sqrt{ax+bx^{2/3}}}{4096b^8x^{4/3}}+\frac{138567a^5\sqrt{ax+bx^{2/3}}}{14336b^7x^{5/3}}-\frac{46189a^4\sqrt{ax+bx^{2/3}}}{5376b^6x^2}$$

```
[Out] 6/(b*x^(8/3)*Sqrt[b*x^(2/3) + a*x]) - (19*Sqrt[b*x^(2/3) + a*x])/(3*b^2*x^(
10/3)) + (323*a*Sqrt[b*x^(2/3) + a*x])/(48*b^3*x^3) - (1615*a^2*Sqrt[b*x^(2
/3) + a*x])/(224*b^4*x^(8/3)) + (20995*a^3*Sqrt[b*x^(2/3) + a*x])/(2688*b^5
*x^(7/3)) - (46189*a^4*Sqrt[b*x^(2/3) + a*x])/(5376*b^6*x^2) + (138567*a^5*
Sqrt[b*x^(2/3) + a*x])/(14336*b^7*x^(5/3)) - (46189*a^6*Sqrt[b*x^(2/3) + a*
x])/(4096*b^8*x^(4/3)) + (230945*a^7*Sqrt[b*x^(2/3) + a*x])/(16384*b^9*x) -
 (692835*a^8*Sqrt[b*x^(2/3) + a*x])/(32768*b^10*x^(2/3)) + (692835*a^9*ArcT
anh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(32768*b^(21/2))
```

---

**Rubi [A]** time = 0.597076, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2023, 2025, 2029, 206}

$$-\frac{692835a^8\sqrt{ax+bx^{2/3}}}{32768b^{10}x^{2/3}}+\frac{230945a^7\sqrt{ax+bx^{2/3}}}{16384b^9x}-\frac{46189a^6\sqrt{ax+bx^{2/3}}}{4096b^8x^{4/3}}+\frac{138567a^5\sqrt{ax+bx^{2/3}}}{14336b^7x^{5/3}}-\frac{46189a^4\sqrt{ax+bx^{2/3}}}{5376b^6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(8/3)*Sqrt[b*x^(2/3) + a*x]) - (19*Sqrt[b*x^(2/3) + a*x])/(3*b^2*x^(
10/3)) + (323*a*Sqrt[b*x^(2/3) + a*x])/(48*b^3*x^3) - (1615*a^2*Sqrt[b*x^(2
/3) + a*x])/(224*b^4*x^(8/3)) + (20995*a^3*Sqrt[b*x^(2/3) + a*x])/(2688*b^5
*x^(7/3)) - (46189*a^4*Sqrt[b*x^(2/3) + a*x])/(5376*b^6*x^2) + (138567*a^5*
Sqrt[b*x^(2/3) + a*x])/(14336*b^7*x^(5/3)) - (46189*a^6*Sqrt[b*x^(2/3) + a*
x])/(4096*b^8*x^(4/3)) + (230945*a^7*Sqrt[b*x^(2/3) + a*x])/(16384*b^9*x) -
 (692835*a^8*Sqrt[b*x^(2/3) + a*x])/(32768*b^10*x^(2/3)) + (692835*a^9*ArcT
anh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(32768*b^(21/2))
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}+\frac{19\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}\\
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}-\frac{(323a)\int \frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{18b^2}\\
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}+\frac{\left(1615a^2\right)\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx}{96b^3}\\
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}-\frac{\left(20995a^3\right)}{1}\\
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{20995a^3\sqrt{}}{2688b}\\
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{20995a^3\sqrt{}}{2688b}\\
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{20995a^3\sqrt{}}{2688b}\\
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{20995a^3\sqrt{}}{2688b}\\
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{20995a^3\sqrt{}}{2688b}\\
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{20995a^3\sqrt{}}{2688b}\\
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{20995a^3\sqrt{}}{2688b}\\
&= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{20995a^3\sqrt{}}{2688b}
\end{aligned}
$$

**Mathematica [C]** time = 0.0566937, size = 48, normalized size = 0.15

$$
-\frac{6a^9\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},10;\frac{1}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^{10}\sqrt{ax+bx^{2/3}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (-6*a^9*x^(1/3)*Hypergeometric2F1[-1/2, 10, 1/2, 1 + (a*x^(1/3))/b])/(b^10*
Sqrt[b*x^(2/3) + a*x])
```

---

**Maple [A]** time = 0.018, size = 159, normalized size = 0.5

$$\frac{1}{688128\,x^{2}}\left(b+a\sqrt[3]{x}\right)\left|\left(-537472\,b^{11/2}x^{4/3}a^{4}+739024\,b^{9/2}x^{5/3}a^{5}-1108536\,b^{7/2}x^{2}a^{6}+1939938\,b^{5/2}x^{7/3}a^{7}-4849845\,b^{3/2}x^{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^(2/3)+a*x)^(3/2),x)
```

```
[Out] 1/688128*(b+a*x^(1/3))*(-537472*b^(11/2)*x^(4/3)*a^4+739024*b^(9/2)*x^(5/3)
*a^5-1108536*b^(7/2)*x^2*a^6+1939938*b^(5/2)*x^(7/3)*a^7-4849845*b^(3/2)*x^
(8/3)*a^8-14549535*x^3*a^9*b^(1/2)+272384*b^(17/2)*x^(1/3)*a-330752*b^(15/2
)*x^(2/3)*a^2+413440*b^(13/2)*x*a^3-229376*b^(19/2)+14549535*(b+a*x^(1/3))^
(1/2)*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*x^3*a^9)/x^2/(b*x^(2/3)+a*x)^(3/
2)/b^(21/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^3), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a*x + b*x**(2/3))**(3/2)), x)
```

---

**Giac [A]** time = 1.36586, size = 279, normalized size = 0.86

$$-\frac{692835\,a^{9}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{32768\,\sqrt{-b}b^{10}}-\frac{6\,a^{9}}{\sqrt{ax^{\frac{1}{3}}+b}\,b^{10}}-\frac{10420767\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{9}-88937058\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{9}b+334408914\,(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -692835/32768*a^9*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^10) - 6*
a^9/(sqrt(a*x^(1/3) + b)*b^10) - 1/688128*(10420767*(a*x^(1/3) + b)^(17/2)*
a^9 - 88937058*(a*x^(1/3) + b)^(15/2)*a^9*b + 334408914*(a*x^(1/3) + b)^(13
/2)*a^9*b^2 - 724860666*(a*x^(1/3) + b)^(11/2)*a^9*b^3 + 993296384*(a*x^(1/
3) + b)^(9/2)*a^9*b^4 - 884769030*(a*x^(1/3) + b)^(7/2)*a^9*b^5 + 503730990
*(a*x^(1/3) + b)^(5/2)*a^9*b^6 - 169799070*(a*x^(1/3) + b)^(3/2)*a^9*b^7 +
26738145*sqrt(a*x^(1/3) + b)*a^9*b^8)/(a^9*b^10*x^3)
```

## 3.202 $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

**Optimal**. Leaf size=412

$$\frac{50702925a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{13}x^{2/3}}-\frac{16900975a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^{12}x}+\frac{3380195a^{9}\sqrt{ax+bx^{2/3}}}{262144b^{11}x^{4/3}}-\frac{1448655a^{8}\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{5/3}}+\frac{482885a^{7}\sqrt{a}}{49152}$$

```
[Out] 6/(b*x^(11/3)*Sqrt[b*x^(2/3) + a*x]) - (25*Sqrt[b*x^(2/3) + a*x])/(4*b^2*x^
(13/3)) + (575*a*Sqrt[b*x^(2/3) + a*x])/(88*b^3*x^4) - (2415*a^2*Sqrt[b*x^(
2/3) + a*x])/(352*b^4*x^(11/3)) + (15295*a^3*Sqrt[b*x^(2/3) + a*x])/(2112*b
^5*x^(10/3)) - (260015*a^4*Sqrt[b*x^(2/3) + a*x])/(33792*b^6*x^3) + (185725
*a^5*Sqrt[b*x^(2/3) + a*x])/(22528*b^7*x^(8/3)) - (2414425*a^6*Sqrt[b*x^(2/
3) + a*x])/(270336*b^8*x^(7/3)) + (482885*a^7*Sqrt[b*x^(2/3) + a*x])/(49152
*b^9*x^2) - (1448655*a^8*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(5/3)) + (33
80195*a^9*Sqrt[b*x^(2/3) + a*x])/(262144*b^11*x^(4/3)) - (16900975*a^10*Sqr
t[b*x^(2/3) + a*x])/(1048576*b^12*x) + (50702925*a^11*Sqrt[b*x^(2/3) + a*x]
)/(2097152*b^13*x^(2/3)) - (50702925*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*
x^(2/3) + a*x]])/(2097152*b^(27/2))
```

---

**Rubi [A]** time = 0.83999, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2023, 2025, 2029, 206}

$$\frac{50702925a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{13}x^{2/3}}-\frac{16900975a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^{12}x}+\frac{3380195a^{9}\sqrt{ax+bx^{2/3}}}{262144b^{11}x^{4/3}}-\frac{1448655a^{8}\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{5/3}}+\frac{482885a^{7}\sqrt{a}}{49152}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(11/3)*Sqrt[b*x^(2/3) + a*x]) - (25*Sqrt[b*x^(2/3) + a*x])/(4*b^2*x^
(13/3)) + (575*a*Sqrt[b*x^(2/3) + a*x])/(88*b^3*x^4) - (2415*a^2*Sqrt[b*x^(
2/3) + a*x])/(352*b^4*x^(11/3)) + (15295*a^3*Sqrt[b*x^(2/3) + a*x])/(2112*b
^5*x^(10/3)) - (260015*a^4*Sqrt[b*x^(2/3) + a*x])/(33792*b^6*x^3) + (185725
*a^5*Sqrt[b*x^(2/3) + a*x])/(22528*b^7*x^(8/3)) - (2414425*a^6*Sqrt[b*x^(2/
3) + a*x])/(270336*b^8*x^(7/3)) + (482885*a^7*Sqrt[b*x^(2/3) + a*x])/(49152
*b^9*x^2) - (1448655*a^8*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(5/3)) + (33
80195*a^9*Sqrt[b*x^(2/3) + a*x])/(262144*b^11*x^(4/3)) - (16900975*a^10*Sqr
t[b*x^(2/3) + a*x])/(1048576*b^12*x) + (50702925*a^11*Sqrt[b*x^(2/3) + a*x]
)/(2097152*b^13*x^(2/3)) - (50702925*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*
x^(2/3) + a*x]])/(2097152*b^(27/2))
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}+\frac{25\int \frac{1}{x^{14/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}-\frac{(575a)\int \frac{1}{x^{13/3}\sqrt{bx^{2/3}+ax}}\,dx}{24b^2}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}+\frac{\left(4025a^2\right)\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx}{176b^3}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}-\frac{\left(15295a^3\right)\int}{70}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2}}}{2112b^5x^{1}}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2}}}{2112b^5x^{1}}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2}}}{2112b^5x^{1}}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2}}}{2112b^5x^{1}}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2}}}{2112b^5x^{1}}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2}}}{2112b^5x^{1}}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2}}}{2112b^5x^{1}}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2}}}{2112b^5x^{1}}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2}}}{2112b^5x^{1}}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2}}}{2112b^5x^{1}}\\
&= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}}-\frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}}+\frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4}-\frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}}+\frac{15295a^3\sqrt{bx^{2}}}{2112b^5x^{1}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0589611, size = 48, normalized size = 0.12

$$\frac{6a^{12}\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},13;\frac{1}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^{13}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (6*a^12*x^(1/3)*Hypergeometric2F1[-1/2, 13, 1/2, 1 + (a*x^(1/3))/b])/(b^13*
Sqrt[b*x^(2/3) + a*x])
```

---

**Maple [A]** time = 0.025, size = 192, normalized size = 0.5

$$-\frac{1}{69206016\,x^3}\left(b+a\sqrt[3]{x}\right)\Bigg(17301504\,b^{\frac{25}{2}}+1673196525\sqrt{b+a\sqrt[3]{x}}\mathrm{Artanh}\left(\frac{\sqrt{b+a\sqrt[3]{x}}}{\sqrt{b}}\right)x^4a^{12}-19660800\,b^{23/2}\sqrt[3]{x}a+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^(2/3)+a*x)^(3/2),x)
```

```
[Out] -1/69206016*(b+a*x^(1/3))*(17301504*b^(25/2)+1673196525*(b+a*x^(1/3))^(1/2)
*arctanh((b+a*x^(1/3))^(1/2)/b^(1/2))*x^4*a^12-19660800*b^(23/2)*x^(1/3)*a+
22609920*b^(21/2)*x^(2/3)*a^2-26378240*b^(19/2)*x*a^3+31324160*b^(17/2)*x^(
4/3)*a^4-38036480*b^(15/2)*x^(5/3)*a^5+47545600*b^(13/2)*x^2*a^6-61809280*b
^(11/2)*x^(7/3)*a^7+84987760*b^(9/2)*x^(8/3)*a^8-127481640*b^(7/2)*x^3*a^9+
223092870*b^(5/2)*x^(10/3)*a^10-557732175*b^(3/2)*x^(11/3)*a^11-1673196525*
x^4*a^12*b^(1/2))/x^3/(b*x^(2/3)+a*x)^(3/2)/b^(27/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^4), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.51867, size = 348, normalized size = 0.84

$$\frac{50702925\,a^{12}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{2097152\,\sqrt{-b}b^{13}} + \frac{6\,a^{12}}{\sqrt{ax^{\frac{1}{3}}+b}\,b^{13}} + \frac{1257960429\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}a^{12} - 14537792973\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{12}b + 76667}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 50702925/2097152*a^12*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^13)
+ 6*a^12/(sqrt(a*x^(1/3) + b)*b^13) + 1/69206016*(1257960429*(a*x^(1/3) + b
)^(23/2)*a^12 - 14537792973*(a*x^(1/3) + b)^(21/2)*a^12*b + 76667241519*(a*
x^(1/3) + b)^(19/2)*a^12*b^2 - 243717614415*(a*x^(1/3) + b)^(17/2)*a^12*b^3
 + 519393101810*(a*x^(1/3) + b)^(15/2)*a^12*b^4 - 780150847218*(a*x^(1/3) +
 b)^(13/2)*a^12*b^5 + 844265343246*(a*x^(1/3) + b)^(11/2)*a^12*b^6 - 659969
685518*(a*x^(1/3) + b)^(9/2)*a^12*b^7 + 366679446705*(a*x^(1/3) + b)^(7/2)*
a^12*b^8 - 138840292305*(a*x^(1/3) + b)^(5/2)*a^12*b^9 + 32660709939*(a*x^(
1/3) + b)^(3/2)*a^12*b^10 - 3724872723*sqrt(a*x^(1/3) + b)*a^12*b^11)/(a^12
*b^13*x^4)
```

## 3.203 $\int x^2 \left(ax^2+bx^3\right) dx$

**Optimal**. Leaf size=17

$$\frac{ax^5}{5}+\frac{bx^6}{6}$$

```
[Out] (a*x^5)/5 + (b*x^6)/6
```

---

**Rubi [A]** time = 0.0088074, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{ax^5}{5}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3),x]
```

```
[Out] (a*x^5)/5 + (b*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax^2+bx^3\right)dx&=\int\left(ax^4+bx^5\right)dx\\&=\frac{ax^5}{5}+\frac{bx^6}{6}\end{aligned}$$

**Mathematica [A]** time = 0.0011068, size = 17, normalized size = 1.

$$\frac{ax^5}{5}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3),x]
```

```
[Out] (a*x^5)/5 + (b*x^6)/6
```

---

**Maple [A]** time = 0.001, size = 14, normalized size = 0.8

$$\frac{ax^5}{5}+\frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2),x)
```

```
[Out] 1/5*a*x^5+1/6*b*x^6
```

---

**Maxima [A]** time = 0.994284, size = 18, normalized size = 1.06

$$\frac{1}{6}\,bx^{6}+\frac{1}{5}\,ax^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/5*a*x^5
```

---

**Fricas [A]** time = 1.06124, size = 31, normalized size = 1.82

$$\frac{1}{6}x^{6}b+\frac{1}{5}x^{5}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b + 1/5*x^5*a
```

---

**Sympy [A]** time = 0.116661, size = 12, normalized size = 0.71

$$\frac{ax^{5}}{5}+\frac{bx^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2),x)
```

```
[Out] a*x**5/5 + b*x**6/6
```

---

**Giac [A]** time = 1.14835, size = 18, normalized size = 1.06

$$\frac{1}{6}\,bx^{6}+\frac{1}{5}\,ax^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6 + 1/5*a*x^5
```

## 3.204 $\int x\left(ax^2+bx^3\right)\,dx$

**Optimal**. Leaf size=17

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

```
[Out] (a*x^4)/4 + (b*x^5)/5
```

---

**Rubi [A]** time = 0.0079168, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3),x]

[Out] (a*x^4)/4 + (b*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(ax^2+bx^3\right)\,dx &= \int \left(ax^3+bx^4\right)\,dx\\ &= \frac{ax^4}{4}+\frac{bx^5}{5}\end{aligned}$$

**Mathematica [A]** time = 0.0010626, size = 17, normalized size = 1.

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3),x]

[Out] (a*x^4)/4 + (b*x^5)/5
```

---

**Maple [A]** time = 0.001, size = 14, normalized size = 0.8

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2),x)
```

```
[Out] 1/4*a*x^4+1/5*b*x^5
```

---

**Maxima [A]** time = 0.988774, size = 18, normalized size = 1.06

$$\frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

---

**Fricas [A]** time = 1.02401, size = 31, normalized size = 1.82

$$\frac{1}{5}x^5b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b + 1/4*x^4*a
```

---

**Sympy [A]** time = 0.142072, size = 12, normalized size = 0.71

$$\frac{ax^4}{4} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2),x)
```

```
[Out] a*x**4/4 + b*x**5/5
```

---

**Giac [A]** time = 1.10421, size = 18, normalized size = 1.06

$$\frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

## 3.205 $\int \left(ax^2+bx^3\right) dx$

**Optimal**. Leaf size=17

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

---

**Rubi [A]** time = 0.0029354, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a*x^2 + b*x^3,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

Rubi steps

$$\int \left(ax^2+bx^3\right) dx=\frac{ax^3}{3}+\frac{bx^4}{4}$$

**Mathematica [A]** time = 0.0000287, size = 17, normalized size = 1.

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*x^2 + b*x^3,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

---

**Maple [A]** time = 0., size = 14, normalized size = 0.8

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^3+a*x^2,x)
```

```
[Out] 1/3*a*x^3+1/4*b*x^4
```

---

**Maxima [A]** time = 0.98957, size = 18, normalized size = 1.06

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

---

**Fricas [A]** time = 1.09005, size = 31, normalized size = 1.82

$$\frac{1}{4}x^4b+\frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x^2,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b + 1/3*x^3*a
```

---

**Sympy [A]** time = 0.217801, size = 12, normalized size = 0.71

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**3+a*x**2,x)
```

```
[Out] a*x**3/3 + b*x**4/4
```

---

**Giac [A]** time = 1.15425, size = 18, normalized size = 1.06

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

## 3.206 $\int \frac{ax^2+bx^3}{x} \, dx$

**Optimal**. Leaf size=17

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

```
[Out] (a*x^2)/2 + (b*x^3)/3
```

---

**Rubi [A]** time = 0.006029, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)/x,x]

[Out] (a*x^2)/2 + (b*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{ax^2+bx^3}{x} \, dx &= \int \left(ax+bx^2\right) \, dx\\ &= \frac{ax^2}{2}+\frac{bx^3}{3}\end{aligned}$$

**Mathematica [A]** time = 0.0011616, size = 17, normalized size = 1.

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)/x,x]

[Out] (a*x^2)/2 + (b*x^3)/3
```

---

**Maple [A]** time = 0., size = 14, normalized size = 0.8

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)/x,x)
```

```
[Out] 1/2*a*x^2+1/3*b*x^3
```

---

**Maxima [A]** time = 0.985271, size = 18, normalized size = 1.06

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

---

**Fricas [A]** time = 0.958315, size = 31, normalized size = 1.82

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

---

**Sympy [A]** time = 0.08106, size = 12, normalized size = 0.71

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)/x,x)
```

```
[Out] a*x**2/2 + b*x**3/3
```

---

**Giac [A]** time = 1.19771, size = 18, normalized size = 1.06

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

## 3.207 $\int \frac{ax^2+bx^3}{x^2} dx$

**Optimal**. Leaf size=12

$$ax+\frac{bx^2}{2}$$

```
[Out] a*x + (b*x^2)/2
```

---

**Rubi [A]** time = 0.0035858, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)/x^2,x]
```

```
[Out] a*x + (b*x^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{ax^2+bx^3}{x^2}\,dx &= \int (a+bx)\,dx\\ &= ax+\frac{bx^2}{2}\end{aligned}$$

**Mathematica [A]** time = 0.000439, size = 12, normalized size = 1.

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)/x^2,x]
```

```
[Out] a*x + (b*x^2)/2
```

---

**Maple [A]** time = 0.001, size = 11, normalized size = 0.9

$$ax+\frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)/x^2,x)
```

```
[Out] a*x+1/2*b*x^2
```

---

**Maxima [A]** time = 0.974904, size = 14, normalized size = 1.17

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

---

**Fricas [A]** time = 0.801145, size = 23, normalized size = 1.92

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

---

**Sympy [A]** time = 0.100523, size = 8, normalized size = 0.67

$$ax + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)/x**2,x)
```

```
[Out] a*x + b*x**2/2
```

---

**Giac [A]** time = 1.13288, size = 14, normalized size = 1.17

$$\frac{1}{2}bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

## 3.208 $\int x^2 \left(ax^2+bx^3\right)^2 \, dx$

**Optimal**. Leaf size=30

$$\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9
```

---

**Rubi [A]** time = 0.0314118, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax^2+bx^3\right)^2\,dx&=\int x^6(a+bx)^2\,dx\\&=\int\left(a^2x^6+2abx^7+b^2x^8\right)dx\\&=\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}\end{aligned}$$

**Mathematica [A]** time = 0.0023497, size = 30, normalized size = 1.

$$\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9
```

---

**Maple [A]** time = 0., size = 25, normalized size = 0.8

$$\frac{a^2x^7}{7}+\frac{abx^8}{4}+\frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2)^2,x)
```

```
[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*b^2*x^9
```

---

**Maxima [A]** time = 0.986406, size = 32, normalized size = 1.07

$$\frac{1}{9}\,b^2x^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

---

**Fricas [A]** time = 0.574869, size = 55, normalized size = 1.83

$$\frac{1}{9}x^9b^2+\frac{1}{4}x^8ba+\frac{1}{7}x^7a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^2 + 1/4*x^8*b*a + 1/7*x^7*a^2
```

---

**Sympy [A]** time = 0.122817, size = 24, normalized size = 0.8

$$\frac{a^2x^7}{7}+\frac{abx^8}{4}+\frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**7/7 + a*b*x**8/4 + b**2*x**9/9
```

---

**Giac [A]** time = 1.1433, size = 32, normalized size = 1.07

$$\frac{1}{9}\,b^2x^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

## 3.209 $\int x\left(ax^2+bx^3\right)^2\,dx$

**Optimal**. Leaf size=30

$$\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8
```

---

**Rubi [A]** time = 0.0170845, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 43}

$$\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(ax^2+bx^3\right)^2\,dx &= \int x^5(a+bx)^2\,dx\\ &= \int\left(a^2x^5+2abx^6+b^2x^7\right)dx\\ &= \frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}\end{aligned}$$

**Mathematica [A]** time = 0.001907, size = 30, normalized size = 1.

$$\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8
```

---

**Maple [A]** time = 0., size = 25, normalized size = 0.8

$$\frac{a^2x^6}{6}+\frac{2\,abx^7}{7}+\frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2)^2,x)
```

```
[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*b^2*x^8
```

---

**Maxima [A]** time = 0.979917, size = 32, normalized size = 1.07

$$\frac{1}{8}\,b^2x^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

---

**Fricas [A]** time = 0.617166, size = 55, normalized size = 1.83

$$\frac{1}{8}x^8b^2+\frac{2}{7}x^7ba+\frac{1}{6}x^6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^2 + 2/7*x^7*b*a + 1/6*x^6*a^2
```

---

**Sympy [A]** time = 0.242338, size = 26, normalized size = 0.87

$$\frac{a^2x^6}{6}+\frac{2abx^7}{7}+\frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**6/6 + 2*a*b*x**7/7 + b**2*x**8/8
```

---

**Giac [A]** time = 1.13299, size = 32, normalized size = 1.07

$$\frac{1}{8}\,b^2x^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

## 3.210 $\int \left(ax^2+bx^3\right)^2 \, dx$

**Optimal**. Leaf size=30

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

---

**Rubi [A]** time = 0.0134057, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 43}

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(ax^2+bx^3\right)^2 \, dx &= \int x^4(a+bx)^2 \, dx\\ &= \int \left(a^2x^4+2abx^5+b^2x^6\right) dx\\ &= \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}\end{aligned}$$

**Mathematica [A]** time = 0.0015421, size = 30, normalized size = 1.

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

---

**Maple [A]** time = 0., size = 25, normalized size = 0.8

$$\frac{a^2x^5}{5}+\frac{abx^6}{3}+\frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^2,x)
```

```
[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*b^2*x^7
```

---

**Maxima [A]** time = 0.993275, size = 32, normalized size = 1.07

$$\frac{1}{7}\,b^2x^7+\frac{1}{3}\,abx^6+\frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

---

**Fricas [A]** time = 0.626779, size = 55, normalized size = 1.83

$$\frac{1}{7}x^7b^2+\frac{1}{3}x^6ba+\frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^2 + 1/3*x^6*b*a + 1/5*x^5*a^2
```

---

**Sympy [A]** time = 0.122, size = 24, normalized size = 0.8

$$\frac{a^2x^5}{5}+\frac{abx^6}{3}+\frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**5/5 + a*b*x**6/3 + b**2*x**7/7
```

---

**Giac [A]** time = 1.15384, size = 32, normalized size = 1.07

$$\frac{1}{7}\,b^2x^7+\frac{1}{3}\,abx^6+\frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

## 3.211 $\int \frac{(ax^2+bx^3)^2}{x}\,dx$

**Optimal**. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

---

**Rubi [A]** time = 0.0174214, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax^2+bx^3\right)^2}{x}\,dx &= \int x^3(a+bx)^2\,dx\\ &= \int \left(a^2x^3+2abx^4+b^2x^5\right)dx\\ &= \frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}\end{aligned}$$

**Mathematica [A]** time = 0.0016031, size = 30, normalized size = 1.

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

---

**Maple [A]** time = 0.002, size = 25, normalized size = 0.8

$$\frac{a^2x^4}{4}+\frac{2\,abx^5}{5}+\frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^2/x,x)
```

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

---

**Maxima [A]** time = 1.01225, size = 32, normalized size = 1.07

$$\frac{1}{6}\,b^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

---

**Fricas [A]** time = 0.704881, size = 55, normalized size = 1.83

$$\frac{1}{6}\,b^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

---

**Sympy [A]** time = 0.239738, size = 26, normalized size = 0.87

$$\frac{a^2x^4}{4}+\frac{2abx^5}{5}+\frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**2/x,x)
```

```
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + b**2*x**6/6
```

---

**Giac [A]** time = 1.14748, size = 32, normalized size = 1.07

$$\frac{1}{6}\,b^2x^6+\frac{2}{5}\,abx^5+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

## 3.212 $\int \frac{\left(ax^2+bx^3\right)^2}{x^2} \, dx$

**Optimal**. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

---

**Rubi [A]** time = 0.0163855, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^2/x^2,x]

[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax^2+bx^3\right)^2}{x^2} \, dx &= \int x^2(a+bx)^2 \, dx\\ &= \int \left(a^2x^2+2abx^3+b^2x^4\right) dx\\ &= \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}\end{aligned}$$

**Mathematica [A]** time = 0.0015831, size = 30, normalized size = 1.

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

---

**Maple [A]** time = 0., size = 25, normalized size = 0.8

$$\frac{x^3a^2}{3}+\frac{abx^4}{2}+\frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^2/x^2,x)
```

```
[Out] 1/3*x^3*a^2+1/2*a*b*x^4+1/5*b^2*x^5
```

---

**Maxima [A]** time = 0.993497, size = 32, normalized size = 1.07

$$\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

---

**Fricas [A]** time = 0.71121, size = 55, normalized size = 1.83

$$\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

---

**Sympy [A]** time = 0.219798, size = 24, normalized size = 0.8

$$\frac{a^2x^3}{3}+\frac{abx^4}{2}+\frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**2/x**2,x)
```

```
[Out] a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5
```

---

**Giac [A]** time = 1.20632, size = 32, normalized size = 1.07

$$\frac{1}{5}\,b^2x^5+\frac{1}{2}\,abx^4+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

## 3.213 $\int \frac{x^6}{ax^2+bx^3}\,dx$

**Optimal**. Leaf size=57

$$\frac{a^2x^2}{2b^3}-\frac{a^3x}{b^4}+\frac{a^4\log(a+bx)}{b^5}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

---

**Rubi [A]** time = 0.0355117, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2x^2}{2b^3}-\frac{a^3x}{b^4}+\frac{a^4\log(a+bx)}{b^5}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{ax^2+bx^3}\,dx &= \int \frac{x^4}{a+bx}\,dx\\ &= \int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx\\ &= -\frac{a^3x}{b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}+\frac{a^4\log(a+bx)}{b^5}\end{aligned}$$

**Mathematica [A]** time = 0.0045484, size = 57, normalized size = 1.

$$\frac{a^2x^2}{2b^3}-\frac{a^3x}{b^4}+\frac{a^4\log(a+bx)}{b^5}-\frac{ax^3}{3b^2}+\frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

---

**Maple [A]** time = 0.003, size = 52, normalized size = 0.9

$$-\frac{xa^{3}}{b^{4}}+\frac{a^{2}x^{2}}{2\,b^{3}}-\frac{ax^{3}}{3\,b^{2}}+\frac{x^{4}}{4\,b}+\frac{a^{4}\ln\left(bx+a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a*x^2),x)
```

```
[Out] -a^3*x/b^4+1/2*a^2*x^2/b^3-1/3*a*x^3/b^2+1/4*x^4/b+a^4*ln(b*x+a)/b^5
```

---

**Maxima [A]** time = 0.961211, size = 70, normalized size = 1.23

$$\frac{a^{4}\log\left(bx+a\right)}{b^{5}}+\frac{3\,b^{3}x^{4}-4\,ab^{2}x^{3}+6\,a^{2}bx^{2}-12\,a^{3}x}{12\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] a^4*log(b*x + a)/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3
*x)/b^4
```

---

**Fricas [A]** time = 0.734538, size = 117, normalized size = 2.05

$$\frac{3\,b^{4}x^{4}-4\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}-12\,a^{3}bx+12\,a^{4}\log\left(bx+a\right)}{12\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))/b^5
```

---

**Sympy [A]** time = 0.721893, size = 49, normalized size = 0.86

$$\frac{a^{4}\log\left(a+bx\right)}{b^{5}}-\frac{a^{3}x}{b^{4}}+\frac{a^{2}x^{2}}{2b^{3}}-\frac{ax^{3}}{3b^{2}}+\frac{x^{4}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x**2),x)
```

```
[Out] a**4*log(a + b*x)/b**5 - a**3*x/b**4 + a**2*x**2/(2*b**3) - a*x**3/(3*b**2)
 + x**4/(4*b)
```

---

**Giac [A]** time = 1.14571, size = 72, normalized size = 1.26

$$\frac{a^4 \log(|bx + a|)}{b^5} + \frac{3\, b^3 x^4 - 4\, ab^2 x^3 + 6\, a^2 b x^2 - 12\, a^3 x}{12\, b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] a^4*log(abs(b*x + a))/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 1
2*a^3*x)/b^4
```

## 3.214 $\int \frac{x^5}{ax^2+bx^3}\,dx$

**Optimal**. Leaf size=44

$$\frac{a^2x}{b^3}-\frac{a^3\log(a+bx)}{b^4}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

---

**Rubi [A]** time = 0.0270089, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2x}{b^3}-\frac{a^3\log(a+bx)}{b^4}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{ax^2+bx^3}\,dx &= \int \frac{x^3}{a+bx}\,dx\\ &= \int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx\\ &= \frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}-\frac{a^3\log(a+bx)}{b^4}\end{aligned}$$

**Mathematica [A]** time = 0.003585, size = 44, normalized size = 1.

$$\frac{a^2x}{b^3}-\frac{a^3\log(a+bx)}{b^4}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

---

**Maple [A]** time = 0.002, size = 41, normalized size = 0.9

$$\frac{a^2x}{b^3} - \frac{ax^2}{2\,b^2} + \frac{x^3}{3\,b} - \frac{a^3\ln\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a*x^2),x)
```

```
[Out] a^2*x/b^3-1/2*a*x^2/b^2+1/3*x^3/b-a^3*ln(b*x+a)/b^4
```

---

**Maxima [A]** time = 0.971426, size = 57, normalized size = 1.3

$$-\frac{a^3\log\left(bx+a\right)}{b^4} + \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] -a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

---

**Fricas [A]** time = 0.872357, size = 92, normalized size = 2.09

$$\frac{2\,b^3x^3 - 3\,ab^2x^2 + 6\,a^2bx - 6\,a^3\log\left(bx+a\right)}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

---

**Sympy [A]** time = 0.486927, size = 37, normalized size = 0.84

$$-\frac{a^3\log\left(a+bx\right)}{b^4} + \frac{a^2x}{b^3} - \frac{ax^2}{2b^2} + \frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x**2),x)
```

```
[Out] -a**3*log(a + b*x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b)
```

---

**Giac [A]** time = 1.12933, size = 58, normalized size = 1.32

$$-\frac{a^3\log\left(|bx+a|\right)}{b^4} + \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

## 3.215 $\int \frac{x^4}{ax^2+bx^3}\,dx$

**Optimal**. Leaf size=31

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

---

**Rubi [A]** time = 0.0205414, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{ax^2+bx^3}\,dx &= \int \frac{x^2}{a+bx}\,dx\\ &= \int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx\\ &= -\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}\end{aligned}$$

**Mathematica [A]** time = 0.003362, size = 31, normalized size = 1.

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

---

**Maple [A]** time = 0.001, size = 30, normalized size = 1.

$$-\frac{ax}{b^2}+\frac{x^2}{2\,b}+\frac{a^2\ln\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2),x)
```

```
[Out] -a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

---

**Maxima [A]** time = 0.977369, size = 39, normalized size = 1.26

$$\frac{a^2\log\left(bx+a\right)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

---

**Fricas [A]** time = 0.689718, size = 68, normalized size = 2.19

$$\frac{b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

---

**Sympy [A]** time = 0.624748, size = 26, normalized size = 0.84

$$\frac{a^2\log\left(a+bx\right)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2),x)
```

```
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

---

**Giac [A]** time = 1.15661, size = 41, normalized size = 1.32

$$\frac{a^2\log\left(|bx+a|\right)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

## 3.216 $\int \frac{x^3}{ax^2+bx^3} dx$

**Optimal**. Leaf size=18

$$\frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

---

**Rubi [A]** time = 0.0163976, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{ax^2+bx^3}\,dx &= \int \frac{x}{a+bx}\,dx\\ &= \int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right)dx\\ &= \frac{x}{b} - \frac{a\log(a+bx)}{b^2}\end{aligned}$$

**Mathematica [A]** time = 0.0025374, size = 18, normalized size = 1.

$$\frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

---

**Maple [A]** time = 0.002, size = 19, normalized size = 1.1

$$\frac{x}{b}-\frac{a\ln\left(bx+a\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2),x)
```

```
[Out] x/b-a*ln(b*x+a)/b^2
```

---

**Maxima [A]** time = 0.989485, size = 24, normalized size = 1.33

$$\frac{x}{b}-\frac{a\log\left(bx+a\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] x/b - a*log(b*x + a)/b^2
```

---

**Fricas [A]** time = 0.83147, size = 38, normalized size = 2.11

$$\frac{bx-a\log\left(bx+a\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] (b*x - a*log(b*x + a))/b^2
```

---

**Sympy [A]** time = 0.647865, size = 14, normalized size = 0.78

$$-\frac{a\log\left(a+bx\right)}{b^{2}}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2),x)
```

```
[Out] -a*log(a + b*x)/b**2 + x/b
```

---

**Giac [A]** time = 1.15236, size = 26, normalized size = 1.44

$$\frac{x}{b}-\frac{a\log\left(|bx+a|\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] x/b - a*log(abs(b*x + a))/b^2
```

## 3.217 $\int \frac{x^2}{ax^2+bx^3}\,dx$

**Optimal**. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

```
[Out] Log[a + b*x]/b
```

---

**Rubi [A]** time = 0.0072511, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 31}

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3),x]
```

```
[Out] Log[a + b*x]/b
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{ax^2+bx^3}\,dx &= \int \frac{1}{a+bx}\,dx\\ &= \frac{\log(a+bx)}{b}\end{aligned}$$

**Mathematica [A]** time = 0.0009937, size = 10, normalized size = 1.

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3),x]
```

```
[Out] Log[a + b*x]/b
```

---

**Maple [A]** time = 0.001, size = 11, normalized size = 1.1

$$\frac{\ln(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2),x)
```

```
[Out] ln(b*x+a)/b
```

---

**Maxima [A]** time = 1.09744, size = 14, normalized size = 1.4

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/b
```

---

**Fricas [A]** time = 0.761894, size = 22, normalized size = 2.2

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

---

**Sympy [A]** time = 0.229684, size = 7, normalized size = 0.7

$$\frac{\log(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2),x)
```

```
[Out] log(a + b*x)/b
```

---

**Giac [A]** time = 1.15313, size = 15, normalized size = 1.5

$$\frac{\log(|bx+a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b
```

## 3.218 $\int \frac{x}{ax^2+bx^3}\,dx$

**Optimal**. Leaf size=18

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

```
[Out] Log[x]/a - Log[a + b*x]/a
```

---

**Rubi [A]** time = 0.0074465, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1584, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{ax^2+bx^3}\,dx&=\int \frac{1}{x(a+bx)}\,dx\\&=\frac{\int \frac{1}{x}\,dx}{a}-\frac{b\int \frac{1}{a+bx}\,dx}{a}\\&=\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}\end{aligned}$$

**Mathematica [A]** time = 0.0035121, size = 18, normalized size = 1.

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

---

**Maple [A]** time = 0.005, size = 19, normalized size = 1.1

$$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2),x)
```

```
[Out] ln(x)/a-ln(b*x+a)/a
```

---

**Maxima [A]** time = 0.978949, size = 24, normalized size = 1.33

$$-\frac{\log(bx+a)}{a}+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] -log(b*x + a)/a + log(x)/a
```

---

**Fricas [A]** time = 0.781125, size = 38, normalized size = 2.11

$$-\frac{\log(bx+a)-\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] -(log(b*x + a) - log(x))/a
```

---

**Sympy [A]** time = 0.542956, size = 10, normalized size = 0.56

$$\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2),x)
```

```
[Out] (log(x) - log(a/b + x))/a
```

---

**Giac [A]** time = 1.15968, size = 27, normalized size = 1.5

$$-\frac{\log\left(|bx+a|\right)}{a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

## 3.219 $\int \frac{1}{ax^2+bx^3}\,dx$

**Optimal**. Leaf size=28

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

---

**Rubi [A]** time = 0.0146112, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 44}

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(-1),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax^2+bx^3}\,dx &= \int \frac{1}{x^2(a+bx)}\,dx\\ &= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\ &= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}\end{aligned}$$

**Mathematica [A]** time = 0.0041116, size = 28, normalized size = 1.

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(-1),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

---

**Maple [A]** time = 0.006, size = 29, normalized size = 1.

$$-\frac{1}{ax}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{b\ln\left(bx+a\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2),x)
```

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

---

**Maxima [A]** time = 1.01665, size = 38, normalized size = 1.36

$$\frac{b\log\left(bx+a\right)}{a^{2}}-\frac{b\log\left(x\right)}{a^{2}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

---

**Fricas [A]** time = 0.795504, size = 61, normalized size = 2.18

$$\frac{bx\log\left(bx+a\right)-bx\log\left(x\right)-a}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

---

**Sympy [A]** time = 0.714865, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2),x)
```

```
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

---

**Giac [A]** time = 1.14083, size = 41, normalized size = 1.46

$$\frac{b\log\left(|bx+a|\right)}{a^{2}}-\frac{b\log\left(|x|\right)}{a^{2}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

## 3.220 $\int \frac{1}{x\left(ax^2+bx^3\right)}\,dx$

**Optimal**. Leaf size=42

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

---

**Rubi [A]** time = 0.0212892, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 44}

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(ax^2+bx^3\right)}\,dx &= \int \frac{1}{x^3(a+bx)}\,dx\\ &= \int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}\end{aligned}$$

**Mathematica [A]** time = 0.0043671, size = 42, normalized size = 1.

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

---

**Maple [A]** time = 0.007, size = 41, normalized size = 1.

$$-\frac{1}{2\,a{x}^{2}}+\frac{b}{{a}^{2}x}+\frac{{b}^{2}\ln \left( x \right)}{{a}^{3}}-\frac{{b}^{2}\ln \left( bx+a \right)}{{a}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2),x)
```

```
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

---

**Maxima [A]** time = 1.04069, size = 54, normalized size = 1.29

$$-\frac{b^{2} \log \left(b x + a\right)}{a^{3}} + \frac{b^{2} \log \left(x\right)}{a^{3}} + \frac{2 \, b x - a}{2 \, a^{2} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

---

**Fricas [A]** time = 0.780534, size = 103, normalized size = 2.45

$$-\frac{2 \, b^{2} x^{2} \log \left(b x + a\right) - 2 \, b^{2} x^{2} \log \left(x\right) - 2 \, a b x + a^{2}}{2 \, a^{3} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

---

**Sympy [A]** time = 1.03191, size = 31, normalized size = 0.74

$$\frac{- a + 2 b x}{2 a^{2} x^{2}} + \frac{b^{2} \left(\log{\left(x \right)} - \log{\left(\frac{a}{b} + x \right)}\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2),x)
```

```
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

---

**Giac [A]** time = 1.15355, size = 61, normalized size = 1.45

$$-\frac{b^2 \log\left(|bx+a|\right)}{a^3}+\frac{b^2 \log\left(|x|\right)}{a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

## 3.221 $\int \frac{1}{x^2(ax^2+bx^3)}\,dx$

**Optimal**. Leaf size=56

$$-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[a
+ b*x])/a^4
```

---

**Rubi [A]** time = 0.0267151, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 44}

$$-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[a
+ b*x])/a^4
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(ax^2+bx^3\right)}\,dx &= \int \frac{1}{x^4(a+bx)}\,dx\\ &= \int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}\end{aligned}$$

**Mathematica [A]** time = 0.0048084, size = 56, normalized size = 1.

$$-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[a
+ b*x])/a^4
```

---

**Maple [A]** time = 0.006, size = 53, normalized size = 1.

$$-\frac{1}{3\,a{x}^{3}}+\frac{b}{2\,{a}^{2}{x}^{2}}-\frac{{b}^{2}}{x{a}^{3}}-\frac{{b}^{3}\ln \left( x \right)}{{a}^{4}}+\frac{{b}^{3}\ln \left( bx+a \right)}{{a}^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x^2),x)
```

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

---

**Maxima [A]** time = 0.971695, size = 69, normalized size = 1.23

$$\frac{b^{3}\log \left(b x + a\right)}{a^{4}} - \frac{b^{3}\log \left(x\right)}{a^{4}} - \frac{6\,b^{2}x^{2} - 3\,abx + 2\,a^{2}}{6\,a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/a^4 - b^3*log(x)/a^4 - 1/6*(6*b^2*x^2 - 3*a*b*x + 2*a^2)/(
a^3*x^3)
```

---

**Fricas [A]** time = 0.725751, size = 126, normalized size = 2.25

$$\frac{6\,b^{3}x^{3}\log \left(b x + a\right) - 6\,b^{3}x^{3}\log \left(x\right) - 6\,ab^{2}x^{2} + 3\,a^{2}bx - 2\,a^{3}}{6\,a^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(b*x + a) - 6*b^3*x^3*log(x) - 6*a*b^2*x^2 + 3*a^2*b*x -
2*a^3)/(a^4*x^3)
```

---

**Sympy [A]** time = 1.20975, size = 44, normalized size = 0.79

$$-\frac{2a^{2} - 3abx + 6b^{2}x^{2}}{6a^{3}x^{3}} + \frac{b^{3}\left(-\log{\left(x \right)} + \log{\left(\frac{a}{b} + x \right)}\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x**2),x)
```

```
[Out] -(2*a**2 - 3*a*b*x + 6*b**2*x**2)/(6*a**3*x**3) + b**3*(-log(x) + log(a/b +
 x))/a**4
```

---

**Giac [A]** time = 1.20949, size = 76, normalized size = 1.36

$$\frac{b^3 \log\left(|bx+a|\right)}{a^4} - \frac{b^3 \log\left(|x|\right)}{a^4} - \frac{6\,ab^2x^2 - 3\,a^2bx + 2\,a^3}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] b^3*log(abs(b*x + a))/a^4 - b^3*log(abs(x))/a^4 - 1/6*(6*a*b^2*x^2 - 3*a^2*
b*x + 2*a^3)/(a^4*x^3)
```

## 3.222 $\int \frac{x^8}{\left(ax^2+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=58

$$-\frac{a^4}{b^5(a+bx)}+\frac{3a^2x}{b^4}-\frac{4a^3\log(a+bx)}{b^5}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

```
[Out] (3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*Lo
g[a + b*x])/b^5
```

---

**Rubi [A]** time = 0.040053, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$-\frac{a^4}{b^5(a+bx)}+\frac{3a^2x}{b^4}-\frac{4a^3\log(a+bx)}{b^5}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*Lo
g[a + b*x])/b^5
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(ax^2+bx^3\right)^2}\,dx &= \int \frac{x^4}{(a+bx)^2}\,dx\\ &= \int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx\\ &= \frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}\end{aligned}$$

**Mathematica [A]** time = 0.0200056, size = 54, normalized size = 0.93

$$\frac{-\frac{3a^4}{a+bx}+9a^2bx-12a^3\log(a+bx)-3ab^2x^2+b^3x^3}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 - (3*a^4)/(a + b*x) - 12*a^3*Log[a + b*x
])/(3*b^5)
```

---

**Maple [A]** time = 0.006, size = 57, normalized size = 1.

$$3\frac{a^{2}x}{b^{4}}-\frac{ax^{2}}{b^{3}}+\frac{x^{3}}{3\,b^{2}}-\frac{a^{4}}{b^{5}\left(bx+a\right)}-4\,\frac{a^{3}\ln\left(bx+a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a*x^2)^2,x)
```

```
[Out] 3*a^2*x/b^4-a*x^2/b^3+1/3*x^3/b^2-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

---

**Maxima [A]** time = 0.976477, size = 80, normalized size = 1.38

$$-\frac{a^{4}}{b^{6}x+ab^{5}}-\frac{4\,a^{3}\log\left(bx+a\right)}{b^{5}}+\frac{b^{2}x^{3}-3\,abx^{2}+9\,a^{2}x}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -a^4/(b^6*x + a*b^5) - 4*a^3*log(b*x + a)/b^5 + 1/3*(b^2*x^3 - 3*a*b*x^2 +
9*a^2*x)/b^4
```

---

**Fricas [A]** time = 0.67851, size = 155, normalized size = 2.67

$$\frac{b^{4}x^{4}-2\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}+9\,a^{3}bx-3\,a^{4}-12\left(a^{3}bx+a^{4}\right)\log\left(bx+a\right)}{3\left(b^{6}x+ab^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

---

**Sympy [A]** time = 1.19462, size = 54, normalized size = 0.93

$$-\frac{a^{4}}{ab^{5}+b^{6}x}-\frac{4a^{3}\log\left(a+bx\right)}{b^{5}}+\frac{3a^{2}x}{b^{4}}-\frac{ax^{2}}{b^{3}}+\frac{x^{3}}{3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a*x**2)**2,x)
```

```
[Out] -a**4/(a*b**5 + b**6*x) - 4*a**3*log(a + b*x)/b**5 + 3*a**2*x/b**4 - a*x**2
/b**3 + x**3/(3*b**2)
```

---

**Giac [A]** time = 1.15278, size = 84, normalized size = 1.45

$$-\frac{4\,a^{3}\log\left(|bx+a|\right)}{b^{5}}-\frac{a^{4}}{(bx+a)b^{5}}+\frac{b^{4}x^{3}-3\,ab^{3}x^{2}+9\,a^{2}b^{2}x}{3\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*a^3*log(abs(b*x + a))/b^5 - a^4/((b*x + a)*b^5) + 1/3*(b^4*x^3 - 3*a*b^3
*x^2 + 9*a^2*b^2*x)/b^6
```

## 3.223 $\int \frac{x^7}{\left(ax^2+bx^3\right)^2} \, dx$

**Optimal**. Leaf size=46

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

```
[Out] (-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^4
```

---

**Rubi [A]** time = 0.0301855, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^4
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(ax^2+bx^3\right)^2} \, dx &= \int \frac{x^3}{(a+bx)^2} \, dx\\ &= \int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right) dx\\ &= -\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}\end{aligned}$$

**Mathematica [A]** time = 0.0136239, size = 43, normalized size = 0.93

$$\frac{\frac{2a^3}{a+bx}+6a^2\log(a+bx)-4abx+b^2x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])/(2*b^4)
```

---

**Maple [A]** time = 0.006, size = 45, normalized size = 1.

$$-2\,{\frac {ax}{{b}^{3}}}+{\frac {{x}^{2}}{2\,{b}^{2}}}+{\frac {{a}^{3}}{{b}^{4} \left( bx+a \right) }}+3\,{\frac {{a}^{2}\ln \left( bx+a \right) }{{b}^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a*x^2)^2,x)
```

```
[Out] -2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

---

**Maxima [A]** time = 0.993047, size = 63, normalized size = 1.37

$$\frac{a^{3}}{b^{5} x + a b^{4}} + \frac{3 \, a^{2} \log\left(b x + a\right)}{b^{4}} + \frac{b x^{2} - 4 \, a x}{2 \, b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] a^3/(b^5*x + a*b^4) + 3*a^2*log(b*x + a)/b^4 + 1/2*(b*x^2 - 4*a*x)/b^3
```

---

**Fricas [A]** time = 0.7575, size = 132, normalized size = 2.87

$$\frac{b^{3} x^{3} - 3 \, a b^{2} x^{2} - 4 \, a^{2} b x + 2 \, a^{3} + 6 \,{\left(a^{2} b x + a^{3}\right)} \log\left(b x + a\right)}{2 \,{\left(b^{5} x + a b^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))/(b^5*x + a*b^4)
```

---

**Sympy [A]** time = 0.969929, size = 44, normalized size = 0.96

$$\frac{a^{3}}{a b^{4} + b^{5} x} + \frac{3 a^{2} \log{\left (a + b x \right )}}{b^{4}} - \frac{2 a x}{b^{3}} + \frac{x^{2}}{2 b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a*x**2)**2,x)
```

```
[Out] a**3/(a*b**4 + b**5*x) + 3*a**2*log(a + b*x)/b**4 - 2*a*x/b**3 + x**2/(2*b*
*2)
```

---

**Giac [A]** time = 1.23639, size = 65, normalized size = 1.41

$$\frac{3\,a^2\log\left(|bx+a|\right)}{b^4}+\frac{a^3}{(bx+a)b^4}+\frac{b^2x^2-4\,abx}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 3*a^2*log(abs(b*x + a))/b^4 + a^3/((b*x + a)*b^4) + 1/2*(b^2*x^2 - 4*a*b*x)
/b^4
```

## 3.224 $\int \frac{x^6}{\left(ax^2+bx^3\right)^2} \, dx$

**Optimal**. Leaf size=33

$$-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}$$

```
[Out] x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

---

**Rubi [A]** time = 0.023916, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3)^2,x]
```

```
[Out] x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(ax^2+bx^3\right)^2} \, dx &= \int \frac{x^2}{(a+bx)^2} \, dx\\ &= \int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right) dx\\ &= \frac{x}{b^2}-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}\end{aligned}$$

**Mathematica [A]** time = 0.0122328, size = 29, normalized size = 0.88

$$\frac{-\frac{a^2}{a+bx}-2a\log(a+bx)+bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3
```

---

**Maple [A]** time = 0.006, size = 34, normalized size = 1.

$$\frac{x}{b^2}-\frac{a^2}{b^3\left(bx+a\right)}-2\,\frac{a\ln\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a*x^2)^2,x)
```

```
[Out] x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

---

**Maxima [A]** time = 0.991994, size = 49, normalized size = 1.48

$$-\frac{a^2}{b^4x+ab^3}+\frac{x}{b^2}-\frac{2\,a\log\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3
```

---

**Fricas [A]** time = 0.780113, size = 97, normalized size = 2.94

$$\frac{b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)}{b^4x+ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

---

**Sympy [A]** time = 0.558101, size = 31, normalized size = 0.94

$$-\frac{a^2}{ab^3+b^4x}-\frac{2a\log\left(a+bx\right)}{b^3}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x**2)**2,x)
```

```
[Out] -a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2
```

---

**Giac [A]** time = 1.16743, size = 46, normalized size = 1.39

$$\frac{x}{b^2} - \frac{2\,a\log\left(|bx + a|\right)}{b^3} - \frac{a^2}{(bx + a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] x/b^2 - 2*a*log(abs(b*x + a))/b^3 - a^2/((b*x + a)*b^3)
```

## 3.225 $\int \frac{x^5}{\left(ax^2+bx^3\right)^2} dx$

**Optimal**. Leaf size=23

$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

```
[Out] a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

---

**Rubi [A]** time = 0.0180811, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3)^2,x]
```

```
[Out] a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(ax^2+bx^3\right)^2} dx &= \int \frac{x}{(a+bx)^2} dx\\ &= \int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right) dx\\ &= \frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}\end{aligned}$$

**Mathematica [A]** time = 0.0063073, size = 20, normalized size = 0.87

$$\frac{\frac{a}{a+bx}+\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a/(a + b*x) + Log[a + b*x])/b^2
```

---

**Maple [A]** time = 0.005, size = 24, normalized size = 1.

$$\frac{a}{b^2\left(bx+a\right)}+\frac{\ln\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a*x^2)^2,x)
```

```
[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

---

**Maxima [A]** time = 0.983391, size = 35, normalized size = 1.52

$$\frac{a}{b^3x+ab^2}+\frac{\log\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

---

**Fricas [A]** time = 0.765177, size = 62, normalized size = 2.7

$$\frac{\left(bx+a\right)\log\left(bx+a\right)+a}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] ((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

---

**Sympy [A]** time = 0.527788, size = 20, normalized size = 0.87

$$\frac{a}{ab^2+b^3x}+\frac{\log\left(a+bx\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x**2)**2,x)
```

```
[Out] a/(a*b**2 + b**3*x) + log(a + b*x)/b**2
```

---

**Giac [A]** time = 1.19752, size = 32, normalized size = 1.39

$$\frac{\log\left(|bx+a|\right)}{b^2}+\frac{a}{(bx+a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^2 + a/((b*x + a)*b^2)
```

## 3.226 $\int \frac{x^4}{\left(ax^2+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

```
[Out] -(1/(b*(a + b*x)))
```

---

**Rubi [A]** time = 0.0073729, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(ax^2+bx^3\right)^2}\,dx &= \int \frac{1}{(a+bx)^2}\,dx\\ &= -\frac{1}{b(a+bx)}\end{aligned}$$

**Mathematica [A]** time = 0.0026073, size = 12, normalized size = 1.

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -(1/(b*(a + b*x)))
```

---

**Maple [A]** time = 0.001, size = 13, normalized size = 1.1

$$-\frac{1}{b\,(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2)^2,x)
```

```
[Out] -1/b/(b*x+a)
```

---

**Maxima [A]** time = 0.988879, size = 18, normalized size = 1.5

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/(b^2*x + a*b)
```

---

**Fricas [A]** time = 0.789475, size = 24, normalized size = 2.

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

---

**Sympy [A]** time = 0.920552, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2)**2,x)
```

```
[Out] -1/(a*b + b**2*x)
```

---

**Giac [A]** time = 1.1492, size = 16, normalized size = 1.33

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/((b*x + a)*b)
```

## 3.227 $\int \frac{x^3}{\left(ax^2+bx^3\right)^2} \, dx$

**Optimal**. Leaf size=29

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

```
[Out] 1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

---

**Rubi [A]** time = 0.0188643, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 44}

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3)^2,x]
```

```
[Out] 1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(ax^2+bx^3\right)^2} \, dx &= \int \frac{1}{x(a+bx)^2} \, dx\\ &= \int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right) dx\\ &= \frac{1}{a(a+bx)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx)}{a^2}\end{aligned}$$

**Mathematica [A]** time = 0.0099306, size = 24, normalized size = 0.83

$$\frac{\frac{a}{a+bx}-\log(a+bx)+\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a/(a + b*x) + Log[x] - Log[a + b*x])/a^2
```

---

**Maple [A]** time = 0.006, size = 30, normalized size = 1.

$$\frac{1}{a\left(bx+a\right)}+\frac{\ln\left(x\right)}{a^{2}}-\frac{\ln\left(bx+a\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2)^2,x)
```

```
[Out] 1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

---

**Maxima [A]** time = 1.01662, size = 38, normalized size = 1.31

$$\frac{1}{abx+a^{2}}-\frac{\log\left(bx+a\right)}{a^{2}}+\frac{\log\left(x\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

---

**Fricas [A]** time = 0.730418, size = 89, normalized size = 3.07

$$-\frac{(bx+a)\log\left(bx+a\right)-(bx+a)\log\left(x\right)-a}{a^{2}bx+a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] -((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

---

**Sympy [A]** time = 1.03874, size = 22, normalized size = 0.76

$$\frac{1}{a^{2}+abx}+\frac{\log\left(x\right)-\log\left(\frac{a}{b}+x\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2)**2,x)
```

```
[Out] 1/(a**2 + a*b*x) + (log(x) - log(a/b + x))/a**2
```

---

**Giac [A]** time = 1.14525, size = 42, normalized size = 1.45

$$-\frac{\log\left(|bx+a|\right)}{a^{2}}+\frac{\log\left(|x|\right)}{a^{2}}+\frac{1}{(bx+a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^2 + log(abs(x))/a^2 + 1/((b*x + a)*a)
```

## 3.228 $\int \frac{x^2}{\left(ax^2+bx^3\right)^2} \, dx$

**Optimal**. Leaf size=42

$$-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{1}{a^2x}$$

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

---

**Rubi [A]** time = 0.0239151, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 44}

$$-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(ax^2+bx^3\right)^2} \, dx &= \int \frac{1}{x^2(a+bx)^2} \, dx\\ &= \int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right) dx\\ &= -\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}\end{aligned}$$

**Mathematica [A]** time = 0.0374035, size = 35, normalized size = 0.83

$$-\frac{a\left(\frac{b}{a+bx}+\frac{1}{x}\right)-2b\log(a+bx)+2b\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

---

**Maple [A]** time = 0.009, size = 43, normalized size = 1.

$$-\frac{1}{a^{2}x}-\frac{b}{a^{2}\left(bx+a\right)}-2\,\frac{b\ln\left(x\right)}{a^{3}}+2\,\frac{b\ln\left(bx+a\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2)^2,x)
```

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

---

**Maxima [A]** time = 1.00111, size = 61, normalized size = 1.45

$$-\frac{2\,bx+a}{a^{2}bx^{2}+a^{3}x}+\frac{2\,b\log\left(bx+a\right)}{a^{3}}-\frac{2\,b\log\left(x\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

---

**Fricas [A]** time = 0.725405, size = 138, normalized size = 3.29

$$-\frac{2\,abx+a^{2}-2\left(b^{2}x^{2}+abx\right)\log\left(bx+a\right)+2\left(b^{2}x^{2}+abx\right)\log\left(x\right)}{a^{3}bx^{2}+a^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

---

**Sympy [A]** time = 1.24111, size = 36, normalized size = 0.86

$$-\frac{a+2bx}{a^{3}x+a^{2}bx^{2}}+\frac{2b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2)**2,x)
```

```
[Out] -(a + 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

---

**Giac [A]** time = 1.16753, size = 61, normalized size = 1.45

$$\frac{2\,b\log\left(|bx+a|\right)}{a^3}-\frac{2\,b\log\left(|x|\right)}{a^3}-\frac{2\,bx+a}{\left(bx^2+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*b*log(abs(b*x + a))/a^3 - 2*b*log(abs(x))/a^3 - (2*b*x + a)/((b*x^2 + a*x
)*a^2)
```

# 3.229 $\int \frac{x}{\left(ax^2+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=58

$$\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

```
[Out] -1/(2*a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^4 -
 (3*b^2*Log[a + b*x])/a^4
```

---

**Rubi [A]** time = 0.0315931, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 44}

$$\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -1/(2*a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^4 -
 (3*b^2*Log[a + b*x])/a^4
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(ax^2+bx^3\right)^2}\,dx &= \int \frac{1}{x^3(a+bx)^2}\,dx\\ &= \int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}\end{aligned}$$

**Mathematica [A]** time = 0.0500844, size = 53, normalized size = 0.91

$$\frac{a\left(\frac{2b^2}{a+bx}-\frac{a}{x^2}+\frac{4b}{x}\right)-6b^2\log(a+bx)+6b^2\log(x)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a*(-(a/x^2) + (4*b)/x + (2*b^2)/(a + b*x)) + 6*b^2*Log[x] - 6*b^2*Log[a +
b*x])/(2*a^4)
```

---

**Maple [A]** time = 0.01, size = 57, normalized size = 1.

$$-\frac{1}{2\,a^{2}x^{2}}+2\,\frac{b}{xa^{3}}+\frac{b^{2}}{a^{3}\left(bx+a\right)}+3\,\frac{b^{2}\ln\left(x\right)}{a^{4}}-3\,\frac{b^{2}\ln\left(bx+a\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2)^2,x)
```

```
[Out] -1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

---

**Maxima [A]** time = 1.01718, size = 86, normalized size = 1.48

$$\frac{6\,b^{2}x^{2}+3\,abx-a^{2}}{2\left(a^{3}bx^{3}+a^{4}x^{2}\right)}-\frac{3\,b^{2}\log\left(bx+a\right)}{a^{4}}+\frac{3\,b^{2}\log\left(x\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)/
a^4 + 3*b^2*log(x)/a^4
```

---

**Fricas [A]** time = 0.791073, size = 177, normalized size = 3.05

$$\frac{6\,ab^{2}x^{2}+3\,a^{2}bx-a^{3}-6\left(b^{3}x^{3}+ab^{2}x^{2}\right)\log\left(bx+a\right)+6\left(b^{3}x^{3}+ab^{2}x^{2}\right)\log\left(x\right)}{2\left(a^{4}bx^{3}+a^{5}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a) +
 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

---

**Sympy [A]** time = 0.759451, size = 54, normalized size = 0.93

$$\frac{-a^{2}+3abx+6b^{2}x^{2}}{2a^{4}x^{2}+2a^{3}bx^{3}}+\frac{3b^{2}\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x + 6*b**2*x**2)/(2*a**4*x**2 + 2*a**3*b*x**3) + 3*b**2*(log
(x) - log(a/b + x))/a**4
```

---

**Giac [A]** time = 1.18427, size = 86, normalized size = 1.48

$$-\frac{3\,b^2\log\left(|bx+a|\right)}{a^4}+\frac{3\,b^2\log\left(|x|\right)}{a^4}+\frac{6\,ab^2x^2+3\,a^2bx-a^3}{2\,(bx+a)a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -3*b^2*log(abs(b*x + a))/a^4 + 3*b^2*log(abs(x))/a^4 + 1/2*(6*a*b^2*x^2 + 3
*a^2*b*x - a^3)/((b*x + a)*a^4*x^2)
```

## 3.230 $\int \frac{1}{\left(ax^2+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=69

$$-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

```
[Out] -1/(3*a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (4*b
^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

---

**Rubi [A]** time = 0.0389185, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 44}

$$-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(-2),x]
```

```
[Out] -1/(3*a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (4*b
^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(ax^2+bx^3\right)^2}\,dx &= \int \frac{1}{x^4(a+bx)^2}\,dx\\ &= \int \left(\frac{1}{a^2x^4}-\frac{2b}{a^3x^3}+\frac{3b^2}{a^4x^2}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{4b^4}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}\end{aligned}$$

**Mathematica [A]** time = 0.0528533, size = 66, normalized size = 0.96

$$-\frac{\frac{a\left(-2a^2bx+a^3+6ab^2x^2+12b^3x^3\right)}{x^3(a+bx)}-12b^3\log(a+bx)+12b^3\log(x)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(-2),x]
```

```
[Out] -((a*(a^3 - 2*a^2*b*x + 6*a*b^2*x^2 + 12*b^3*x^3))/(x^3*(a + b*x)) + 12*b^3
*Log[x] - 12*b^3*Log[a + b*x])/(3*a^5)
```

---

**Maple [A]** time = 0.01, size = 68, normalized size = 1.

$$-\frac{1}{3\,x^3a^2}+\frac{b}{x^2a^3}-3\,\frac{b^2}{a^4x}-\frac{b^3}{a^4\,(bx+a)}-4\,\frac{b^3\ln(x)}{a^5}+4\,\frac{b^3\ln(bx+a)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2)^2,x)
```

```
[Out] -1/3/x^3/a^2+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*ln
(b*x+a)/a^5
```

---

**Maxima [A]** time = 0.986677, size = 99, normalized size = 1.43

$$-\frac{12\,b^3x^3+6\,ab^2x^2-2\,a^2bx+a^3}{3\left(a^4bx^4+a^5x^3\right)}+\frac{4\,b^3\log(bx+a)}{a^5}-\frac{4\,b^3\log(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 6*a*b^2*x^2 - 2*a^2*b*x + a^3)/(a^4*b*x^4 + a^5*x^3) + 4
*b^3*log(b*x + a)/a^5 - 4*b^3*log(x)/a^5
```

---

**Fricas [A]** time = 0.8104, size = 204, normalized size = 2.96

$$-\frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4-12\left(b^4x^4+ab^3x^3\right)\log(bx+a)+12\left(b^4x^4+ab^3x^3\right)\log(x)}{3\left(a^5bx^4+a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 6*a^2*b^2*x^2 - 2*a^3*b*x + a^4 - 12*(b^4*x^4 + a*b^3*
x^3)*log(b*x + a) + 12*(b^4*x^4 + a*b^3*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3)
```

---

**Sympy [A]** time = 1.76697, size = 66, normalized size = 0.96

$$-\frac{a^3-2a^2bx+6ab^2x^2+12b^3x^3}{3a^5x^3+3a^4bx^4}+\frac{4b^3\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2)**2,x)
```

```
[Out] -(a**3 - 2*a**2*b*x + 6*a*b**2*x**2 + 12*b**3*x**3)/(3*a**5*x**3 + 3*a**4*b
*x**4) + 4*b**3*(-log(x) + log(a/b + x))/a**5
```

---

**Giac [A]** time = 1.16329, size = 99, normalized size = 1.43

$$\frac{4\, b^3 \log\left(|bx + a|\right)}{a^5} - \frac{4\, b^3 \log\left(|x|\right)}{a^5} - \frac{12\, ab^3x^3 + 6\, a^2b^2x^2 - 2\, a^3bx + a^4}{3\,(bx + a)a^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 4*b^3*log(abs(b*x + a))/a^5 - 4*b^3*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 + 6
*a^2*b^2*x^2 - 2*a^3*b*x + a^4)/((b*x + a)*a^5*x^3)
```

## 3.231 $\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx$

**Optimal**. Leaf size=84

$$-\frac{3b^2}{2a^4x^2}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

```
[Out] -1/(4*a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*x)
+ b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6
```

---

**Rubi [A]** time = 0.052285, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 44}

$$-\frac{3b^2}{2a^4x^2}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3)^2),x]
```

```
[Out] -1/(4*a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*x)
+ b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx&=\int \frac{1}{x^5(a+bx)^2}\,dx\\&=\int \left(\frac{1}{a^2x^5}-\frac{2b}{a^3x^4}+\frac{3b^2}{a^4x^3}-\frac{4b^3}{a^5x^2}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{5b^5}{a^6(a+bx)}\right)dx\\&=-\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}\end{aligned}$$

**Mathematica [A]** time = 0.0426332, size = 79, normalized size = 0.94

$$\frac{\frac{a\left(-10a^2b^2x^2+5a^3bx-3a^4+30ab^3x^3+60b^4x^4\right)}{x^4(a+bx)}-60b^4\log(a+bx)+60b^4\log(x)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3)^2),x]
```

```
[Out] ((a*(-3*a^4 + 5*a^3*b*x - 10*a^2*b^2*x^2 + 30*a*b^3*x^3 + 60*b^4*x^4))/(x^4
*(a + b*x)) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(12*a^6)
```

---

**Maple [A]** time = 0.01, size = 79, normalized size = 0.9

$$-\frac{1}{4\,a^{2}x^{4}}+\frac{2\,b}{3\,a^{3}x^{3}}-\frac{3\,b^{2}}{2\,a^{4}x^{2}}+4\,\frac{b^{3}}{a^{5}x}+\frac{b^{4}}{a^{5}\left(bx+a\right)}+5\,\frac{b^{4}\ln\left(x\right)}{a^{6}}-5\,\frac{b^{4}\ln\left(bx+a\right)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2)^2,x)
```

```
[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b^
4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

---

**Maxima [A]** time = 1.10895, size = 116, normalized size = 1.38

$$\frac{60\,b^{4}x^{4}+30\,ab^{3}x^{3}-10\,a^{2}b^{2}x^{2}+5\,a^{3}bx-3\,a^{4}}{12\left(a^{5}bx^{5}+a^{6}x^{4}\right)}-\frac{5\,b^{4}\log\left(bx+a\right)}{a^{6}}+\frac{5\,b^{4}\log\left(x\right)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*b^4*x^4 + 30*a*b^3*x^3 - 10*a^2*b^2*x^2 + 5*a^3*b*x - 3*a^4)/(a^5*
b*x^5 + a^6*x^4) - 5*b^4*log(b*x + a)/a^6 + 5*b^4*log(x)/a^6
```

---

**Fricas [A]** time = 0.677263, size = 231, normalized size = 2.75

$$\frac{60\,ab^{4}x^{4}+30\,a^{2}b^{3}x^{3}-10\,a^{3}b^{2}x^{2}+5\,a^{4}bx-3\,a^{5}-60\left(b^{5}x^{5}+ab^{4}x^{4}\right)\log\left(bx+a\right)+60\left(b^{5}x^{5}+ab^{4}x^{4}\right)\log\left(x\right)}{12\left(a^{6}bx^{5}+a^{7}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*x^4 + 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5 -
60*(b^5*x^5 + a*b^4*x^4)*log(b*x + a) + 60*(b^5*x^5 + a*b^4*x^4)*log(x))/(a
^6*b*x^5 + a^7*x^4)
```

---

**Sympy [A]** time = 1.7031, size = 80, normalized size = 0.95

$$\frac{-3a^{4}+5a^{3}bx-10a^{2}b^{2}x^{2}+30ab^{3}x^{3}+60b^{4}x^{4}}{12a^{6}x^{4}+12a^{5}bx^{5}}+\frac{5b^{4}\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-3*a**4 + 5*a**3*b*x - 10*a**2*b**2*x**2 + 30*a*b**3*x**3 + 60*b**4*x**4)/
(12*a**6*x**4 + 12*a**5*b*x**5) + 5*b**4*(log(x) - log(a/b + x))/a**6
```

---

**Giac [A]** time = 1.14102, size = 116, normalized size = 1.38

$$-\frac{5\,b^4\log\left(|bx+a|\right)}{a^6}+\frac{5\,b^4\log\left(|x|\right)}{a^6}+\frac{60\,ab^4x^4+30\,a^2b^3x^3-10\,a^3b^2x^2+5\,a^4bx-3\,a^5}{12\,(bx+a)a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -5*b^4*log(abs(b*x + a))/a^6 + 5*b^4*log(abs(x))/a^6 + 1/12*(60*a*b^4*x^4 +
 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5)/((b*x + a)*a^6*x^4)
```

# 3.232 $\int x^2\sqrt{ax^2+bx^3}\,dx$

**Optimal**. Leaf size=105

$$-\frac{32a^3\left(ax^2+bx^3\right)^{3/2}}{315b^4x^3}+\frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{21b^2x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}$$

```
[Out] (2*(a*x^2 + b*x^3)^(3/2))/(9*b) - (32*a^3*(a*x^2 + b*x^3)^(3/2))/(315*b^4*x
^3) + (16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^2) - (4*a*(a*x^2 + b*x^3)^(
3/2))/(21*b^2*x)
```

---

**Rubi [A]** time = 0.119729, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{32a^3\left(ax^2+bx^3\right)^{3/2}}{315b^4x^3}+\frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{21b^2x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(3/2))/(9*b) - (32*a^3*(a*x^2 + b*x^3)^(3/2))/(315*b^4*x
^3) + (16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^2) - (4*a*(a*x^2 + b*x^3)^(
3/2))/(21*b^2*x)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{ax^2+bx^3}\,dx &= \frac{2\left(ax^2+bx^3\right)^{3/2}}{9b} - \frac{(2a)\int x\sqrt{ax^2+bx^3}\,dx}{3b}\\ &= \frac{2\left(ax^2+bx^3\right)^{3/2}}{9b} - \frac{4a\left(ax^2+bx^3\right)^{3/2}}{21b^2x} + \frac{\left(8a^2\right)\int\sqrt{ax^2+bx^3}\,dx}{21b^2}\\ &= \frac{2\left(ax^2+bx^3\right)^{3/2}}{9b} + \frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^2} - \frac{4a\left(ax^2+bx^3\right)^{3/2}}{21b^2x} - \frac{\left(16a^3\right)\int\frac{\sqrt{ax^2+bx^3}}{x}\,dx}{105b^3}\\ &= \frac{2\left(ax^2+bx^3\right)^{3/2}}{9b} - \frac{32a^3\left(ax^2+bx^3\right)^{3/2}}{315b^4x^3} + \frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^2} - \frac{4a\left(ax^2+bx^3\right)^{3/2}}{21b^2x}\end{aligned}$$

**Mathematica [A]** time = 0.0281838, size = 53, normalized size = 0.5

$$\frac{2\left(x^2(a+bx)\right)^{3/2}\left(24a^2bx-16a^3-30ab^2x^2+35b^3x^3\right)}{315b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(x^2*(a + b*x))^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x^3)
)/(315*b^4*x^3)
```

---

**Maple [A]** time = 0.004, size = 57, normalized size = 0.5

$$-\frac{(2\,bx+2\,a)\left(-35\,x^3b^3+30\,ab^2x^2-24\,a^2xb+16\,a^3\right)}{315\,b^4x}\sqrt{bx^3+ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/315*(b*x+a)*(-35*b^3*x^3+30*a*b^2*x^2-24*a^2*b*x+16*a^3)*(b*x^3+a*x^2)^(
1/2)/b^4/x
```

---

**Maxima [A]** time = 1.00444, size = 72, normalized size = 0.69

$$\frac{2\left(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4\right)\sqrt{bx+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

---

**Fricas [A]** time = 0.819516, size = 134, normalized size = 1.28

$$\frac{2\left(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4\right)\sqrt{bx^3+ax^2}}{315\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x^3 + a*x^2)/(b^4*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{x^2\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(a + b*x)), x)
```

---

**Giac [A]** time = 1.14418, size = 84, normalized size = 0.8

$$\frac{32\,a^{\frac{9}{2}}\mathrm{sgn}\left(x\right)}{315\,b^4}+\frac{2\left(35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^2-105\,(bx+a)^{\frac{3}{2}}a^3\right)\mathrm{sgn}\left(x\right)}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 32/315*a^(9/2)*sgn(x)/b^4 + 2/315*(35*(b*x + a)^(9/2) - 135*(b*x + a)^(7/2)
*a + 189*(b*x + a)^(5/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)*sgn(x)/b^4
```

## 3.233 $\int x\sqrt{ax^2+bx^3}\,dx$

**Optimal**. Leaf size=80

$$\frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^3}-\frac{8a\left(ax^2+bx^3\right)^{3/2}}{35b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}$$

```
[Out] (16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^3) - (8*a*(a*x^2 + b*x^3)^(3/2))/
(35*b^2*x^2) + (2*(a*x^2 + b*x^3)^(3/2))/(7*b*x)
```

---

**Rubi [A]** time = 0.073556, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2016, 2002, 2014}

$$\frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^3}-\frac{8a\left(ax^2+bx^3\right)^{3/2}}{35b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^3) - (8*a*(a*x^2 + b*x^3)^(3/2))/
(35*b^2*x^2) + (2*(a*x^2 + b*x^3)^(3/2))/(7*b*x)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int x\sqrt{ax^2+bx^3}\,dx &= \frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx} - \frac{(4a)\int \sqrt{ax^2+bx^3}\,dx}{7b} \\
&= -\frac{8a\left(ax^2+bx^3\right)^{3/2}}{35b^2x^2} + \frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx} + \frac{\left(8a^2\right)\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx}{35b^2} \\
&= \frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^3} - \frac{8a\left(ax^2+bx^3\right)^{3/2}}{35b^2x^2} + \frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}
\end{aligned}
$$

**Mathematica [A]** time = 0.0196859, size = 42, normalized size = 0.52

$$
\frac{2\left(x^2(a+bx)\right)^{3/2}\left(8a^2-12abx+15b^2x^2\right)}{105b^3x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(x^2*(a + b*x))^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3*x^3)
```

---

**Maple [A]** time = 0.003, size = 46, normalized size = 0.6

$$
\frac{(2\,bx+2\,a)\left(15\,b^2x^2-12\,abx+8\,a^2\right)}{105\,b^3x}\sqrt{bx^3+ax^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 2/105*(b*x+a)*(15*b^2*x^2-12*a*b*x+8*a^2)*(b*x^3+a*x^2)^(1/2)/b^3/x
```

---

**Maxima [A]** time = 1.01651, size = 57, normalized size = 0.71

$$
\frac{2\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{105\,b^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

---

**Fricas [A]** time = 0.81328, size = 111, normalized size = 1.39

$$
\frac{2\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx^3+ax^2}}{105\,b^3x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2)/(b
^3*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x^2\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(a + b*x)), x)
```

---

**Giac [A]** time = 1.15079, size = 68, normalized size = 0.85

$$-\frac{16\,a^{\frac{7}{2}}\operatorname{sgn}(x)}{105\,b^3}+\frac{2\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)\operatorname{sgn}(x)}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -16/105*a^(7/2)*sgn(x)/b^3 + 2/105*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)
*a + 35*(b*x + a)^(3/2)*a^2)*sgn(x)/b^3
```

## 3.234 $\int \sqrt{ax^2+bx^3}\,dx$

**Optimal**. Leaf size=52

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}$$

```
[Out] (-4*a*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/2))/(5*b*
x^2)
```

---

**Rubi [A]** time = 0.0430886, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2002, 2014}

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-4*a*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/2))/(5*b*
x^2)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{ax^2+bx^3}\,dx &= \frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{(2a)\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx}{5b}\\ &= -\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}\end{aligned}$$

**Mathematica [A]** time = 0.0139493, size = 31, normalized size = 0.6

$$\frac{2\left(x^2(a+bx)\right)^{3/2}(3bx-2a)}{15b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(x^2*(a + b*x))^(3/2)*(-2*a + 3*b*x))/(15*b^2*x^3)
```

---

**Maple [A]** time = 0.003, size = 35, normalized size = 0.7

$$-\frac{(2\,bx+2\,a)\,(-3\,bx+2\,a)}{15\,b^2x}\sqrt{bx^3+ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/15*(b*x+a)*(-3*b*x+2*a)*(b*x^3+a*x^2)^(1/2)/b^2/x
```

---

**Maxima [A]** time = 0.976278, size = 41, normalized size = 0.79

$$\frac{2\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

---

**Fricas [A]** time = 0.873243, size = 84, normalized size = 1.62

$$\frac{2\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx^3+ax^2}}{15\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x^3 + a*x^2)/(b^2*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\sqrt{ax^2+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2 + b*x**3), x)
```

---

**Giac [A]** time = 1.1859, size = 51, normalized size = 0.98

$$\frac{4\,a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}{15\,b^2}+\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)\mathrm{sgn}\left(x\right)}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*a^(5/2)*sgn(x)/b^2 + 2/15*(3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)*sg
n(x)/b^2
```

## 3.235 $\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx$

**Optimal**. Leaf size=25

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

```
[Out] (2*(a*x^2 + b*x^3)^(3/2))/(3*b*x^3)
```

---

**Rubi [A]** time = 0.0358451, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x,x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(3/2))/(3*b*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx=\frac{2\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

**Mathematica [A]** time = 0.0090109, size = 23, normalized size = 0.92

$$\frac{2\left(x^2(a+bx)\right)^{3/2}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x,x]
```

```
[Out] (2*(x^2*(a + b*x))^(3/2))/(3*b*x^3)
```

---

**Maple [A]** time = 0.001, size = 27, normalized size = 1.1

$$\frac{2\,bx+2\,a}{3\,bx}\sqrt{bx^3+ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x,x)
```

```
[Out] 2/3*(b*x+a)*(b*x^3+a*x^2)^(1/2)/b/x
```

---

**Maxima [A]** time = 1.00775, size = 16, normalized size = 0.64

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

---

**Fricas [A]** time = 0.823993, size = 55, normalized size = 2.2

$$\frac{2\,\sqrt{bx^3+ax^2}(bx+a)}{3\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^3 + a*x^2)*(b*x + a)/(b*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2\,(a+bx)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x, x)
```

---

**Giac [A]** time = 1.17417, size = 34, normalized size = 1.36

$$\frac{2\,(bx+a)^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,b}-\frac{2\,a^{\frac{3}{2}}\operatorname{sgn}(x)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2/3*(b*x + a)^(3/2)*sgn(x)/b - 2/3*a^(3/2)*sgn(x)/b
```

# 3.236 $\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx$

**Optimal**. Leaf size=51

$$\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]]
```

---

**Rubi [A]** time = 0.0490906, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2021, 2008, 206}

$$\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx &= \frac{2\sqrt{ax^2+bx^3}}{x}+a\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx\\ &= \frac{2\sqrt{ax^2+bx^3}}{x}-(2a)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\ &= \frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\end{aligned}$$

**Mathematica [A]** time = 0.0278593, size = 53, normalized size = 1.04

$$\frac{2x\left(-\sqrt{a}\sqrt{a+bx}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+a+bx\right)}{\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

```
[Out] (2*x*(a + b*x - Sqrt[a]*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/Sqrt
[x^2*(a + b*x)]
```

---

**Maple [A]** time = 0.006, size = 52, normalized size = 1.

$$-2\,\frac{\sqrt{bx^3+ax^2}}{x\sqrt{bx+a}}\left(\sqrt{a}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\sqrt{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^2,x)
```

```
[Out] -2*(b*x^3+a*x^2)^(1/2)*(a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))-(b*x+a)^(1/2
))/x/(b*x+a)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{bx^3+ax^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^2, x)
```

---

**Fricas [A]** time = 0.850497, size = 247, normalized size = 4.84

$$\left[\frac{\sqrt{a}x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}}{x},\ \frac{2\left(\sqrt{-a}x\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*sq
rt(b*x^3 + a*x^2))/x, 2*(sqrt(-a)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*
x)) + sqrt(b*x^3 + a*x^2))/x]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 (a + bx)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**2, x)
```

---

**Giac [A]** time = 1.21336, size = 88, normalized size = 1.73

$$2\left(\frac{a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{bx + a}\right)\operatorname{sgn}(x) - \frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) + \sqrt{-a}\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*(a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x + a))*sgn(x) - 2*(a
*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

## 3.237 $\int \frac{\sqrt{ax^2+bx^3}}{x^3} dx$

**Optimal**. Leaf size=52

$$-\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

```
[Out] -(Sqrt[a*x^2 + b*x^3]/x^2) - (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/S
qrt[a]
```

---

**Rubi [A]** time = 0.0519346, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2020, 2008, 206}

$$-\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3]/x^2) - (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/S
qrt[a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^2+bx^3}}{x^3} dx &= -\frac{\sqrt{ax^2+bx^3}}{x^2}+\frac{1}{2}b\int \frac{1}{\sqrt{ax^2+bx^3}} dx\\ &= -\frac{\sqrt{ax^2+bx^3}}{x^2}-b\,\text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)\\ &= -\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}\end{aligned}$$

**Mathematica [A]** time = 0.0360123, size = 48, normalized size = 0.92

$$-\frac{bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx}{\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

```
[Out] -((a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[x^2*(a
+ b*x)])
```

---

**Maple [A]** time = 0.01, size = 56, normalized size = 1.1

$$-\frac{1}{x^2}\sqrt{bx^3+ax^2}\left(\mathrm{Artanh}\left(\sqrt{bx+a}\frac{1}{\sqrt{a}}\right)bx+\sqrt{bx+a}\sqrt{a}\right)\frac{1}{\sqrt{bx+a}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^3,x)
```

```
[Out] -(b*x^3+a*x^2)^(1/2)*(arctanh((b*x+a)^(1/2)/a^(1/2))*b*x+(b*x+a)^(1/2)*a^(1
/2))/x^2/(b*x+a)^(1/2)/a^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3+ax^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^3, x)
```

---

**Fricas [A]** time = 0.869586, size = 282, normalized size = 5.42

$$\left[\frac{\sqrt{a}bx^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}a}{2\,ax^2},\ \frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)-\sqrt{bx^3+ax^2}a}{ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2
) - 2*sqrt(b*x^3 + a*x^2)*a)/(a*x^2), (sqrt(-a)*b*x^2*arctan(sqrt(b*x^3 + a
*x^2)*sqrt(-a)/(a*x)) - sqrt(b*x^3 + a*x^2)*a)/(a*x^2)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 (a+bx)}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**3, x)
```

---

**Giac [A]** time = 1.20562, size = 58, normalized size = 1.12

$$\frac{\left(\frac{b^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx+a}b}{x}\right)\operatorname{sgn}(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x + a)*b/x)*sgn(x)/b
```

# 3.238 $\int \frac{\sqrt{ax^2+bx^3}}{x^4} dx$

**Optimal**. Leaf size=84

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}} - \frac{b\sqrt{ax^2+bx^3}}{4ax^2} - \frac{\sqrt{ax^2+bx^3}}{2x^3}$$

```
[Out] -Sqrt[a*x^2 + b*x^3]/(2*x^3) - (b*Sqrt[a*x^2 + b*x^3])/(4*a*x^2) + (b^2*Arc
Tanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*a^(3/2))
```

---

**Rubi [A]** time = 0.0915473, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2008, 206}

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}} - \frac{b\sqrt{ax^2+bx^3}}{4ax^2} - \frac{\sqrt{ax^2+bx^3}}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3]/(2*x^3) - (b*Sqrt[a*x^2 + b*x^3])/(4*a*x^2) + (b^2*Arc
Tanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*a^(3/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{2x^3}+\frac{1}{4}b\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx \\
&= -\frac{\sqrt{ax^2+bx^3}}{2x^3}-\frac{b\sqrt{ax^2+bx^3}}{4ax^2}-\frac{b^2\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{8a} \\
&= -\frac{\sqrt{ax^2+bx^3}}{2x^3}-\frac{b\sqrt{ax^2+bx^3}}{4ax^2}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{4a} \\
&= -\frac{\sqrt{ax^2+bx^3}}{2x^3}-\frac{b\sqrt{ax^2+bx^3}}{4ax^2}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0118765, size = 42, normalized size = 0.5

$$
-\frac{2b^2\left(x^2(a+bx)\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx}{a}+1\right)}{3a^3x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

```
[Out] (-2*b^2*(x^2*(a + b*x))^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x)/a])/
(3*a^3*x^3)
```

---

**Maple [A]** time = 0.01, size = 73, normalized size = 0.9

$$
-\frac{1}{4\,x^3}\sqrt{bx^3+ax^2}\left(a^{\frac{3}{2}}\,(bx+a)^{\frac{3}{2}}-\text{Artanh}\left(\sqrt{bx+a}\,\frac{1}{\sqrt{a}}\right)ab^2x^2+a^{\frac{5}{2}}\sqrt{bx+a}\right)a^{-\frac{5}{2}}\frac{1}{\sqrt{bx+a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/4*(b*x^3+a*x^2)^(1/2)*(a^(3/2)*(b*x+a)^(3/2)-arctanh((b*x+a)^(1/2)/a^(1/
2))*a*b^2*x^2+a^(5/2)*(b*x+a)^(1/2))/x^3/(b*x+a)^(1/2)/a^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{bx^3+ax^2}}{x^4}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^4, x)
```

---

**Fricas [A]** time = 0.865725, size = 338, normalized size = 4.02

$$\left[\frac{\sqrt{a}b^2x^3\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}\left(abx+2\,a^2\right)}{8\,a^2x^3},-\frac{\sqrt{-a}b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\left(abx+2\right.}{4\,a^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x
^2) - 2*sqrt(b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/4*(sqrt(-a)*b^2*
x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(a*b*x
 + 2*a^2))/(a^2*x^3)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2\,(a+bx)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**4, x)
```

---

**Giac [A]** time = 1.21381, size = 92, normalized size = 1.1

$$-\frac{\left(\frac{b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{(bx+a)^{\frac{3}{2}}b^3+\sqrt{bx+a}ab^3}{ab^2x^2}\right)\mathrm{sgn}\left(x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x + a)^(3/2)*b^
3 + sqrt(b*x + a)*a*b^3)/(a*b^2*x^2))*sgn(x)/b
```

## 3.239 $\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx$

**Optimal**. Leaf size=112

$$\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

```
[Out] -Sqrt[a*x^2 + b*x^3]/(3*x^4) - (b*Sqrt[a*x^2 + b*x^3])/(12*a*x^3) + (b^2*Sq
rt[a*x^2 + b*x^3])/(8*a^2*x^2) - (b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]])/(8*a^(5/2))
```

---

**Rubi [A]** time = 0.137877, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2008, 206}

$$\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3]/(3*x^4) - (b*Sqrt[a*x^2 + b*x^3])/(12*a*x^3) + (b^2*Sq
rt[a*x^2 + b*x^3])/(8*a^2*x^2) - (b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]])/(8*a^(5/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{3x^4}+\frac{1}{6}b\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}-\frac{b^2\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{8a}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}+\frac{b^3\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a^2}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{8a^2}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.0124094, size = 42, normalized size = 0.38

$$\frac{2b^3\left(x^2(a+bx)\right)^{3/2}{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{bx}{a}+1\right)}{3a^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

```
[Out] (2*b^3*(x^2*(a + b*x))^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + (b*x)/a])/(
3*a^4*x^3)
```

---

**Maple [A]** time = 0.012, size = 89, normalized size = 0.8

$$-\frac{1}{24\,x^4}\sqrt{bx^3+ax^2}\left(3\,a^{9/2}\sqrt{bx+a}+8\,a^{7/2}\,(bx+a)^{3/2}-3\,a^{5/2}\,(bx+a)^{5/2}+3\,\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^2b^3x^3\right)a^{-\frac{9}{2}}\frac{1}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^5,x)
```

```
[Out] -1/24*(b*x^3+a*x^2)^(1/2)*(3*a^(9/2)*(b*x+a)^(1/2)+8*a^(7/2)*(b*x+a)^(3/2)-
3*a^(5/2)*(b*x+a)^(5/2)+3*arctanh((b*x+a)^(1/2)/a^(1/2))*a^2*b^3*x^3)/x^4/(
b*x+a)^(1/2)/a^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^3+ax^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^5, x)
```

---

**Fricas [A]** time = 0.933463, size = 393, normalized size = 3.51

$$\left[\frac{3\,\sqrt{a}b^3x^4\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(3\,ab^2x^2-2\,a^2bx-8\,a^3\right)\sqrt{bx^3+ax^2}}{48\,a^3x^4},\frac{3\,\sqrt{-a}b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\left(3\,a}{24\,a^3x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) + 2*(3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^4)
, 1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (3*
a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^4)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\,(a+bx)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**5, x)
```

---

**Giac [A]** time = 1.18835, size = 116, normalized size = 1.04

$$\frac{\left(\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3\,(bx+a)^{\frac{5}{2}}b^4-8\,(bx+a)^{\frac{3}{2}}ab^4-3\,\sqrt{bx+a}a^2b^4}{a^2b^3x^3}\right)\mathrm{sgn}\,(x)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(5
/2)*b^4 - 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)*a^2*b^4)/(a^2*b^3*x^3))
*sgn(x)/b
```

# 3.240 $\int x^2\left(ax^2+bx^3\right)^{3/2}\,dx$

**Optimal**. Leaf size=161

$$-\frac{512a^5\left(ax^2+bx^3\right)^{5/2}}{45045b^6x^5}+\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{9009b^5x^4}-\frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x}+\frac{2\left(ax^2\right.}{1}$$

```
[Out] (2*(a*x^2 + b*x^3)^(5/2))/(15*b) - (512*a^5*(a*x^2 + b*x^3)^(5/2))/(45045*b
^6*x^5) + (256*a^4*(a*x^2 + b*x^3)^(5/2))/(9009*b^5*x^4) - (64*a^3*(a*x^2 +
 b*x^3)^(5/2))/(1287*b^4*x^3) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^2
) - (4*a*(a*x^2 + b*x^3)^(5/2))/(39*b^2*x)
```

---

**Rubi [A]** time = 0.23122, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{512a^5\left(ax^2+bx^3\right)^{5/2}}{45045b^6x^5}+\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{9009b^5x^4}-\frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x}+\frac{2\left(ax^2\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(5/2))/(15*b) - (512*a^5*(a*x^2 + b*x^3)^(5/2))/(45045*b
^6*x^5) + (256*a^4*(a*x^2 + b*x^3)^(5/2))/(9009*b^5*x^4) - (64*a^3*(a*x^2 +
 b*x^3)^(5/2))/(1287*b^4*x^3) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^2
) - (4*a*(a*x^2 + b*x^3)^(5/2))/(39*b^2*x)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\left(ax^2+bx^3\right)^{3/2}dx &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b} - \frac{(2a)\int x\left(ax^2+bx^3\right)^{3/2}dx}{3b} \\
&= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b} - \frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x} + \frac{\left(16a^2\right)\int\left(ax^2+bx^3\right)^{3/2}dx}{39b^2} \\
&= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b} + \frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2} - \frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x} - \frac{\left(32a^3\right)\int\frac{\left(ax^2+bx^3\right)^{3/2}}{x}dx}{143b^3} \\
&= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b} - \frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3} + \frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2} - \frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x} + \\
&= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b} + \frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{9009b^5x^4} - \frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3} + \frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2} - \\
&= \frac{2\left(ax^2+bx^3\right)^{5/2}}{15b} - \frac{512a^5\left(ax^2+bx^3\right)^{5/2}}{45045b^6x^5} + \frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{9009b^5x^4} - \frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3} +
\end{aligned}$$

**Mathematica [A]** time = 0.0414666, size = 80, normalized size = 0.5

$$\frac{2x(a+bx)^3\left(-1120a^3b^2x^2+1680a^2b^3x^3+640a^4bx-256a^5-2310ab^4x^4+3003b^5x^5\right)}{45045b^6\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a + b*x)^3*(-256*a^5 + 640*a^4*b*x - 1120*a^3*b^2*x^2 + 1680*a^2*b^3*
x^3 - 2310*a*b^4*x^4 + 3003*b^5*x^5))/(45045*b^6*Sqrt[x^2*(a + b*x)])
```

---

**Maple [A]** time = 0.005, size = 79, normalized size = 0.5

$$-\frac{(2\,bx+2\,a)\left(-3003\,x^5b^5+2310\,ax^4b^4-1680\,x^3a^2b^3+1120\,x^2a^3b^2-640\,a^4xb+256\,a^5\right)}{45045\,b^6x^3}\left(bx^3+ax^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -2/45045*(b*x+a)*(-3003*b^5*x^5+2310*a*b^4*x^4-1680*a^2*b^3*x^3+1120*a^3*b^
2*x^2-640*a^4*b*x+256*a^5)*(b*x^3+a*x^2)^(3/2)/b^6/x^3
```

---

**Maxima [A]** time = 1.00878, size = 116, normalized size = 0.72

$$\frac{2\left(3003\,b^7x^7+3696\,ab^6x^6+63\,a^2b^5x^5-70\,a^3b^4x^4+80\,a^4b^3x^3-96\,a^5b^2x^2+128\,a^6bx-256\,a^7\right)\sqrt{bx+a}}{45045\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x + a)/b^6
```

---

**Fricas [A]** time = 0.868384, size = 217, normalized size = 1.35

$$\frac{2\left(3003\,b^7x^7+3696\,ab^6x^6+63\,a^2b^5x^5-70\,a^3b^4x^4+80\,a^4b^3x^3-96\,a^5b^2x^2+128\,a^6bx-256\,a^7\right)\sqrt{bx^3+ax^2}}{45045\,b^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x^3 + a*x^2
)/(b^6*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2\left(x^2\,(a+bx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(x**2*(a + b*x))**(3/2), x)
```

---

**Giac [A]** time = 1.13546, size = 242, normalized size = 1.5

$$\frac{512\,a^{\frac{15}{2}}\operatorname{sgn}(x)}{45045\,b^6}+\frac{2\left(\frac{5\left(693\,(bx+a)^{\frac{13}{2}}-4095\,(bx+a)^{\frac{11}{2}}a+10010\,(bx+a)^{\frac{9}{2}}a^2-12870\,(bx+a)^{\frac{7}{2}}a^3+9009\,(bx+a)^{\frac{5}{2}}a^4-3003\,(bx+a)^{\frac{3}{2}}a^5\right)a\operatorname{sgn}(x)}{b^5}+\frac{\left(3003\,(bx+a)\right.}{}\right.}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 512/45045*a^(15/2)*sgn(x)/b^6 + 2/45045*(5*(693*(b*x + a)^(13/2) - 4095*(b*
x + a)^(11/2)*a + 10010*(b*x + a)^(9/2)*a^2 - 12870*(b*x + a)^(7/2)*a^3 + 9
009*(b*x + a)^(5/2)*a^4 - 3003*(b*x + a)^(3/2)*a^5)*a*sgn(x)/b^5 + (3003*(b
*x + a)^(15/2) - 20790*(b*x + a)^(13/2)*a + 61425*(b*x + a)^(11/2)*a^2 - 10
0100*(b*x + a)^(9/2)*a^3 + 96525*(b*x + a)^(7/2)*a^4 - 54054*(b*x + a)^(5/2
)*a^5 + 15015*(b*x + a)^(3/2)*a^6)*sgn(x)/b^5)/b
```

## 3.241 $\int x\left(ax^2+bx^3\right)^{3/2}\,dx$

**Optimal**. Leaf size=136

$$\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{15015b^5x^5}-\frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}$$

```
[Out] (256*a^4*(a*x^2 + b*x^3)^(5/2))/(15015*b^5*x^5) - (128*a^3*(a*x^2 + b*x^3)^
(5/2))/(3003*b^4*x^4) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^3) - (16*
a*(a*x^2 + b*x^3)^(5/2))/(143*b^2*x^2) + (2*(a*x^2 + b*x^3)^(5/2))/(13*b*x)
```

---

**Rubi [A]** time = 0.171624, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2016, 2002, 2014}

$$\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{15015b^5x^5}-\frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (256*a^4*(a*x^2 + b*x^3)^(5/2))/(15015*b^5*x^5) - (128*a^3*(a*x^2 + b*x^3)^
(5/2))/(3003*b^4*x^4) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^3) - (16*
a*(a*x^2 + b*x^3)^(5/2))/(143*b^2*x^2) + (2*(a*x^2 + b*x^3)^(5/2))/(13*b*x)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int x\left(ax^2+bx^3\right)^{3/2}dx &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{(8a)\int\left(ax^2+bx^3\right)^{3/2}dx}{13b}\\
&= -\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}+\frac{\left(48a^2\right)\int\frac{\left(ax^2+bx^3\right)^{3/2}}{x}dx}{143b^2}\\
&= \frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{\left(64a^3\right)\int\frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}dx}{429b^3}\\
&= -\frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}+\left(128a^4\right)\\
&= \frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{15015b^5x^5}-\frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+2\left(ax\right.
\end{aligned}
$$

**Mathematica [A]** time = 0.0332815, size = 69, normalized size = 0.51

$$\frac{2x(a+bx)^3\left(560a^2b^2x^2-320a^3bx+128a^4-840ab^3x^3+1155b^4x^4\right)}{15015b^5\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a + b*x)^3*(128*a^4 - 320*a^3*b*x + 560*a^2*b^2*x^2 - 840*a*b^3*x^3 +
 1155*b^4*x^4))/(15015*b^5*Sqrt[x^2*(a + b*x)])
```

---

**Maple [A]** time = 0.004, size = 68, normalized size = 0.5

$$\frac{(2\,bx+2\,a)\left(1155\,x^4b^4-840\,ab^3x^3+560\,a^2x^2b^2-320\,xa^3b+128\,a^4\right)}{15015\,b^5x^3}\left(bx^3+ax^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 2/15015*(b*x+a)*(1155*b^4*x^4-840*a*b^3*x^3+560*a^2*b^2*x^2-320*a^3*b*x+128
*a^4)*(b*x^3+a*x^2)^(3/2)/b^5/x^3
```

---

**Maxima [A]** time = 1.05686, size = 101, normalized size = 0.74

$$\frac{2\left(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6\right)\sqrt{bx+a}}{15015\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x + a)/b^5
```

---

**Fricas [A]** time = 0.808423, size = 193, normalized size = 1.42

$$\frac{2\left(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6\right)\sqrt{bx^3+ax^2}}{15015\,b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x^3 + a*x^2)/(b^5*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(a + b*x))**(3/2), x)
```

---

**Giac [A]** time = 1.17089, size = 211, normalized size = 1.55

$$-\frac{256\,a^{\frac{13}{2}}\,\mathrm{sgn}\left(x\right)}{15015\,b^5}+\frac{2\left(\frac{13\left(315\,(bx+a)^{\frac{11}{2}}-1540\,(bx+a)^{\frac{9}{2}}a+2970\,(bx+a)^{\frac{7}{2}}a^2-2772\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4\right)a\,\mathrm{sgn}(x)}{b^4}+\frac{5\left(693\,(bx+a)^{\frac{13}{2}}-4095\,(bx\right.}{}\right.}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -256/15015*a^(13/2)*sgn(x)/b^5 + 2/45045*(13*(315*(b*x + a)^(11/2) - 1540*(
b*x + a)^(9/2)*a + 2970*(b*x + a)^(7/2)*a^2 - 2772*(b*x + a)^(5/2)*a^3 + 11
55*(b*x + a)^(3/2)*a^4)*a*sgn(x)/b^4 + 5*(693*(b*x + a)^(13/2) - 4095*(b*x
+ a)^(11/2)*a + 10010*(b*x + a)^(9/2)*a^2 - 12870*(b*x + a)^(7/2)*a^3 + 900
9*(b*x + a)^(5/2)*a^4 - 3003*(b*x + a)^(3/2)*a^5)*sgn(x)/b^4)/b
```

## 3.242 $\int \left(ax^2+bx^3\right)^{3/2}\,dx$

**Optimal**. Leaf size=108

$$-\frac{32a^3\left(ax^2+bx^3\right)^{5/2}}{1155b^4x^5}+\frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{231b^3x^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}$$

```
[Out] (-32*a^3*(a*x^2 + b*x^3)^(5/2))/(1155*b^4*x^5) + (16*a^2*(a*x^2 + b*x^3)^(5
/2))/(231*b^3*x^4) - (4*a*(a*x^2 + b*x^3)^(5/2))/(33*b^2*x^3) + (2*(a*x^2 +
 b*x^3)^(5/2))/(11*b*x^2)
```

---

**Rubi [A]** time = 0.141219, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2002, 2016, 2014}

$$-\frac{32a^3\left(ax^2+bx^3\right)^{5/2}}{1155b^4x^5}+\frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{231b^3x^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-32*a^3*(a*x^2 + b*x^3)^(5/2))/(1155*b^4*x^5) + (16*a^2*(a*x^2 + b*x^3)^(5
/2))/(231*b^3*x^4) - (4*a*(a*x^2 + b*x^3)^(5/2))/(33*b^2*x^3) + (2*(a*x^2 +
 b*x^3)^(5/2))/(11*b*x^2)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \left(ax^2+bx^3\right)^{3/2}\,dx &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{(6a)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x}\,dx}{11b}\\ &= -\frac{4a\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}+\frac{\left(8a^2\right)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx}{33b^2}\\ &= \frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{231b^3x^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{\left(16a^3\right)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx}{231b^3}\\ &= -\frac{32a^3\left(ax^2+bx^3\right)^{5/2}}{1155b^4x^5}+\frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{231b^3x^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}\end{aligned}$$

**Mathematica [A]** time = 0.0270002, size = 58, normalized size = 0.54

$$\frac{2x(a+bx)^3\left(40a^2bx-16a^3-70ab^2x^2+105b^3x^3\right)}{1155b^4\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a + b*x)^3*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(1155
*b^4*Sqrt[x^2*(a + b*x)])
```

---

**Maple [A]** time = 0.003, size = 57, normalized size = 0.5

$$-\frac{(2\,bx+2\,a)\left(-105\,x^3b^3+70\,ab^2x^2-40\,a^2xb+16\,a^3\right)}{1155\,b^4x^3}\left(bx^3+ax^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -2/1155*(b*x+a)*(-105*b^3*x^3+70*a*b^2*x^2-40*a^2*b*x+16*a^3)*(b*x^3+a*x^2)
^(3/2)/b^4/x^3
```

---

**Maxima [A]** time = 0.987819, size = 86, normalized size = 0.8

$$\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx+a}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

---

**Fricas [A]** time = 0.740348, size = 161, normalized size = 1.49

$$\frac{2\left(105\,b^{5}x^{5}+140\,ab^{4}x^{4}+5\,a^{2}b^{3}x^{3}-6\,a^{3}b^{2}x^{2}+8\,a^{4}bx-16\,a^{5}\right)\sqrt{bx^{3}+ax^{2}}}{1155\,b^{4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2)/(b^4*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(ax^{2}+bx^{3}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3)**(3/2), x)
```

---

**Giac [A]** time = 1.26395, size = 177, normalized size = 1.64

$$\frac{32\,a^{\frac{11}{2}}\operatorname{sgn}(x)}{1155\,b^{4}}+\frac{2\left(\frac{11\left(35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^{2}-105\,(bx+a)^{\frac{3}{2}}a^{3}\right)a\operatorname{sgn}(x)}{b^{3}}+\frac{(315\,(bx+a)^{\frac{11}{2}}-1540\,(bx+a)^{\frac{9}{2}}a+2970\,(bx+a)^{\frac{7}{2}}a^{2}-2772\,(b}{b^{3}}\right.}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 32/1155*a^(11/2)*sgn(x)/b^4 + 2/3465*(11*(35*(b*x + a)^(9/2) - 135*(b*x + a
)^(7/2)*a + 189*(b*x + a)^(5/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)*a*sgn(x)/b^3
 + (315*(b*x + a)^(11/2) - 1540*(b*x + a)^(9/2)*a + 2970*(b*x + a)^(7/2)*a^
2 - 2772*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4)*sgn(x)/b^3)/b
```

## 3.243 $\int \frac{(ax^2+bx^3)^{3/2}}{x} \, dx$

**Optimal**. Leaf size=80

$$\frac{16a^2(ax^2+bx^3)^{5/2}}{315b^3x^5}-\frac{8a(ax^2+bx^3)^{5/2}}{63b^2x^4}+\frac{2(ax^2+bx^3)^{5/2}}{9bx^3}$$

```
[Out] (16*a^2*(a*x^2 + b*x^3)^(5/2))/(315*b^3*x^5) - (8*a*(a*x^2 + b*x^3)^(5/2))/
(63*b^2*x^4) + (2*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3)
```

---

**Rubi [A]** time = 0.132562, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{16a^2(ax^2+bx^3)^{5/2}}{315b^3x^5}-\frac{8a(ax^2+bx^3)^{5/2}}{63b^2x^4}+\frac{2(ax^2+bx^3)^{5/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x,x]
```

```
[Out] (16*a^2*(a*x^2 + b*x^3)^(5/2))/(315*b^3*x^5) - (8*a*(a*x^2 + b*x^3)^(5/2))/
(63*b^2*x^4) + (2*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2+bx^3)^{3/2}}{x} \, dx &= \frac{2(ax^2+bx^3)^{5/2}}{9bx^3}-\frac{(4a)\int \frac{(ax^2+bx^3)^{3/2}}{x^2} \, dx}{9b}\\
&= -\frac{8a(ax^2+bx^3)^{5/2}}{63b^2x^4}+\frac{2(ax^2+bx^3)^{5/2}}{9bx^3}+\frac{(8a^2)\int \frac{(ax^2+bx^3)^{3/2}}{x^3} \, dx}{63b^2}\\
&= \frac{16a^2(ax^2+bx^3)^{5/2}}{315b^3x^5}-\frac{8a(ax^2+bx^3)^{5/2}}{63b^2x^4}+\frac{2(ax^2+bx^3)^{5/2}}{9bx^3}
\end{aligned}$$

**Mathematica [A]** time = 0.0224445, size = 47, normalized size = 0.59

$$\frac{2x(a+bx)^3\left(8a^2-20abx+35b^2x^2\right)}{315b^3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x,x]
```

```
[Out] (2*x*(a + b*x)^3*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b^3*Sqrt[x^2*(a + b*
x)])
```

---

**Maple [A]** time = 0.004, size = 46, normalized size = 0.6

$$\frac{(2\,bx+2\,a)\left(35\,b^2x^2-20\,abx+8\,a^2\right)}{315\,b^3x^3}\left(bx^3+ax^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x,x)
```

```
[Out] 2/315*(b*x+a)*(35*b^2*x^2-20*a*b*x+8*a^2)*(b*x^3+a*x^2)^(3/2)/b^3/x^3
```

---

**Maxima [A]** time = 1.00761, size = 72, normalized size = 0.9

$$\frac{2\left(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4\right)\sqrt{bx+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x + a)/b^3
```

---

**Fricas [A]** time = 0.829455, size = 134, normalized size = 1.68

$$\frac{2\left(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4\right)\sqrt{bx^3+ax^2}}{315\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x^3 + a*x^2)/(b^3*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{2}\left(a+bx\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x, x)
```

---

**Giac [A]** time = 1.28919, size = 144, normalized size = 1.8

$$-\frac{16\,a^{\frac{9}{2}}\operatorname{sgn}\left(x\right)}{315\,b^{3}}+\frac{2\left(\frac{3\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}\right)a\operatorname{sgn}(x)}{b^{2}}+\frac{\left(35\,(bx+a)^{\frac{9}{2}}-135\,(bx+a)^{\frac{7}{2}}a+189\,(bx+a)^{\frac{5}{2}}a^{2}-105\,(bx+a)^{\frac{3}{2}}a^{3}\right)\operatorname{sgn}(x)}{b^{2}}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -16/315*a^(9/2)*sgn(x)/b^3 + 2/315*(3*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5
/2)*a + 35*(b*x + a)^(3/2)*a^2)*a*sgn(x)/b^2 + (35*(b*x + a)^(9/2) - 135*(b
*x + a)^(7/2)*a + 189*(b*x + a)^(5/2)*a^2 - 105*(b*x + a)^(3/2)*a^3)*sgn(x)
/b^2)/b
```

## 3.244 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx$

**Optimal**. Leaf size=52

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}$$

```
[Out] (-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b*
x^4)
```

---

**Rubi [A]** time = 0.0826054, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b*
x^4)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{(2a)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx}{7b}\\ &= -\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}\end{aligned}$$

**Mathematica [A]** time = 0.01721, size = 36, normalized size = 0.69

$$\frac{2x(a+bx)^3(5bx-2a)}{35b^2\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (2*x*(a + b*x)^3*(-2*a + 5*b*x))/(35*b^2*Sqrt[x^2*(a + b*x)])
```

---

**Maple [A]** time = 0.003, size = 35, normalized size = 0.7

$$-\frac{(2\,bx+2\,a)\,(-5\,bx+2\,a)}{35\,b^2x^3}\left(bx^3+ax^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^2,x)
```

```
[Out] -2/35*(b*x+a)*(-5*b*x+2*a)*(b*x^3+a*x^2)^(3/2)/b^2/x^3
```

---

**Maxima [A]** time = 1.02657, size = 55, normalized size = 1.06

$$\frac{2\left(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3\right)\sqrt{bx+a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

---

**Fricas [A]** time = 0.817854, size = 105, normalized size = 2.02

$$\frac{2\left(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3\right)\sqrt{bx^3+ax^2}}{35\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*x^2)/(b^2*x
)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**2,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**2, x)
```

---

**Giac [A]** time = 1.27493, size = 112, normalized size = 2.15

$$\frac{4\,a^{\frac{7}{2}}\operatorname{sgn}(x)}{35\,b^2}+\frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}\,a\right)a\operatorname{sgn}(x)}{b}+\frac{\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}\,a+35\,(bx+a)^{\frac{3}{2}}\,a^2\right)\operatorname{sgn}(x)}{b}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 4/35*a^(7/2)*sgn(x)/b^2 + 2/105*(7*(3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a
)*a*sgn(x)/b + (15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3
/2)*a^2)*sgn(x)/b)/b
```

## 3.245 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3} dx$

**Optimal**. Leaf size=25

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{5bx^5}$$

```
[Out] (2*(a*x^2 + b*x^3)^(5/2))/(5*b*x^5)
```

---

**Rubi [A]** time = 0.0406793, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(5/2))/(5*b*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx=\frac{2\left(ax^2+bx^3\right)^{5/2}}{5bx^5}$$

**Mathematica [A]** time = 0.0128377, size = 23, normalized size = 0.92

$$\frac{2\left(x^2(a+bx)\right)^{5/2}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (2*(x^2*(a + b*x))^(5/2))/(5*b*x^5)
```

---

**Maple [A]** time = 0.002, size = 27, normalized size = 1.1

$$\frac{2\,bx+2\,a}{5\,bx^3}\left(bx^3+ax^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^3,x)
```

```
[Out] 2/5*(b*x+a)*(b*x^3+a*x^2)^(3/2)/b/x^3
```

---

**Maxima [A]** time = 0.988703, size = 38, normalized size = 1.52

$$\frac{2\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

---

**Fricas [A]** time = 0.811251, size = 77, normalized size = 3.08

$$\frac{2\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx^3+ax^2}}{5\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x^3 + a*x^2)/(b*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**3, x)
```

---

**Giac [B]** time = 1.24223, size = 70, normalized size = 2.8

$$-\frac{2\,a^{\frac{5}{2}}\mathrm{sgn}\,(x)}{5\,b}+\frac{2\left(5\,(bx+a)^{\frac{3}{2}}a\mathrm{sgn}\,(x)+\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)\mathrm{sgn}\,(x)\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -2/5*a^(5/2)*sgn(x)/b + 2/15*(5*(b*x + a)^(3/2)*a*sgn(x) + (3*(b*x + a)^(5/
2) - 5*(b*x + a)^(3/2)*a)*sgn(x))/b
```

## 3.246 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}\,dx$

**Optimal**. Leaf size=74

$$-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)+\frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}$$

```
[Out] (2*a*Sqrt[a*x^2 + b*x^3])/x + (2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) - 2*a^(3/2)
*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
```

---

**Rubi [A]** time = 0.0958373, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2021, 2008, 206}

$$-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)+\frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (2*a*Sqrt[a*x^2 + b*x^3])/x + (2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) - 2*a^(3/2)
*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}\,dx &= \frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}+a\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx \\
&= \frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}+a^2\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx \\
&= \frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}-\left(2a^2\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right) \\
&= \frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.0426705, size = 68, normalized size = 0.92

$$
\frac{2x\sqrt{a+bx}\left(\sqrt{a+bx}(4a+bx)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3\sqrt{x^2(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (2*x*Sqrt[a + b*x]*(Sqrt[a + b*x]*(4*a + b*x) - 3*a^(3/2)*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]]))/(3*Sqrt[x^2*(a + b*x)])
```

---

**Maple [A]** time = 0.007, size = 63, normalized size = 0.9

$$
-\frac{2}{3\,x^3}\left(bx^3+ax^2\right)^{\frac{3}{2}}\left(3\,a^{3/2}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-(bx+a)^{\frac{3}{2}}-3\,a\sqrt{bx+a}\right)(bx+a)^{-\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^4,x)
```

```
[Out] -2/3*(b*x^3+a*x^2)^(3/2)*(3*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))-(b*x+a)^
(3/2)-3*a*(b*x+a)^(1/2))/x^3/(b*x+a)^(3/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}}{x^4}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^4, x)
```

---

**Fricas [A]** time = 0.810448, size = 296, normalized size = 4.

$$\left[\frac{3\,a^{\frac{3}{2}}x\log\left(\frac{bx^{2}+2\,ax-2\,\sqrt{bx^{3}+ax^{2}}\sqrt{a}}{x^{2}}\right)+2\,\sqrt{bx^{3}+ax^{2}}(bx+4\,a)}{3\,x},\ \frac{2\left(3\,\sqrt{-a}ax\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\sqrt{-a}}{ax}\right)+\sqrt{bx^{3}+ax^{2}}(bx+4\,a)\right.}{3\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)
+ 2*sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x, 2/3*(3*sqrt(-a)*a*x*arctan(sqrt(b*x
^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{2}\left(a+bx\right)\right)^{\frac{3}{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**4, x)
```

---

**Giac [A]** time = 1.19226, size = 115, normalized size = 1.55

$$\frac{2\,a^{2}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}\left(x\right)}{\sqrt{-a}}+\frac{2}{3}\left(bx+a\right)^{\frac{3}{2}}\operatorname{sgn}\left(x\right)+2\,\sqrt{bx+a}a\operatorname{sgn}\left(x\right)-\frac{2\left(3\,a^{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+4\,\sqrt{-a}a^{\frac{3}{2}}\right)\operatorname{sgn}\left(x\right)}{3\,\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*(b*x + a)^(3/2)*
sgn(x) + 2*sqrt(b*x + a)*a*sgn(x) - 2/3*(3*a^2*arctan(sqrt(a)/sqrt(-a)) + 4
*sqrt(-a)*a^(3/2))*sgn(x)/sqrt(-a)
```

## 3.247 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^5}\,dx$

**Optimal**. Leaf size=73

$$-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}+\frac{3b\sqrt{ax^2+bx^3}}{x}-3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

```
[Out] (3*b*Sqrt[a*x^2 + b*x^3])/x - (a*x^2 + b*x^3)^(3/2)/x^4 - 3*Sqrt[a]*b*ArcTa
nh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
```

---

**Rubi [A]** time = 0.0931158, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2021, 2008, 206}

$$-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}+\frac{3b\sqrt{ax^2+bx^3}}{x}-3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (3*b*Sqrt[a*x^2 + b*x^3])/x - (a*x^2 + b*x^3)^(3/2)/x^4 - 3*Sqrt[a]*b*ArcTa
nh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^5}\,dx &= -\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}+\frac{1}{2}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx\\
&= \frac{3b\sqrt{ax^2+bx^3}}{x}-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}+\frac{1}{2}(3ab)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx\\
&= \frac{3b\sqrt{ax^2+bx^3}}{x}-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}-(3ab)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\
&= \frac{3b\sqrt{ax^2+bx^3}}{x}-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}-3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)
\end{aligned}$$

**Mathematica [C]** time = 0.0134288, size = 40, normalized size = 0.55

$$\frac{2b\left(x^2(a+bx)\right)^{5/2}{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx}{a}+1\right)}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (2*b*(x^2*(a + b*x))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x)/a])/(5*
a^2*x^5)
```

---

**Maple [A]** time = 0.011, size = 72, normalized size = 1.

$$-\frac{1}{x^4}\left(bx^3+ax^2\right)^{\frac{3}{2}}\left(a^{\frac{3}{2}}\sqrt{bx+a}-2\sqrt{bx+a}bx\sqrt{a}+3\,\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)xab\right)(bx+a)^{-\frac{3}{2}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^5,x)
```

```
[Out] -(b*x^3+a*x^2)^(3/2)*(a^(3/2)*(b*x+a)^(1/2)-2*(b*x+a)^(1/2)*b*x*a^(1/2)+3*a
rctanh((b*x+a)^(1/2)/a^(1/2))*x*a*b)/x^4/(b*x+a)^(3/2)/a^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^5, x)
```

---

**Fricas [A]** time = 0.898792, size = 304, normalized size = 4.16

$$\left[\frac{3\,\sqrt{a}bx^{2}\log\left(\frac{bx^{2}+2\,ax-2\,\sqrt{bx^{3}+ax^{2}}\sqrt{a}}{x^{2}}\right)+2\,\sqrt{bx^{3}+ax^{2}}(2\,bx-a)}{2\,x^{2}},\ \frac{3\,\sqrt{-a}bx^{2}\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\sqrt{-a}}{ax}\right)+\sqrt{bx^{3}+ax^{2}}(2\,bx-a)}{x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x
^2) + 2*sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2, (3*sqrt(-a)*b*x^2*arctan(sqrt
(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{2}\left(a+bx\right)\right)^{\frac{3}{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**5, x)
```

---

**Giac [A]** time = 1.45642, size = 84, normalized size = 1.15

$$\frac{\frac{3\,ab^{2}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}+2\,\sqrt{bx+a}b^{2}\mathrm{sgn}\left(x\right)-\frac{\sqrt{bx+a}ab\mathrm{sgn}(x)}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] (3*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*b
^2*sgn(x) - sqrt(b*x + a)*a*b*sgn(x)/x)/b
```

## 3.248 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^6}\,dx$

**Optimal**. Leaf size=81

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}-\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{2x^5}$$

```
[Out] (-3*b*Sqrt[a*x^2 + b*x^3])/(4*x^2) - (a*x^2 + b*x^3)^(3/2)/(2*x^5) - (3*b^2
*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*Sqrt[a])
```

---

**Rubi [A]** time = 0.0921833, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2020, 2008, 206}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}-\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

```
[Out] (-3*b*Sqrt[a*x^2 + b*x^3])/(4*x^2) - (a*x^2 + b*x^3)^(3/2)/(2*x^5) - (3*b^2
*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*Sqrt[a])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx &= -\frac{(ax^2+bx^3)^{3/2}}{2x^5}+\frac{1}{4}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}+\frac{1}{8}\left(3b^2\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}-\frac{1}{4}\left(3b^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}\end{aligned}$$

**Mathematica [A]** time = 0.0463637, size = 72, normalized size = 0.89

$$-\frac{2a^2+3b^2x^2\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+7abx+5b^2x^2}{4x\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

```
[Out] -(2*a^2 + 7*a*b*x + 5*b^2*x^2 + 3*b^2*x^2*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1
+ (b*x)/a]])/(4*x*Sqrt[x^2*(a + b*x)])
```

---

**Maple [A]** time = 0.01, size = 74, normalized size = 0.9

$$\frac{1}{4\,x^5}\left(bx^3+ax^2\right)^{\frac{3}{2}}\left(-3\,\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)x^2b^2+3\,a^{3/2}\sqrt{bx+a}-5\,\sqrt{a}\,(bx+a)^{3/2}\right)\frac{1}{\sqrt{a}}\,(bx+a)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^6,x)
```

```
[Out] 1/4*(b*x^3+a*x^2)^(3/2)*(-3*arctanh((b*x+a)^(1/2)/a^(1/2))*x^2*b^2+3*a^(3/2
)*(b*x+a)^(1/2)-5*a^(1/2)*(b*x+a)^(3/2))/x^5/(b*x+a)^(3/2)/a^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^6, x)
```

---

**Fricas [A]** time = 0.928418, size = 342, normalized size = 4.22

$$\left[\frac{3\sqrt{a}b^2x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}\left(5\,abx+2\,a^2\right)}{8\,ax^3},\frac{3\,\sqrt{-a}b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)-\sqrt{bx^3+ax^2}\bigl(5}{4\,ax^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) - 2*sqrt(b*x^3 + a*x^2)*(5*a*b*x + 2*a^2))/(a*x^3), 1/4*(3*sqrt(-a)*b
^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) - sqrt(b*x^3 + a*x^2)*(5*
a*b*x + 2*a^2))/(a*x^3)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**6, x)
```

---

**Giac [A]** time = 1.27062, size = 95, normalized size = 1.17

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}-\frac{5\,(bx+a)^{\frac{3}{2}}b^3\mathrm{sgn}(x)-3\,\sqrt{bx+a}ab^3\mathrm{sgn}(x)}{b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - (5*(b*x + a)^(3
/2)*b^3*sgn(x) - 3*sqrt(b*x + a)*a*b^3*sgn(x))/(b^2*x^2))/b
```

## 3.249 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx$

**Optimal**. Leaf size=109

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}$$

```
[Out] -(b*Sqrt[a*x^2 + b*x^3])/(4*x^3) - (b^2*Sqrt[a*x^2 + b*x^3])/(8*a*x^2) - (a
*x^2 + b*x^3)^(3/2)/(3*x^6) + (b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
)/(8*a^(3/2))
```

---

**Rubi [A]** time = 0.134152, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2008, 206}

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

```
[Out] -(b*Sqrt[a*x^2 + b*x^3])/(4*x^3) - (b^2*Sqrt[a*x^2 + b*x^3])/(8*a*x^2) - (a
*x^2 + b*x^3)^(3/2)/(3*x^6) + (b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
)/(8*a^(3/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx &= -\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}+\frac{1}{2}b\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx\\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}+\frac{1}{8}b^2\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}-\frac{b^3\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a}\\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}+\frac{b^3\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{8a}\\
&= -\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}+\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0157798, size = 42, normalized size = 0.39

$$
\frac{2b^3\left(x^2(a+bx)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{bx}{a}+1\right)}{5a^4x^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

```
[Out] (2*b^3*(x^2*(a + b*x))^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, 1 + (b*x)/a])/(
5*a^4*x^5)
```

---

**Maple [A]** time = 0.012, size = 87, normalized size = 0.8

$$
\frac{1}{24\,x^6}\left(bx^3+ax^2\right)^{\frac{3}{2}}\left(3\,a^{7/2}\sqrt{bx+a}-8\,a^{5/2}\left(bx+a\right)^{3/2}-3\,a^{3/2}\left(bx+a\right)^{5/2}+3\,\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)x^3ab^3\right)a^{-\frac{5}{2}}\left(bx+a\right)^{-\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^7,x)
```

```
[Out] 1/24*(b*x^3+a*x^2)^(3/2)*(3*a^(7/2)*(b*x+a)^(1/2)-8*a^(5/2)*(b*x+a)^(3/2)-3
*a^(3/2)*(b*x+a)^(5/2)+3*arctanh((b*x+a)^(1/2)/a^(1/2))*x^3*a*b^3)/x^6/(b*x
+a)^(3/2)/a^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}}{x^7}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^7, x)
```

---

**Fricas [A]** time = 0.785699, size = 397, normalized size = 3.64

$$\left[\frac{3\sqrt{a}b^3x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\left(3\,ab^2x^2+14\,a^2bx+8\,a^3\right)\sqrt{bx^3+ax^2}}{48\,a^2x^4},\,-\frac{3\,\sqrt{-a}b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\left(3\,a\right.}{24\,a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) - 2*(3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x^4
), -1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (
3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x^4)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**7, x)
```

---

**Giac [A]** time = 1.21501, size = 124, normalized size = 1.14

$$-\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a}+\frac{3\,(bx+a)^{\frac{5}{2}}b^4\mathrm{sgn}(x)+8\,(bx+a)^{\frac{3}{2}}ab^4\mathrm{sgn}(x)-3\,\sqrt{bx+a}a^2b^4\mathrm{sgn}(x)}{ab^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + (3*(b*x +
 a)^(5/2)*b^4*sgn(x) + 8*(b*x + a)^(3/2)*a*b^4*sgn(x) - 3*sqrt(b*x + a)*a^2
*b^4*sgn(x))/(a*b^3*x^3))/b
```

## 3.250 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^8}\,dx$

**Optimal**. Leaf size=137

$$\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}-\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

```
[Out] -(b*Sqrt[a*x^2 + b*x^3])/(8*x^4) - (b^2*Sqrt[a*x^2 + b*x^3])/(32*a*x^3) + (
3*b^3*Sqrt[a*x^2 + b*x^3])/(64*a^2*x^2) - (a*x^2 + b*x^3)^(3/2)/(4*x^7) - (
3*b^4*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(64*a^(5/2))
```

---

**Rubi [A]** time = 0.18419, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2008, 206}

$$\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}-\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

```
[Out] -(b*Sqrt[a*x^2 + b*x^3])/(8*x^4) - (b^2*Sqrt[a*x^2 + b*x^3])/(32*a*x^3) + (
3*b^3*Sqrt[a*x^2 + b*x^3])/(64*a^2*x^2) - (a*x^2 + b*x^3)^(3/2)/(4*x^7) - (
3*b^4*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(64*a^(5/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^8}\,dx &= -\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}+\frac{1}{8}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx\\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}+\frac{1}{16}b^2\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}-\frac{\left(3b^3\right)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{64a}\\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}+\frac{\left(3b^4\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{128a^2}\\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}-\frac{\left(3b^4\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\right.}{64a^2}\\
&= -\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.0145932, size = 42, normalized size = 0.31

$$-\frac{2b^4\left(x^2(a+bx)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{bx}{a}+1\right)}{5a^5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

```
[Out] (-2*b^4*(x^2*(a + b*x))^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, 1 + (b*x)/a])/
(5*a^5*x^5)
```

---

**Maple [A]** time = 0.01, size = 101, normalized size = 0.7

$$\frac{1}{64\,x^7}\left(bx^3+ax^2\right)^{\frac{3}{2}}\left(3\,a^{5/2}\,(bx+a)^{7/2}-11\,a^{7/2}\,(bx+a)^{5/2}-3\operatorname{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^2b^4x^4-11\,a^{9/2}\,(bx+a)^{3/2}+3\,a^{11/2}\sqrt{bx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^8,x)
```

```
[Out] 1/64*(b*x^3+a*x^2)^(3/2)*(3*a^(5/2)*(b*x+a)^(7/2)-11*a^(7/2)*(b*x+a)^(5/2)-
3*arctanh((b*x+a)^(1/2)/a^(1/2))*a^2*b^4*x^4-11*a^(9/2)*(b*x+a)^(3/2)+3*a^(
11/2)*(b*x+a)^(1/2))/x^7/(b*x+a)^(3/2)/a^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^8, x)
```

---

**Fricas [A]** time = 0.893233, size = 443, normalized size = 3.23

$$\left[\frac{3\sqrt{a}b^4x^5\log\left(\frac{bx^2+2ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(3ab^3x^3-2a^2b^2x^2-24a^3bx-16a^4\right)\sqrt{bx^3+ax^2}}{128a^3x^5},\ 3\sqrt{-a}b^4x^5\arctan\left(\frac{\sqrt{bx^3+}}{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/128*(3*sqrt(a)*b^4*x^5*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) + 2*(3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*x^3
+ a*x^2))/(a^3*x^5), 1/64*(3*sqrt(-a)*b^4*x^5*arctan(sqrt(b*x^3 + a*x^2)*sq
rt(-a)/(a*x)) + (3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*
x^3 + a*x^2))/(a^3*x^5)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**8, x)
```

---

**Giac [A]** time = 1.23712, size = 147, normalized size = 1.07

$$\frac{\frac{3\,b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^2}+\frac{3\,(bx+a)^{\frac{7}{2}}b^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{5}{2}}ab^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{3}{2}}a^2b^5\mathrm{sgn}(x)+3\,\sqrt{bx+a}a^3b^5\mathrm{sgn}(x)}{a^2b^4x^4}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/64*(3*b^5*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x
+ a)^(7/2)*b^5*sgn(x) - 11*(b*x + a)^(5/2)*a*b^5*sgn(x) - 11*(b*x + a)^(3/2
)*a^2*b^5*sgn(x) + 3*sqrt(b*x + a)*a^3*b^5*sgn(x))/(a^2*b^4*x^4))/b
```

## 3.251 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx$

**Optimal**. Leaf size=165

$$-\frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2}+\frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3}+\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{128a^{7/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}-\frac{3b\sqrt{ax^2+bx^3}}{40x^5}-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

```
[Out] (-3*b*Sqrt[a*x^2 + b*x^3])/(40*x^5) - (b^2*Sqrt[a*x^2 + b*x^3])/(80*a*x^4)
+ (b^3*Sqrt[a*x^2 + b*x^3])/(64*a^2*x^3) - (3*b^4*Sqrt[a*x^2 + b*x^3])/(128
*a^3*x^2) - (a*x^2 + b*x^3)^(3/2)/(5*x^8) + (3*b^5*ArcTanh[(Sqrt[a]*x)/Sqrt
[a*x^2 + b*x^3]])/(128*a^(7/2))
```

---

**Rubi [A]** time = 0.236105, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2020, 2025, 2008, 206}

$$-\frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2}+\frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3}+\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{128a^{7/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}-\frac{3b\sqrt{ax^2+bx^3}}{40x^5}-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^9,x]
```

```
[Out] (-3*b*Sqrt[a*x^2 + b*x^3])/(40*x^5) - (b^2*Sqrt[a*x^2 + b*x^3])/(80*a*x^4)
+ (b^3*Sqrt[a*x^2 + b*x^3])/(64*a^2*x^3) - (3*b^4*Sqrt[a*x^2 + b*x^3])/(128
*a^3*x^2) - (a*x^2 + b*x^3)^(3/2)/(5*x^8) + (3*b^5*ArcTanh[(Sqrt[a]*x)/Sqrt
[a*x^2 + b*x^3]])/(128*a^(7/2))
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx &= -\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}+\frac{1}{10}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^6}\,dx\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5}-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}+\frac{1}{80}\left(3b^2\right)\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}-\frac{b^3\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{32a}\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}+\frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3}-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}+\frac{\left(3b^4\right)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{128a^2}\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}+\frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3}-\frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}-\frac{\left(3b^5\right)}{}\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}+\frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3}-\frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}+\frac{\left(3b^5\right)}{}\\
&= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}+\frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3}-\frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2}-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}+\frac{3b^5\,\mathrm{ta}}{}
\end{aligned}
$$

**Mathematica [C]** time = 0.014527, size = 42, normalized size = 0.25

$$\frac{2b^5\left(x^2(a+bx)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};\frac{bx}{a}+1\right)}{5a^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^9,x]
```

```
[Out] (2*b^5*(x^2*(a + b*x))^(5/2)*Hypergeometric2F1[5/2, 6, 7/2, 1 + (b*x)/a])/(
5*a^6*x^5)
```

---

**Maple [A]** time = 0.013, size = 113, normalized size = 0.7

$$\frac{1}{640\,x^8}\left(bx^3+ax^2\right)^{\frac{3}{2}}\left(15\,a^{15/2}\sqrt{bx+a}-70\,a^{13/2}\left(bx+a\right)^{3/2}-128\,a^{11/2}\left(bx+a\right)^{5/2}+70\,a^{9/2}\left(bx+a\right)^{7/2}-15\,a^{7/2}\left(bx+a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^9,x)
```

```
[Out] 1/640*(b*x^3+a*x^2)^(3/2)*(15*a^(15/2)*(b*x+a)^(1/2)-70*a^(13/2)*(b*x+a)^(3
/2)-128*a^(11/2)*(b*x+a)^(5/2)+70*a^(9/2)*(b*x+a)^(7/2)-15*a^(7/2)*(b*x+a)^
(9/2)+15*arctanh((b*x+a)^(1/2)/a^(1/2))*a^3*b^5*x^5)/x^8/(b*x+a)^(3/2)/a^(1
3/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + ax^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^9, x)
```

---

**Fricas [A]** time = 0.909493, size = 504, normalized size = 3.05

$$\left[\frac{15\,\sqrt{a}b^5x^6\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right) - 2\left(15\,ab^4x^4 - 10\,a^2b^3x^3 + 8\,a^3b^2x^2 + 176\,a^4bx + 128\,a^5\right)\sqrt{bx^3+ax^2}}{1280\,a^4x^6}, -\frac{15\,\sqrt{-a}b^5x}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/1280*(15*sqrt(a)*b^5*x^6*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt
(a))/x^2) - 2*(15*a*b^4*x^4 - 10*a^2*b^3*x^3 + 8*a^3*b^2*x^2 + 176*a^4*b*x
+ 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^4*x^6), -1/640*(15*sqrt(-a)*b^5*x^6*arct
an(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (15*a*b^4*x^4 - 10*a^2*b^3*x^3 + 8
*a^3*b^2*x^2 + 176*a^4*b*x + 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^4*x^6)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(a + bx\right)\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**9, x)
```

---

**Giac [A]** time = 1.32584, size = 170, normalized size = 1.03

$$-\frac{\frac{15\,b^6\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}a^3} + \frac{15\,(bx+a)^{\frac{9}{2}}b^6\mathrm{sgn}(x) - 70\,(bx+a)^{\frac{7}{2}}ab^6\mathrm{sgn}(x) + 128\,(bx+a)^{\frac{5}{2}}a^2b^6\mathrm{sgn}(x) + 70\,(bx+a)^{\frac{3}{2}}a^3b^6\mathrm{sgn}(x) - 15\,\sqrt{bx+a}a^4b^6\mathrm{sgn}(x)}{a^3b^5x^5}}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/640*(15*b^6*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^3) + (15*(
b*x + a)^(9/2)*b^6*sgn(x) - 70*(b*x + a)^(7/2)*a*b^6*sgn(x) + 128*(b*x + a)
```

```
^(5/2)*a^2*b^6*sgn(x) + 70*(b*x + a)^(3/2)*a^3*b^6*sgn(x) - 15*sqrt(b*x + a
)*a^4*b^6*sgn(x))/(a^3*b^5*x^5))/b
```

## 3.252 $\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx$

**Optimal**. Leaf size=103

$$-\frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x}+\frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}$$

```
[Out] (16*a^2*Sqrt[a*x^2 + b*x^3])/(35*b^3) - (32*a^3*Sqrt[a*x^2 + b*x^3])/(35*b^
4*x) - (12*a*x*Sqrt[a*x^2 + b*x^3])/(35*b^2) + (2*x^2*Sqrt[a*x^2 + b*x^3])/
(7*b)
```

---

**Rubi [A]** time = 0.148466, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$-\frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x}+\frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (16*a^2*Sqrt[a*x^2 + b*x^3])/(35*b^3) - (32*a^3*Sqrt[a*x^2 + b*x^3])/(35*b^
4*x) - (12*a*x*Sqrt[a*x^2 + b*x^3])/(35*b^2) + (2*x^2*Sqrt[a*x^2 + b*x^3])/
(7*b)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx &= \frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{(6a)\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx}{7b}\\ &= -\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}+\frac{\left(24a^2\right)\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{35b^2}\\ &= \frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{\left(16a^3\right)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{35b^3}\\ &= \frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}\end{aligned}$$

**Mathematica [A]** time = 0.0324024, size = 53, normalized size = 0.51

$$\frac{2\sqrt{x^2(a+bx)}\left(8a^2bx-16a^3-6ab^2x^2+5b^3x^3\right)}{35b^4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x)]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(35
*b^4*x)
```

---

**Maple [A]** time = 0.004, size = 55, normalized size = 0.5

$$-\frac{(2\,bx+2\,a)\left(-5\,x^3b^3+6\,ab^2x^2-8\,a^2xb+16\,a^3\right)x}{35\,b^4}\frac{1}{\sqrt{bx^3+ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/35*(b*x+a)*(-5*b^3*x^3+6*a*b^2*x^2-8*a^2*b*x+16*a^3)*x/b^4/(b*x^3+a*x^2)
^(1/2)
```

---

**Maxima [A]** time = 1.06037, size = 72, normalized size = 0.7

$$\frac{2\left(5\,b^4x^4-ab^3x^3+2\,a^2b^2x^2-8\,a^3bx-16\,a^4\right)}{35\,\sqrt{bx+a}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)/(sqrt(b*x
 + a)*b^4)
```

---

**Fricas [A]** time = 0.805416, size = 109, normalized size = 1.06

$$\frac{2\left(5\,b^3x^3-6\,ab^2x^2+8\,a^2bx-16\,a^3\right)\sqrt{bx^3+ax^2}}{35\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^
4*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**2*(a + b*x)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(b*x^3 + a*x^2), x)
```

## 3.253 $\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx$

**Optimal**. Leaf size=75

$$\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x}-\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{2x\sqrt{ax^2+bx^3}}{5b}$$

```
[Out] (-8*a*Sqrt[a*x^2 + b*x^3])/(15*b^2) + (16*a^2*Sqrt[a*x^2 + b*x^3])/(15*b^3*
x) + (2*x*Sqrt[a*x^2 + b*x^3])/(5*b)
```

---

**Rubi [A]** time = 0.0999528, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x}-\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{2x\sqrt{ax^2+bx^3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-8*a*Sqrt[a*x^2 + b*x^3])/(15*b^2) + (16*a^2*Sqrt[a*x^2 + b*x^3])/(15*b^3*
x) + (2*x*Sqrt[a*x^2 + b*x^3])/(5*b)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx &= \frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{(4a)\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{5b}\\ &= -\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{2x\sqrt{ax^2+bx^3}}{5b}+\frac{\left(8a^2\right)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{15b^2}\\ &= -\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x}+\frac{2x\sqrt{ax^2+bx^3}}{5b}\end{aligned}$$

**Mathematica [A]** time = 0.0196869, size = 42, normalized size = 0.56

$$\frac{2\sqrt{x^2(a+bx)}\left(8a^2-4abx+3b^2x^2\right)}{15b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x)]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3*x)
```

---

**Maple [A]** time = 0.004, size = 44, normalized size = 0.6

$$\frac{(2\,bx+2\,a)\left(3\,b^2x^2-4\,abx+8\,a^2\right)x}{15\,b^3}\frac{1}{\sqrt{bx^3+ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 2/15*(b*x+a)*(3*b^2*x^2-4*a*b*x+8*a^2)*x/b^3/(b*x^3+a*x^2)^(1/2)
```

---

**Maxima [A]** time = 1.15133, size = 57, normalized size = 0.76

$$\frac{2\left(3\,b^3x^3-ab^2x^2+4\,a^2bx+8\,a^3\right)}{15\,\sqrt{bx+a}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)/(sqrt(b*x + a)*b^3)
```

---

**Fricas [A]** time = 0.792813, size = 86, normalized size = 1.15

$$\frac{2\left(3\,b^2x^2-4\,abx+8\,a^2\right)\sqrt{bx^3+ax^2}}{15\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^3*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(a + b*x)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^3 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(b*x^3 + a*x^2), x)
```

## 3.254 $\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx$

**Optimal**. Leaf size=49

$$\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)
```

---

**Rubi [A]** time = 0.0545702, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx &= \frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{(2a)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{3b}\\ &= \frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\end{aligned}$$

**Mathematica [A]** time = 0.0148682, size = 30, normalized size = 0.61

$$\frac{2(bx-2a)\sqrt{x^2(a+bx)}}{3b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(-2*a + b*x)*Sqrt[x^2*(a + b*x)])/(3*b^2*x)
```

---

**Maple [A]** time = 0.003, size = 33, normalized size = 0.7

$$-\frac{(2\,bx+2\,a)\,(-bx+2\,a)\,x}{3\,b^2}\frac{1}{\sqrt{bx^3+ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/3*(b*x+a)*(-b*x+2*a)*x/b^2/(b*x^3+a*x^2)^(1/2)
```

---

**Maxima [A]** time = 1.03395, size = 41, normalized size = 0.84

$$\frac{2\left(b^2x^2-abx-2\,a^2\right)}{3\,\sqrt{bx+a}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2*x^2 - a*b*x - 2*a^2)/(sqrt(b*x + a)*b^2)
```

---

**Fricas [A]** time = 0.858497, size = 61, normalized size = 1.24

$$\frac{2\,\sqrt{bx^3+ax^2}(bx-2\,a)}{3\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^3 + a*x^2)*(b*x - 2*a)/(b^2*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(a + b*x)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^3 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*x^3 + a*x^2), x)
```

## 3.255 $\int \frac{x}{\sqrt{ax^2+bx^3}} \, dx$

**Optimal**. Leaf size=23

$$\frac{2\sqrt{ax^2 + bx^3}}{bx}$$

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/(b*x)
```

---

**Rubi [A]** time = 0.0101149, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{2\sqrt{ax^2 + bx^3}}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/(b*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{ax^2 + bx^3}} \, dx = \frac{2\sqrt{ax^2 + bx^3}}{bx}$$

**Mathematica [A]** time = 0.0073949, size = 21, normalized size = 0.91

$$\frac{2\sqrt{x^2(a + bx)}}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x)])/(b*x)
```

---

**Maple [A]** time = 0.001, size = 25, normalized size = 1.1

$$2\,\frac{x\,(bx + a)}{b\sqrt{bx^3 + ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 2*x*(b*x+a)/b/(b*x^3+a*x^2)^(1/2)
```

---

**Maxima [A]** time = 1.16918, size = 16, normalized size = 0.7

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b
```

---

**Fricas [A]** time = 0.797656, size = 39, normalized size = 1.7

$$\frac{2\sqrt{bx^3+ax^2}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x^3 + a*x^2)/(b*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(a + b*x)), x)
```

---

**Giac [A]** time = 1.18953, size = 35, normalized size = 1.52

$$\frac{2}{\sqrt{\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/(sqrt(b/x + a/x^2) - sqrt(a)/x)
```

# 3.256 $\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$

**Optimal**. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

```
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]
```

---

**Rubi [A]** time = 0.0105473, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2008, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}\end{aligned}$$

**Mathematica [A]** time = 0.0089751, size = 46, normalized size = 1.53

$$-\frac{2x\sqrt{a+bx}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-2*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*Sqrt[x^2*(a +
b*x)])
```

---

**Maple [A]** time = 0.004, size = 39, normalized size = 1.3

$$-2\,\frac{x\sqrt{bx+a}}{\sqrt{bx^3+ax^2}\sqrt{a}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/(b*x^3+a*x^2)^(1/2)*x*(b*x+a)^(1/2)/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2
))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^3 + a*x^2), x)
```

---

**Fricas [A]** time = 0.803761, size = 171, normalized size = 5.7

$$\left[\frac{\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)}{\sqrt{a}},\ \frac{2\,\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)/sqrt(a), 2*sqrt(-
a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x))/a]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**3), x)
```

---

**Giac [A]** time = 1.16818, size = 61, normalized size = 2.03

$$-\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}}+\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(sqrt(b*x + a)/sqrt(-
a))/(sqrt(-a)*sgn(x))
```

## 3.257 $\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx$

**Optimal**. Leaf size=54

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$

```
[Out] -(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]
])/a^(3/2)
```

---

**Rubi [A]** time = 0.0489975, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]
])/a^(3/2)
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{ax^2}-\frac{b\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{2a}\\ &= -\frac{\sqrt{ax^2+bx^3}}{ax^2}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{a}\\ &= -\frac{\sqrt{ax^2+bx^3}}{ax^2}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.0445731, size = 66, normalized size = 1.22

$$\frac{2bx(a+bx)\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)}{2\sqrt{\frac{bx}{a}+1}}-\frac{a}{2bx}\right)}{a^2\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (2*b*x*(a + b*x)*(-a/(2*b*x) + ArcTanh[Sqrt[1 + (b*x)/a]]/(2*Sqrt[1 + (b*x)
/a])))/(a^2*Sqrt[x^2*(a + b*x)])
```

---

**Maple [A]** time = 0.007, size = 55, normalized size = 1.

$$-\sqrt{bx+a}\left(a^{\frac{3}{2}}\sqrt{bx+a}-\mathrm{Artanh}\left(\sqrt{bx+a}\frac{1}{\sqrt{a}}\right)xab\right)\frac{1}{\sqrt{bx^3+ax^2}}a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -(b*x+a)^(1/2)*(a^(3/2)*(b*x+a)^(1/2)-arctanh((b*x+a)^(1/2)/a^(1/2))*x*a*b)
/(b*x^3+a*x^2)^(1/2)/a^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+ax^2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x), x)
```

---

**Fricas [A]** time = 0.828951, size = 289, normalized size = 5.35

$$\left[\frac{\sqrt{a}bx^2\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}a}{2\,a^2x^2}, -\frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}a}{a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2
) - 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2), -(sqrt(-a)*b*x^2*arctan(sqrt(b*x^3
+ a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(a + b*x))), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

## 3.258 $\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx$

**Optimal**. Leaf size=87

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

```
[Out] -Sqrt[a*x^2 + b*x^3]/(2*a*x^3) + (3*b*Sqrt[a*x^2 + b*x^3])/(4*a^2*x^2) - (3
*b^2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*a^(5/2))
```

---

**Rubi [A]** time = 0.0911053, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3]/(2*a*x^3) + (3*b*Sqrt[a*x^2 + b*x^3])/(4*a^2*x^2) - (3
*b^2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*a^(5/2))
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{2ax^3} - \frac{(3b)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{4a} \\
&= -\frac{\sqrt{ax^2+bx^3}}{2ax^3} + \frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2} + \frac{\left(3b^2\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{8a^2} \\
&= -\frac{\sqrt{ax^2+bx^3}}{2ax^3} + \frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2} - \frac{\left(3b^2\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{4a^2} \\
&= -\frac{\sqrt{ax^2+bx^3}}{2ax^3} + \frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2} - \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0103942, size = 40, normalized size = 0.46

$$-\frac{2b^2\sqrt{x^2(a+bx)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx}{a}+1\right)}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*b^2*Sqrt[x^2*(a + b*x)]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x)/a])/(a
^3*x)
```

---

**Maple [A]** time = 0.007, size = 77, normalized size = 0.9

$$-\frac{1}{4x}\sqrt{bx+a}\left(2\,a^{5/2}\sqrt{bx+a}-3\,a^{3/2}\sqrt{bx+a}\,xb+3\,\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)ab^2x^2\right)\frac{1}{\sqrt{bx^3+ax^2}}a^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/4*(b*x+a)^(1/2)*(2*a^(5/2)*(b*x+a)^(1/2)-3*a^(3/2)*(b*x+a)^(1/2)*x*b+3*a
rctanh((b*x+a)^(1/2)/a^(1/2))*a*b^2*x^2)/x/(b*x^3+a*x^2)^(1/2)/a^(7/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+ax^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^2), x)
```

---

**Fricas [A]** time = 0.901449, size = 347, normalized size = 3.99

$$\left[\frac{3\sqrt{a}b^{2}x^{3}\log\left(\frac{bx^{2}+2\,ax-2\,\sqrt{bx^{3}+ax^{2}}\sqrt{a}}{x^{2}}\right)+2\,\sqrt{bx^{3}+ax^{2}}\left(3\,abx-2\,a^{2}\right)}{8\,a^{3}x^{3}},\ \frac{3\,\sqrt{-a}b^{2}x^{3}\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\sqrt{-a}}{ax}\right)+\sqrt{bx^{3}+ax^{2}}\left(3\right.}{4\,a^{3}x^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) + 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/4*(3*sqrt(-a)
*b^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(
3*a*b*x - 2*a^2))/(a^3*x^3)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^{2}\sqrt{x^{2}\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x))), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

## 3.259 $\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx$

**Optimal**. Leaf size=115

$$-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

```
[Out] -Sqrt[a*x^2 + b*x^3]/(3*a*x^4) + (5*b*Sqrt[a*x^2 + b*x^3])/(12*a^2*x^3) - (
5*b^2*Sqrt[a*x^2 + b*x^3])/(8*a^3*x^2) + (5*b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*
x^2 + b*x^3]])/(8*a^(7/2))
```

---

**Rubi [A]** time = 0.135388, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3]/(3*a*x^4) + (5*b*Sqrt[a*x^2 + b*x^3])/(12*a^2*x^3) - (
5*b^2*Sqrt[a*x^2 + b*x^3])/(8*a^3*x^2) + (5*b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*
x^2 + b*x^3]])/(8*a^(7/2))
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{3ax^4}-\frac{(5b)\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{6a}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}+\frac{\left(5b^2\right)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{8a^2}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}-\frac{\left(5b^3\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a^3}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{\left(5b^3\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{8a^3}\\
&= -\frac{\sqrt{ax^2+bx^3}}{3ax^4}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0102403, size = 40, normalized size = 0.35

$$
\frac{2b^3\sqrt{x^2(a+bx)}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{bx}{a}+1\right)}{a^4x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (2*b^3*Sqrt[x^2*(a + b*x)]*Hypergeometric2F1[1/2, 4, 3/2, 1 + (b*x)/a])/(a^
4*x)
```

---

**Maple [A]** time = 0.006, size = 95, normalized size = 0.8

$$
-\frac{1}{24\,x^2}\sqrt{bx+a}\left(15\,a^{3/2}\sqrt{bx+a}x^2b^2-15\,\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)x^3ab^3-10\,a^{5/2}\sqrt{bx+a}xb+8\,a^{7/2}\sqrt{bx+a}\right)\frac{1}{\sqrt{bx^3+ax^2}}a
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/24/x^2*(b*x+a)^(1/2)*(15*a^(3/2)*(b*x+a)^(1/2)*x^2*b^2-15*arctanh((b*x+a
)^(1/2)/a^(1/2))*x^3*a*b^3-10*a^(5/2)*(b*x+a)^(1/2)*x*b+8*a^(7/2)*(b*x+a)^(
1/2))/(b*x^3+a*x^2)^(1/2)/a^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{1}{\sqrt{bx^3+ax^2}x^3}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^3), x)
```

---

**Fricas [A]** time = 0.809012, size = 402, normalized size = 3.5

$$\left[\frac{15\sqrt{a}b^3x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\left(15\,ab^2x^2-10\,a^2bx+8\,a^3\right)\sqrt{bx^3+ax^2}}{48\,a^4x^4},-\frac{15\,\sqrt{-a}b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)+(}{24}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) - 2*(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*x
^4), -1/24*(15*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x))
+ (15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*x^4)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(a + b*x))), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

# 3.260 $\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=98

$$\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

```
[Out] (-2*x^4)/(b*Sqrt[a*x^2 + b*x^3]) - (16*a*Sqrt[a*x^2 + b*x^3])/(5*b^3) + (32
*a^2*Sqrt[a*x^2 + b*x^3])/(5*b^4*x) + (12*x*Sqrt[a*x^2 + b*x^3])/(5*b^2)
```

---

**Rubi [A]** time = 0.150882, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 1588}

$$\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^4)/(b*Sqrt[a*x^2 + b*x^3]) - (16*a*Sqrt[a*x^2 + b*x^3])/(5*b^3) + (32
*a^2*Sqrt[a*x^2 + b*x^3])/(5*b^4*x) + (12*x*Sqrt[a*x^2 + b*x^3])/(5*b^2)
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx &= -\frac{2x^4}{b\sqrt{ax^2+bx^3}}+\frac{6\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx}{b}\\
&= -\frac{2x^4}{b\sqrt{ax^2+bx^3}}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}-\frac{(24a)\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{5b^2}\\
&= -\frac{2x^4}{b\sqrt{ax^2+bx^3}}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}+\frac{\left(16a^2\right)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{5b^3}\\
&= -\frac{2x^4}{b\sqrt{ax^2+bx^3}}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}+\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}
\end{aligned}
$$

**Mathematica [A]** time = 0.0242158, size = 50, normalized size = 0.51

$$
\frac{2x\left(8a^2bx+16a^3-2ab^2x^2+b^3x^3\right)}{5b^4\sqrt{x^2(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt[x^2*(a + b*x
)])
```

---

**Maple [A]** time = 0.005, size = 56, normalized size = 0.6

$$
\frac{(2\,bx+2\,a)\left(x^3b^3-2\,ab^2x^2+8\,a^2xb+16\,a^3\right)x^3}{5\,b^4}\left(bx^3+ax^2\right)^{-\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 2/5*(b*x+a)*(b^3*x^3-2*a*b^2*x^2+8*a^2*b*x+16*a^3)*x^3/b^4/(b*x^3+a*x^2)^(3
/2)
```

---

**Maxima [A]** time = 1.15932, size = 55, normalized size = 0.56

$$
\frac{2\left(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3\right)}{5\,\sqrt{bx+a}b^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)/(sqrt(b*x + a)*b^4)
```

---

**Fricas [A]** time = 0.905986, size = 122, normalized size = 1.24

$$\frac{2\left(b^{3}x^{3}-2\,ab^{2}x^{2}+8\,a^{2}bx+16\,a^{3}\right)\sqrt{bx^{3}+ax^{2}}}{5\left(b^{5}x^{2}+ab^{4}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^5*x
^2 + a*b^4*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{6}}{\left(x^{2}\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**6/(x**2*(a + b*x))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{6}}{\left(bx^{3}+ax^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^3 + a*x^2)^(3/2), x)
```

## 3.261 $\int \frac{x^5}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=72

$$\frac{8\sqrt{ax^2+bx^3}}{3b^2}-\frac{16a\sqrt{ax^2+bx^3}}{3b^3x}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

```
[Out] (-2*x^3)/(b*Sqrt[a*x^2 + b*x^3]) + (8*Sqrt[a*x^2 + b*x^3])/(3*b^2) - (16*a*
Sqrt[a*x^2 + b*x^3])/(3*b^3*x)
```

---

**Rubi [A]** time = 0.105215, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 1588}

$$\frac{8\sqrt{ax^2+bx^3}}{3b^2}-\frac{16a\sqrt{ax^2+bx^3}}{3b^3x}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^3)/(b*Sqrt[a*x^2 + b*x^3]) + (8*Sqrt[a*x^2 + b*x^3])/(3*b^2) - (16*a*
Sqrt[a*x^2 + b*x^3])/(3*b^3*x)
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\left(ax^2+bx^3\right)^{3/2}}\,dx &= -\frac{2x^3}{b\sqrt{ax^2+bx^3}} + \frac{4\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{b} \\
&= -\frac{2x^3}{b\sqrt{ax^2+bx^3}} + \frac{8\sqrt{ax^2+bx^3}}{3b^2} - \frac{(8a)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{3b^2} \\
&= -\frac{2x^3}{b\sqrt{ax^2+bx^3}} + \frac{8\sqrt{ax^2+bx^3}}{3b^2} - \frac{16a\sqrt{ax^2+bx^3}}{3b^3x}
\end{aligned}
$$

**Mathematica [A]** time = 0.0172749, size = 39, normalized size = 0.54

$$\frac{2x\left(-8a^2-4abx+b^2x^2\right)}{3b^3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[x^2*(a + b*x)])
```

---

**Maple [A]** time = 0.006, size = 46, normalized size = 0.6

$$-\frac{(2\,bx+2\,a)\left(-b^2x^2+4\,abx+8\,a^2\right)x^3}{3\,b^3}\left(bx^3+ax^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -2/3*(b*x+a)*(-b^2*x^2+4*a*b*x+8*a^2)*x^3/b^3/(b*x^3+a*x^2)^(3/2)
```

---

**Maxima [A]** time = 1.14535, size = 41, normalized size = 0.57

$$\frac{2\left(b^2x^2-4\,abx-8\,a^2\right)}{3\,\sqrt{bx+a}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)/(sqrt(b*x + a)*b^3)
```

---

**Fricas [A]** time = 0.829334, size = 99, normalized size = 1.38

$$\frac{2\left(b^2x^2-4\,abx-8\,a^2\right)\sqrt{bx^3+ax^2}}{3\left(b^4x^2+ab^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^4*x^2 + a*b^3*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(x**2*(a + b*x))**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*x^3 + a*x^2)^(3/2), x)
```

# 3.262 $\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=47

$$\frac{4\sqrt{ax^2+bx^3}}{b^2x}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

```
[Out] (-2*x^2)/(b*Sqrt[a*x^2 + b*x^3]) + (4*Sqrt[a*x^2 + b*x^3])/(b^2*x)
```

---

**Rubi [A]** time = 0.0567734, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 1588}

$$\frac{4\sqrt{ax^2+bx^3}}{b^2x}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^2)/(b*Sqrt[a*x^2 + b*x^3]) + (4*Sqrt[a*x^2 + b*x^3])/(b^2*x)
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx &= -\frac{2x^2}{b\sqrt{ax^2+bx^3}}+\frac{2\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{b}\\ &= -\frac{2x^2}{b\sqrt{ax^2+bx^3}}+\frac{4\sqrt{ax^2+bx^3}}{b^2x}\end{aligned}$$

**Mathematica [A]** time = 0.0124749, size = 26, normalized size = 0.55

$$\frac{2x(2a+bx)}{b^2\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(2*a + b*x))/(b^2*Sqrt[x^2*(a + b*x)])
```

---

**Maple [A]** time = 0.003, size = 34, normalized size = 0.7

$$2\,\frac{(bx+a)\,(bx+2\,a)\,x^3}{b^2\left(bx^3+ax^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 2*(b*x+a)*(b*x+2*a)*x^3/b^2/(b*x^3+a*x^2)^(3/2)
```

---

**Maxima [A]** time = 1.11266, size = 26, normalized size = 0.55

$$\frac{2\,(bx+2\,a)}{\sqrt{bx+a}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(b*x + 2*a)/(sqrt(b*x + a)*b^2)
```

---

**Fricas [A]** time = 0.82862, size = 74, normalized size = 1.57

$$\frac{2\,\sqrt{bx^3+ax^2}(bx+2\,a)}{b^3x^2+ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x^3 + a*x^2)*(b*x + 2*a)/(b^3*x^2 + a*b^2*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**4/(x**2*(a + b*x))**(3/2), x)
```

---

**Giac [A]** time = 1.13415, size = 38, normalized size = 0.81

$$\frac{2\left(\frac{1}{b}+\frac{2a}{b^2x}\right)}{\sqrt{\frac{b}{x}+\frac{a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(1/b + 2*a/(b^2*x))/sqrt(b/x + a/x^2)
```

## 3.263 $\int \frac{x^3}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=21

$$-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

```
[Out] (-2*x)/(b*Sqrt[a*x^2 + b*x^3])
```

---

**Rubi [A]** time = 0.0175054, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[a*x^2 + b*x^3])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(ax^2+bx^3\right)^{3/2}}\,dx=-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

---

**Mathematica [A]** time = 0.0059717, size = 19, normalized size = 0.9

$$-\frac{2x}{b\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x)/(b*Sqrt[x^2*(a + b*x)])
```

---

**Maple [A]** time = 0.002, size = 27, normalized size = 1.3

$$-2\,\frac{(bx+a)\,x^3}{b\left(bx^3+ax^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -2*(b*x+a)*x^3/b/(b*x^3+a*x^2)^(3/2)
```

---

**Maxima [A]** time = 1.07839, size = 16, normalized size = 0.76

$$-\frac{2}{\sqrt{bx+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*x + a)*b)
```

---

**Fricas [A]** time = 0.83839, size = 57, normalized size = 2.71

$$-\frac{2\sqrt{bx^3+ax^2}}{b^2x^2+abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x^3 + a*x^2)/(b^2*x^2 + a*b*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**3/(x**2*(a + b*x))**(3/2), x)
```

---

**Giac [A]** time = 1.18193, size = 50, normalized size = 2.38

$$\frac{2}{\left(\sqrt{a}\left(\sqrt{\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}\right)-b\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/((sqrt(a)*(sqrt(b/x + a/x^2) - sqrt(a)/x) - b)*sqrt(a))
```

## 3.264 $\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=52

$$\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

```
[Out] (2*x)/(a*Sqrt[a*x^2 + b*x^3]) - (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
)/a^(3/2)
```

---

**Rubi [A]** time = 0.0583374, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2023, 2008, 206}

$$\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x)/(a*Sqrt[a*x^2 + b*x^3]) - (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
)/a^(3/2)
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/2}}\,dx &= \frac{2x}{a\sqrt{ax^2+bx^3}}+\frac{\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{a}\\ &= \frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{a}\\ &= \frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}\end{aligned}$$

**Mathematica [C]** time = 0.0085071, size = 35, normalized size = 0.67

$$\frac{2x\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx}{a}+1\right)}{a\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x)/a])/(a*Sqrt[x^2*(a + b*x)])
```

---

**Maple [A]** time = 0.007, size = 53, normalized size = 1.

$$2\,\frac{x^3\left(bx+a\right)}{\left(bx^3+ax^2\right)^{3/2}a^{5/2}}\left(a^{3/2}-\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\sqrt{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 2*x^3*(b*x+a)*(a^(3/2)-arctanh((b*x+a)^(1/2)/a^(1/2))*a*(b*x+a)^(1/2))/(b*x
^3+a*x^2)^(3/2)/a^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^3 + a*x^2)^(3/2), x)
```

---

**Fricas [A]** time = 0.801286, size = 333, normalized size = 6.4

$$\left[\frac{\left(bx^{2}+ax\right)\sqrt{a}\log\left(\frac{bx^{2}+2\,ax-2\,\sqrt{bx^{3}+ax^{2}}\sqrt{a}}{x^{2}}\right)+2\,\sqrt{bx^{3}+ax^{2}}a}{a^{2}bx^{2}+a^{3}x},\ \frac{2\left(\left(bx^{2}+ax\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\sqrt{-a}}{ax}\right)+\sqrt{bx^{3}+ax^{2}}a\right)}{a^{2}bx^{2}+a^{3}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x^2 + a*x)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) + 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*b*x^2 + a^3*x), 2*((b*x^2 + a*x)*sqrt
(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*a)/(a
^2*b*x^2 + a^3*x)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(x^{2}\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(x**2*(a + b*x))**(3/2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.265 $\int \frac{x}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=75

$$-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

```
[Out] 2/(a*Sqrt[a*x^2 + b*x^3]) - (3*Sqrt[a*x^2 + b*x^3])/(a^2*x^2) + (3*b*ArcTan
h[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(5/2)
```

---

**Rubi [A]** time = 0.0857002, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2023, 2025, 2008, 206}

$$-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] 2/(a*Sqrt[a*x^2 + b*x^3]) - (3*Sqrt[a*x^2 + b*x^3])/(a^2*x^2) + (3*b*ArcTan
h[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(5/2)
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(ax^2+bx^3\right)^{3/2}}\,dx &= \frac{2}{a\sqrt{ax^2+bx^3}} + \frac{3\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{a}\\ &= \frac{2}{a\sqrt{ax^2+bx^3}} - \frac{3\sqrt{ax^2+bx^3}}{a^2x^2} - \frac{(3b)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{2a^2}\\ &= \frac{2}{a\sqrt{ax^2+bx^3}} - \frac{3\sqrt{ax^2+bx^3}}{a^2x^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{a^2}\\ &= \frac{2}{a\sqrt{ax^2+bx^3}} - \frac{3\sqrt{ax^2+bx^3}}{a^2x^2} + \frac{3b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}\end{aligned}$$

**Mathematica [C]** time = 0.0087374, size = 36, normalized size = 0.48

$$-\frac{2bx\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx}{a}+1\right)}{a^2\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*b*x*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x)/a])/(a^2*Sqrt[x^2*(a + b*
x)])
```

---

**Maple [A]** time = 0.012, size = 62, normalized size = 0.8

$$x^2\,(bx+a)\left(3\,\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,xb - 3\,bx\sqrt{a} - a^{\frac{3}{2}}\right)\left(bx^3+ax^2\right)^{-\frac{3}{2}} a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] x^2*(b*x+a)*(3*arctanh((b*x+a)^(1/2)/a^(1/2))*(b*x+a)^(1/2)*x*b-3*b*x*a^(1/
2)-a^(3/2))/(b*x^3+a*x^2)^(3/2)/a^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^3 + a*x^2)^(3/2), x)
```

---

**Fricas [A]** time = 0.804222, size = 402, normalized size = 5.36

$$\left[\frac{3\left(b^2x^3+abx^2\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}\left(3\,abx+a^2\right)}{2\left(a^3bx^3+a^4x^2\right)},-\frac{3\left(b^2x^3+abx^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right.}{a^3bx^3+a^4x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^3 + a*b*x^2)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x
^2)*sqrt(a))/x^2) - 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a^4
*x^2), -(3*(b^2*x^3 + a*b*x^2)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)
/(a*x)) + sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a^4*x^2)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x/(x**2*(a + b*x))**(3/2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.266 $\int \frac{1}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=110

$$-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

```
[Out] 2/(a*x*Sqrt[a*x^2 + b*x^3]) - (5*Sqrt[a*x^2 + b*x^3])/(2*a^2*x^3) + (15*b*S
qrt[a*x^2 + b*x^3])/(4*a^3*x^2) - (15*b^2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 +
b*x^3]])/(4*a^(7/2))
```

---

**Rubi [A]** time = 0.105175, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2006, 2025, 2008, 206}

$$-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(-3/2),x]
```

```
[Out] 2/(a*x*Sqrt[a*x^2 + b*x^3]) - (5*Sqrt[a*x^2 + b*x^3])/(2*a^2*x^3) + (15*b*S
qrt[a*x^2 + b*x^3])/(4*a^3*x^2) - (15*b^2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 +
b*x^3]])/(4*a^(7/2))
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(ax^2+bx^3\right)^{3/2}}\,dx &= \frac{2}{ax\sqrt{ax^2+bx^3}}+\frac{5\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{a}\\
&= \frac{2}{ax\sqrt{ax^2+bx^3}}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}-\frac{(15b)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{4a^2}\\
&= \frac{2}{ax\sqrt{ax^2+bx^3}}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}+\frac{\left(15b^2\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{8a^3}\\
&= \frac{2}{ax\sqrt{ax^2+bx^3}}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{\left(15b^2\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{4a^3}\\
&= \frac{2}{ax\sqrt{ax^2+bx^3}}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.0072385, size = 38, normalized size = 0.35

$$\frac{2b^2x\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{bx}{a}+1\right)}{a^3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(-3/2),x]
```

```
[Out] (2*b^2*x*Hypergeometric2F1[-1/2, 3, 1/2, 1 + (b*x)/a])/(a^3*Sqrt[x^2*(a + b
*x)])
```

---

**Maple [A]** time = 0.012, size = 76, normalized size = 0.7

$$-\frac{x(bx+a)}{4}\left(15\,\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}x^2b^2-5\,a^{3/2}xb-15\,x^2b^2\sqrt{a}+2\,a^{5/2}\right)\left(bx^3+ax^2\right)^{-\frac{3}{2}}a^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/4*x*(b*x+a)*(15*arctanh((b*x+a)^(1/2)/a^(1/2))*(b*x+a)^(1/2)*x^2*b^2-5*a
^(3/2)*x*b-15*x^2*b^2*a^(1/2)+2*a^(5/2))/(b*x^3+a*x^2)^(3/2)/a^(7/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(-3/2), x)
```

---

**Fricas [A]** time = 0.901782, size = 466, normalized size = 4.24

$$\left[\frac{15\left(b^{3}x^{4}+ab^{2}x^{3}\right)\sqrt{a}\log\left(\frac{bx^{2}+2\,ax-2\,\sqrt{bx^{3}+ax^{2}}\sqrt{a}}{x^{2}}\right)+2\left(15\,ab^{2}x^{2}+5\,a^{2}bx-2\,a^{3}\right)\sqrt{bx^{3}+ax^{2}}}{8\left(a^{4}bx^{4}+a^{5}x^{3}\right)},\frac{15\left(b^{3}x^{4}+ab^{2}x^{3}\right)\sqrt{-a}\arctan}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 +
a*x^2)*sqrt(a))/x^2) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*
x^2))/(a^4*b*x^4 + a^5*x^3), 1/4*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(-a)*arctan(
sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sq
rt(b*x^3 + a*x^2))/(a^4*b*x^4 + a^5*x^3)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax^{2}+bx^{3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3)**(-3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

## 3.267 $\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=138

$$-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

```
[Out] 2/(a*x^2*Sqrt[a*x^2 + b*x^3]) - (7*Sqrt[a*x^2 + b*x^3])/(3*a^2*x^4) + (35*b
*Sqrt[a*x^2 + b*x^3])/(12*a^3*x^3) - (35*b^2*Sqrt[a*x^2 + b*x^3])/(8*a^4*x^
2) + (35*b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(8*a^(9/2))
```

---

**Rubi [A]** time = 0.186272, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2023, 2025, 2008, 206}

$$-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] 2/(a*x^2*Sqrt[a*x^2 + b*x^3]) - (7*Sqrt[a*x^2 + b*x^3])/(3*a^2*x^4) + (35*b
*Sqrt[a*x^2 + b*x^3])/(12*a^3*x^3) - (35*b^2*Sqrt[a*x^2 + b*x^3])/(8*a^4*x^
2) + (35*b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(8*a^(9/2))
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx &= \frac{2}{ax^2\sqrt{ax^2+bx^3}}+\frac{7\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx}{a}\\
&= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}-\frac{(35b)\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{6a^2}\\
&= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}+\frac{\left(35b^2\right)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{8a^3}\\
&= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}-\frac{\left(35b^3\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a^4}\\
&= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{\left(35b^3\right)\text{Subst}\left(\int \frac{1}{1-ax^2}}{8a^4}\\
&= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0087413, size = 38, normalized size = 0.28

$$-\frac{2b^3x\,{}_2F_1\left(-\frac{1}{2},4;\frac{1}{2};\frac{bx}{a}+1\right)}{a^4\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] (-2*b^3*x*Hypergeometric2F1[-1/2, 4, 1/2, 1 + (b*x)/a])/(a^4*Sqrt[x^2*(a +
b*x)])
```

---

**Maple [A]** time = 0.013, size = 86, normalized size = 0.6

$$-\frac{bx+a}{24}\left(-105\,\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x^3b^3-14\,a^{5/2}xb+35\,a^{3/2}x^2b^2+105\,b^3x^3\sqrt{a}+8\,a^{7/2}\right)\left(bx^3+ax^2\right)^{-\frac{3}{2}}a^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/24*(b*x+a)*(-105*arctanh((b*x+a)^(1/2)/a^(1/2))*(b*x+a)^(1/2)*x^3*b^3-14
*a^(5/2)*x*b+35*a^(3/2)*x^2*b^2+105*b^3*x^3*a^(1/2)+8*a^(7/2))/(b*x^3+a*x^2
)^(3/2)/a^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x), x)
```

---

**Fricas [A]** time = 0.816872, size = 524, normalized size = 3.8

$$\left[\frac{105\left(b^4x^5+ab^3x^4\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\left(105\,ab^3x^3+35\,a^2b^2x^2-14\,a^3bx+8\,a^4\right)\sqrt{bx^3+ax^2}}{48\left(a^5bx^5+a^6x^4\right)},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^4*x^5 + a*b^3*x^4)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3
+ a*x^2)*sqrt(a))/x^2) - 2*(105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8
*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 + a^6*x^4), -1/24*(105*(b^4*x^5 + a*b
^3*x^4)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (105*a*b^3*x^
3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 +
a^6*x^4)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(x**2*(a + b*x))**(3/2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x), x)
```

## 3.268 $\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx$

**Optimal**. Leaf size=166

$$\frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2}-\frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3}-\frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{11/2}}+\frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4}-\frac{9\sqrt{ax^2+bx^3}}{4a^2x^5}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

```
[Out] 2/(a*x^3*Sqrt[a*x^2 + b*x^3]) - (9*Sqrt[a*x^2 + b*x^3])/(4*a^2*x^5) + (21*b
*Sqrt[a*x^2 + b*x^3])/(8*a^3*x^4) - (105*b^2*Sqrt[a*x^2 + b*x^3])/(32*a^4*x
^3) + (315*b^3*Sqrt[a*x^2 + b*x^3])/(64*a^5*x^2) - (315*b^4*ArcTanh[(Sqrt[a
]*x)/Sqrt[a*x^2 + b*x^3]])/(64*a^(11/2))
```

---

**Rubi [A]** time = 0.232448, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2023, 2025, 2008, 206}

$$\frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2}-\frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3}-\frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{11/2}}+\frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4}-\frac{9\sqrt{ax^2+bx^3}}{4a^2x^5}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] 2/(a*x^3*Sqrt[a*x^2 + b*x^3]) - (9*Sqrt[a*x^2 + b*x^3])/(4*a^2*x^5) + (21*b
*Sqrt[a*x^2 + b*x^3])/(8*a^3*x^4) - (105*b^2*Sqrt[a*x^2 + b*x^3])/(32*a^4*x
^3) + (315*b^3*Sqrt[a*x^2 + b*x^3])/(64*a^5*x^2) - (315*b^4*ArcTanh[(Sqrt[a
]*x)/Sqrt[a*x^2 + b*x^3]])/(64*a^(11/2))
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx &= \frac{2}{ax^3\sqrt{ax^2+bx^3}}+\frac{9\int \frac{1}{x^4\sqrt{ax^2+bx^3}}\,dx}{a}\\
&= \frac{2}{ax^3\sqrt{ax^2+bx^3}}-\frac{9\sqrt{ax^2+bx^3}}{4a^2x^5}-\frac{(63b)\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx}{8a^2}\\
&= \frac{2}{ax^3\sqrt{ax^2+bx^3}}-\frac{9\sqrt{ax^2+bx^3}}{4a^2x^5}+\frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4}+\frac{\left(105b^2\right)\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{16a^3}\\
&= \frac{2}{ax^3\sqrt{ax^2+bx^3}}-\frac{9\sqrt{ax^2+bx^3}}{4a^2x^5}+\frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4}-\frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3}-\frac{\left(315b^3\right)\int \frac{1}{x\sqrt{ax^2+bx}}}{64a^4}\\
&= \frac{2}{ax^3\sqrt{ax^2+bx^3}}-\frac{9\sqrt{ax^2+bx^3}}{4a^2x^5}+\frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4}-\frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3}+\frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2}\\
&= \frac{2}{ax^3\sqrt{ax^2+bx^3}}-\frac{9\sqrt{ax^2+bx^3}}{4a^2x^5}+\frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4}-\frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3}+\frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2}\\
&= \frac{2}{ax^3\sqrt{ax^2+bx^3}}-\frac{9\sqrt{ax^2+bx^3}}{4a^2x^5}+\frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4}-\frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3}+\frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2}
\end{aligned}
$$

**Mathematica [C]** time = 0.0099285, size = 38, normalized size = 0.23

$$\frac{2b^4x\,{}_2F_1\left(-\frac{1}{2},5;\frac{1}{2};\frac{bx}{a}+1\right)}{a^5\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] (2*b^4*x*Hypergeometric2F1[-1/2, 5, 1/2, 1 + (b*x)/a])/(a^5*Sqrt[x^2*(a + b
*x)])
```

---

**Maple [A]** time = 0.014, size = 100, normalized size = 0.6

$$-\frac{bx+a}{64\,x}\left(315\,\sqrt{bx+a}\,\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)x^4b^4-24\,a^{7/2}xb+42\,a^{5/2}x^2b^2-105\,a^{3/2}x^3b^3-315\,b^4x^4\sqrt{a}+16\,a^{9/2}\right)\left(bx^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/64*(b*x+a)*(315*(b*x+a)^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*x^4*b^4-24*
a^(7/2)*x*b+42*a^(5/2)*x^2*b^2-105*a^(3/2)*x^3*b^3-315*b^4*x^4*a^(1/2)+16*a
^(9/2))/x/(b*x^3+a*x^2)^(3/2)/a^(11/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x^2), x)
```

---

**Fricas [A]** time = 0.821706, size = 575, normalized size = 3.46

$$\left[\frac{315\left(b^5x^6+ab^4x^5\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(315\,ab^4x^4+105\,a^2b^3x^3-42\,a^3b^2x^2+24\,a^4bx-16\,a^5\right)\sqrt{bx^3+ax^2}}{128\left(a^6bx^6+a^7x^5\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(315*(b^5*x^6 + a*b^4*x^5)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3
 + a*x^2)*sqrt(a))/x^2) + 2*(315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2*x
^2 + 24*a^4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5), 1/64*
(315*(b^5*x^6 + a*b^4*x^5)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*
x)) + (315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2*x^2 + 24*a^4*b*x - 16*a
^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2*(a + b*x))**(3/2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x^2), x)
```

## 3.269 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx$

**Optimal**. Leaf size=125

$$\frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}-\frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2}+\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}$$

```
[Out] (5*a^2*Sqrt[a*x^2 + b*x^3])/(8*b^3*Sqrt[x]) - (5*a*Sqrt[x]*Sqrt[a*x^2 + b*x
^3])/(12*b^2) + (x^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a^3*ArcTanh[(Sqrt[
b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(8*b^(7/2))
```

---

**Rubi [A]** time = 0.169449, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2024, 2029, 206}

$$\frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}-\frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2}+\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (5*a^2*Sqrt[a*x^2 + b*x^3])/(8*b^3*Sqrt[x]) - (5*a*Sqrt[x]*Sqrt[a*x^2 + b*x
^3])/(12*b^2) + (x^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a^3*ArcTanh[(Sqrt[
b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(8*b^(7/2))
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx &= \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{(5a)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx}{6b} \\
&= -\frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} + \frac{\left(5a^2\right)\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx}{8b^2} \\
&= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{\left(5a^3\right)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx}{16b^3} \\
&= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^3} \\
&= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.108556, size = 104, normalized size = 0.83

$$\frac{\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{x}\sqrt{\frac{bx}{a}+1}\left(15a^2-10abx+8b^2x^2\right)-15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\right)}{24b^{7/2}x\sqrt{\frac{bx}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[1 + (b*x)/a]*(15*a^2 - 10*a*b*x
+ 8*b^2*x^2) - 15*a^(5/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(24*b^(7/2)*
x*Sqrt[1 + (b*x)/a])
```

---

**Maple [A]** time = 0.006, size = 103, normalized size = 0.8

$$-\frac{1}{48}\sqrt{x}\left(-16\,b^{9/2}x^4+4\,b^{7/2}x^3a-10\,b^{5/2}x^2a^2-30\,b^{3/2}xa^3+15\,\sqrt{x\,(bx+a)}\ln\left(1/2\,\frac{2\,\sqrt{bx^2+ax}\sqrt{b}+2\,bx+a}{\sqrt{b}}\right)a^3b\right)\frac{1}{\sqrt{bx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/48*x^(1/2)*(-16*b^(9/2)*x^4+4*b^(7/2)*x^3*a-10*b^(5/2)*x^2*a^2-30*b^(3/2
)*x*a^3+15*(x*(b*x+a))^(1/2)*ln(1/2*(2*(b*x^2+a*x)^(1/2)*b^(1/2)+2*b*x+a)/b
^(1/2))*a^3*b)/(b*x^3+a*x^2)^(1/2)/b^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/sqrt(b*x^3 + a*x^2), x)
```

---

**Fricas [A]** time = 0.792716, size = 428, normalized size = 3.42

$$\left[\frac{15\,a^3\sqrt{b}x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right)+2\left(8\,b^3x^2-10\,ab^2x+15\,a^2b\right)\sqrt{bx^3+ax^2}\sqrt{x}}{48\,b^4x},\ 15\,a^3\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-b}}{bx^{\frac{3}{2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*
sqrt(x))/x) + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^3 + a*x^2)*sqr
t(x))/(b^4*x), 1/24*(15*a^3*sqrt(-b)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/
(b*x^(3/2))) + (8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^3 + a*x^2)*sqrt
(x))/(b^4*x)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(7/2)/sqrt(x**2*(a + b*x)), x)
```

---

**Giac [A]** time = 1.30755, size = 86, normalized size = 0.69

$$\frac{1}{24}\,\sqrt{bx+a}\left(2\,x\left(\frac{4\,x}{b}-\frac{5\,a}{b^2}\right)+\frac{15\,a^2}{b^3}\right)\sqrt{x}+\frac{5\,a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*x + a)*(2*x*(4*x/b - 5*a/b^2) + 15*a^2/b^3)*sqrt(x) + 5/8*a^3*l
og(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

## 3.270 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx$

**Optimal**. Leaf size=95

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}-\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}}+\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}$$

```
[Out] (-3*a*Sqrt[a*x^2 + b*x^3])/(4*b^2*Sqrt[x]) + (Sqrt[x]*Sqrt[a*x^2 + b*x^3])/
(2*b) + (3*a^2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(4*b^(5/2))
```

---

**Rubi [A]** time = 0.125441, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2024, 2029, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}-\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}}+\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-3*a*Sqrt[a*x^2 + b*x^3])/(4*b^2*Sqrt[x]) + (Sqrt[x]*Sqrt[a*x^2 + b*x^3])/
(2*b) + (3*a^2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(4*b^(5/2))
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx &= \frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}-\frac{(3a)\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx}{4b}\\ &= -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}}+\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}+\frac{\left(3a^2\right)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx}{8b^2}\\ &= -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}}+\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}+\frac{\left(3a^2\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^2}\\ &= -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}}+\frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}\end{aligned}$$

**Mathematica [A]** time = 0.0519435, size = 90, normalized size = 0.95

$$\frac{\sqrt{b}x^{3/2}\left(-3a^2-abx+2b^2x^2\right)+3a^{5/2}x\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{5/2}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (Sqrt[b]*x^(3/2)*(-3*a^2 - a*b*x + 2*b^2*x^2) + 3*a^(5/2)*x*Sqrt[1 + (b*x)/
a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(5/2)*Sqrt[x^2*(a + b*x)])
```

---

**Maple [A]** time = 0.005, size = 92, normalized size = 1.

$$\frac{1}{8}\sqrt{x}\left(4\,b^{7/2}x^3-2\,b^{5/2}x^2a-6\,b^{3/2}xa^2+3\,\sqrt{x\,(bx+a)}\ln\left(1/2\,\frac{2\,\sqrt{bx^2+ax}\sqrt{b}+2\,bx+a}{\sqrt{b}}\right)a^2b\right)\frac{1}{\sqrt{bx^3+ax^2}}b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/8*x^(1/2)*(4*b^(7/2)*x^3-2*b^(5/2)*x^2*a-6*b^(3/2)*x*a^2+3*(x*(b*x+a))^(1
/2)*ln(1/2*(2*(b*x^2+a*x)^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*a^2*b)/(b*x^3+a*x
^2)^(1/2)/b^(7/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/sqrt(b*x^3 + a*x^2), x)
```

---

**Fricas [A]** time = 0.830067, size = 375, normalized size = 3.95

$$\left[\frac{3\,a^{2}\sqrt{b}x\log\left(\frac{2\,bx^{2}+ax+2\,\sqrt{bx^{3}+ax^{2}}\sqrt{b}\sqrt{x}}{x}\right)+2\,\sqrt{bx^{3}+ax^{2}}\left(2\,b^{2}x-3\,ab\right)\sqrt{x}}{8\,b^{3}x},-\frac{3\,a^{2}\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\sqrt{-b}}{bx^{\frac{3}{2}}}\right)-\sqrt{bx^{3}+ax^{2}}(}{4\,b^{3}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*x*log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sq
rt(x))/x) + 2*sqrt(b*x^3 + a*x^2)*(2*b^2*x - 3*a*b)*sqrt(x))/(b^3*x), -1/4*
(3*a^2*sqrt(-b)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2))) - sqrt(b
*x^3 + a*x^2)*(2*b^2*x - 3*a*b)*sqrt(x))/(b^3*x)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{x^{2}\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)/sqrt(x**2*(a + b*x)), x)
```

---

**Giac [A]** time = 1.29259, size = 70, normalized size = 0.74

$$\frac{1}{4}\,\sqrt{bx+a}\sqrt{x}\left(\frac{2\,x}{b}-\frac{3\,a}{b^{2}}\right)-\frac{3\,a^{2}\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{4\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x + a)*sqrt(x)*(2*x/b - 3*a/b^2) - 3/4*a^2*log(abs(-sqrt(b)*sqrt
(x) + sqrt(b*x + a)))/b^(5/2)
```

## 3.271 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx$

**Optimal**. Leaf size=60

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

```
[Out] Sqrt[a*x^2 + b*x^3]/(b*Sqrt[x]) - (a*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 +
 b*x^3]])/b^(3/2)
```

---

**Rubi [A]** time = 0.0833389, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2024, 2029, 206}

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3]/(b*Sqrt[x]) - (a*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 +
 b*x^3]])/b^(3/2)
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx &= \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}} - \frac{a\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx}{2b} \\ &= \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b} \\ &= \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.0392275, size = 73, normalized size = 1.22

$$\frac{\sqrt{b}x^{3/2}(a+bx) - a^{3/2}x\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (Sqrt[b]*x^(3/2)*(a + b*x) - a^(3/2)*x*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*S
qrt[x])/Sqrt[a]])/(b^(3/2)*Sqrt[x^2*(a + b*x)])
```

---

**Maple [A]** time = 0.005, size = 79, normalized size = 1.3

$$\frac{1}{2}\sqrt{x}\left(2\,b^{5/2}x^2 + 2\,b^{3/2}xa - a\sqrt{x\,(bx+a)}\ln\left(\frac{1}{2}\left(2\,\sqrt{bx^2+ax}\sqrt{b} + 2\,bx + a\right)\frac{1}{\sqrt{b}}\right)b\right)\frac{1}{\sqrt{bx^3+ax^2}}b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/2*x^(1/2)*(2*b^(5/2)*x^2+2*b^(3/2)*x*a-a*(x*(b*x+a))^(1/2)*ln(1/2*(2*(b*x
^2+a*x)^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*b)/(b*x^3+a*x^2)^(1/2)/b^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(b*x^3 + a*x^2), x)
```

---

**Fricas [A]** time = 0.885123, size = 315, normalized size = 5.25

$$\left[\frac{a\sqrt{b}x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right)+2\,\sqrt{bx^3+ax^2}b\sqrt{x}}{2\,b^2x}, \frac{a\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-b}}{bx^{\frac{3}{2}}}\right)+\sqrt{bx^3+ax^2}b\sqrt{x}}{b^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(x
))/x) + 2*sqrt(b*x^3 + a*x^2)*b*sqrt(x))/(b^2*x), (a*sqrt(-b)*x*arctan(sqrt
(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2))) + sqrt(b*x^3 + a*x^2)*b*sqrt(x))/(b^2
*x)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{x^2\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x**2*(a + b*x)), x)
```

---

**Giac [A]** time = 1.35451, size = 51, normalized size = 0.85

$$\frac{a\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}+\frac{\sqrt{bx+a}\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2) + sqrt(b*x + a)*sqrt(x
)/b
```

## 3.272 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx$

**Optimal**. Leaf size=34

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/Sqrt[b]
```

---

**Rubi [A]** time = 0.0424737, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2029, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/Sqrt[b]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}\end{aligned}$$

**Mathematica [A]** time = 0.0161086, size = 55, normalized size = 1.62

$$\frac{2\sqrt{a}x\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[a]*x*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]
*Sqrt[x^2*(a + b*x)])
```

---

**Maple [B]** time = 0.005, size = 58, normalized size = 1.7

$$\sqrt{x}\sqrt{x\,(bx+a)}\ln\left(\frac{1}{2}\left(2\sqrt{bx^2+ax}\sqrt{b}+2\,bx+a\right)\frac{1}{\sqrt{b}}\right)\frac{1}{\sqrt{bx^3+ax^2}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/(b*x^3+a*x^2)^(1/2)*x^(1/2)*(x*(b*x+a))^(1/2)*ln(1/2*(2*(b*x^2+a*x)^(1/2)
*b^(1/2)+2*b*x+a)/b^(1/2))/b^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(b*x^3 + a*x^2), x)
```

---

**Fricas [A]** time = 0.840399, size = 189, normalized size = 5.56

$$\left[\frac{\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\sqrt{b}\sqrt{x}}{x}\right)}{\sqrt{b}},-\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-b}}{bx^{\frac{3}{2}}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(x))/x)/sqrt(b), -2
*sqrt(-b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2)))/b]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x}}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(x**2*(a + b*x)), x)
```

---

**Giac [A]** time = 1.39691, size = 31, normalized size = 0.91

$$-\frac{2\,\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/sqrt(b)
```

## 3.273 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx$

**Optimal**. Leaf size=25

$$-\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(a*x^(3/2))
```

---

**Rubi [A]** time = 0.0377593, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2014}

$$-\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(a*x^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx=-\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

**Mathematica [A]** time = 0.0082031, size = 23, normalized size = 0.92

$$-\frac{2\sqrt{x^2(a+bx)}}{ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[x^2*(a + b*x)])/(a*x^(3/2))
```

---

**Maple [A]** time = 0.003, size = 27, normalized size = 1.1

$$-2\,\frac{\sqrt{x}\,(bx+a)}{a\sqrt{bx^3+ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2*x^(1/2)*(b*x+a)/a/(b*x^3+a*x^2)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3 + ax^2}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*sqrt(x)), x)
```

---

**Fricas [A]** time = 0.713405, size = 49, normalized size = 1.96

$$-\frac{2\sqrt{bx^3 + ax^2}}{ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x^3 + a*x^2)/(a*x^(3/2))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\sqrt{x^2\,(a + bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(x**2*(a + b*x))), x)
```

---

**Giac [A]** time = 1.26793, size = 41, normalized size = 1.64

$$\frac{4\sqrt{b}}{\left(\sqrt{b}\sqrt{x} - \sqrt{bx + a}\right)^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 4*sqrt(b)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)
```

# 3.274 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx$

**Optimal**. Leaf size=56

$$\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}$$

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(3*a*x^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*x
^(3/2))
```

---

**Rubi [A]** time = 0.0766247, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(3*a*x^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*x
^(3/2))
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx &= -\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}-\frac{(2b)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}+\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.0150395, size = 31, normalized size = 0.55

$$-\frac{2(a-2bx)\sqrt{x^2(a+bx)}}{3a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*(a - 2*b*x)*Sqrt[x^2*(a + b*x)])/(3*a^2*x^(5/2))
```

---

**Maple [A]** time = 0.003, size = 33, normalized size = 0.6

$$-\frac{(2\,bx+2\,a)\,(-2\,bx+a)}{3\,a^2}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx^3+ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/3*(b*x+a)*(-2*b*x+a)/x^(1/2)/a^2/(b*x^3+a*x^2)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+ax^2}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(3/2)), x)
```

---

**Fricas [A]** time = 0.890541, size = 69, normalized size = 1.23

$$\frac{2\,\sqrt{bx^3+ax^2}(2\,bx-a)}{3\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^3 + a*x^2)*(2*b*x - a)/(a^2*x^(5/2))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x))), x)
```

---

**Giac [A]** time = 1.20807, size = 74, normalized size = 1.32

$$\frac{8\left(3\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)*b^(3/2)/((sqrt(b)*sqrt(x) -
 sqrt(b*x + a))^2 - a)^3
```

## 3.275 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx$

**Optimal**. Leaf size=86

$$-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(5*a*x^(7/2)) + (8*b*Sqrt[a*x^2 + b*x^3])/(15*a^2*
x^(5/2)) - (16*b^2*Sqrt[a*x^2 + b*x^3])/(15*a^3*x^(3/2))
```

---

**Rubi [A]** time = 0.1178, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(5*a*x^(7/2)) + (8*b*Sqrt[a*x^2 + b*x^3])/(15*a^2*
x^(5/2)) - (16*b^2*Sqrt[a*x^2 + b*x^3])/(15*a^3*x^(3/2))
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx &= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}-\frac{(4b)\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx}{5a}\\ &= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}+\frac{\left(8b^2\right)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx}{15a^2}\\ &= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}\end{aligned}$$

**Mathematica [A]** time = 0.0152781, size = 44, normalized size = 0.51

$$-\frac{2\sqrt{x^2(a+bx)}\left(3a^2-4abx+8b^2x^2\right)}{15a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[x^2*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(7/2))
```

---

**Maple [A]** time = 0.004, size = 46, normalized size = 0.5

$$-\frac{(2\,bx+2\,a)\left(8\,b^2x^2-4\,abx+3\,a^2\right)}{15\,a^3}x^{-\frac{3}{2}}\frac{1}{\sqrt{bx^3+ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/15*(b*x+a)*(8*b^2*x^2-4*a*b*x+3*a^2)/x^(3/2)/a^3/(b*x^3+a*x^2)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+ax^2}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(5/2)), x)
```

---

**Fricas [A]** time = 0.789147, size = 96, normalized size = 1.12

$$-\frac{2\left(8\,b^2x^2-4\,abx+3\,a^2\right)\sqrt{bx^3+ax^2}}{15\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x^3 + a*x^2)/(a^3*x^(7/2))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(x**2*(a + b*x))), x)
```

---

**Giac [A]** time = 1.15967, size = 104, normalized size = 1.21

$$\frac{32\left(10\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{4}-5\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}+a^{2}\right)b^{\frac{5}{2}}}{15\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 32/15*(10*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 5*a*(sqrt(b)*sqrt(x) - sqrt
(b*x + a))^2 + a^2)*b^(5/2)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5
```

## 3.276 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx$

**Optimal**. Leaf size=116

$$\frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(7*a*x^(9/2)) + (12*b*Sqrt[a*x^2 + b*x^3])/(35*a^2
*x^(7/2)) - (16*b^2*Sqrt[a*x^2 + b*x^3])/(35*a^3*x^(5/2)) + (32*b^3*Sqrt[a*
x^2 + b*x^3])/(35*a^4*x^(3/2))
```

---

**Rubi [A]** time = 0.162766, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(7*a*x^(9/2)) + (12*b*Sqrt[a*x^2 + b*x^3])/(35*a^2
*x^(7/2)) - (16*b^2*Sqrt[a*x^2 + b*x^3])/(35*a^3*x^(5/2)) + (32*b^3*Sqrt[a*
x^2 + b*x^3])/(35*a^4*x^(3/2))
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx &= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}-\frac{(6b)\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx}{7a}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}+\frac{\left(24b^2\right)\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx}{35a^2}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}-\frac{\left(16b^3\right)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx}{35a^3}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}+\frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0177761, size = 55, normalized size = 0.47

$$\frac{2\sqrt{x^2(a+bx)}\left(6a^2bx-5a^3-8ab^2x^2+16b^3x^3\right)}{35a^4x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35
*a^4*x^(9/2))
```

---

**Maple [A]** time = 0.004, size = 57, normalized size = 0.5

$$-\frac{(2\,bx+2\,a)\left(-16\,b^3x^3+8\,ab^2x^2-6\,bxa^2+5\,a^3\right)}{35\,a^4}x^{-\frac{5}{2}}\frac{1}{\sqrt{bx^3+ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/35*(b*x+a)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/x^(5/2)/a^4/(b*x^3+
a*x^2)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^3+ax^2}x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(7/2)), x)
```

---

**Fricas [A]** time = 0.756968, size = 117, normalized size = 1.01

$$\frac{2\left(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3\right)\sqrt{bx^3+ax^2}}{35\,a^4x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^3 + a*x^2)/(a^
4*x^(9/2))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{7}{2}}\sqrt{x^{2}\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(x**2*(a + b*x))), x)
```

---

**Giac [A]** time = 1.26918, size = 139, normalized size = 1.2

$$\frac{64\left(35\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{6}-21\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{4}+7\,a^{2}\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a^{3}\right)b^{\frac{7}{2}}}{35\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^{2}-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 64/35*(35*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 - 21*a*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^4 + 7*a^2*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^3)*b^(7/2)/((
sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7
```

## 3.277 $\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx$

**Optimal**. Leaf size=61

$$\frac{x^{2-3n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(2-n,-n;3-n;-\frac{bx}{a}\right)}{2-n}$$

```
[Out] (x^(2 - 3*n)*(a*x^2 + b*x^3)^n*Hypergeometric2F1[2 - n, -n, 3 - n, -((b*x)/
a)])/((2 - n)*(1 + (b*x)/a)^n)
```

---

**Rubi [A]** time = 0.038824, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 66, 64}

$$\frac{x^{2-3n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(2-n,-n;3-n;-\frac{bx}{a}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] (x^(2 - 3*n)*(a*x^2 + b*x^3)^n*Hypergeometric2F1[2 - n, -n, 3 - n, -((b*x)/
a)])/((2 - n)*(1 + (b*x)/a)^n)
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int x^{1-3n}\left(ax^2+bx^3\right)^n\,dx &= \left(x^{-2n}(a+bx)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{1-n}(a+bx)^n\,dx\\ &= \left(x^{-2n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{1-n}\left(1+\frac{bx}{a}\right)^n\,dx\\ &= \frac{x^{2-3n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(2-n,-n;3-n;-\frac{bx}{a}\right)}{2-n}\end{aligned}$$

**Mathematica [A]** time = 0.0175152, size = 59, normalized size = 0.97

$$\frac{x^{2-3n}\left(x^2(a+bx)\right)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(2-n,-n;3-n;-\frac{bx}{a}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] (x^(2 - 3*n)*(x^2*(a + b*x))^n*Hypergeometric2F1[2 - n, -n, 3 - n, -((b*x)/
a)])/((2 - n)*(1 + (b*x)/a)^n)
```

**Maple [F]** time = 0.296, size = 0, normalized size = 0.

$$\int x^{1-3\,n}\left(bx^3+ax^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1-3*n)*(b*x^3+a*x^2)^n,x)
```

```
[Out] int(x^(1-3*n)*(b*x^3+a*x^2)^n,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+ax^2\right)^n x^{-3\,n+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^3+ax^2\right)^n x^{-3\,n+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + ax^2\right)^n x^{-3\,n+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

## 3.278 $\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx$

**Optimal**. Leaf size=48

$$\frac{x^{-3n-1}\left(ax^2+bx^3\right)^{n+1}\,{}_2F_1\left(1,2;2-n;-\frac{bx}{a}\right)}{a(1-n)}$$

```
[Out] (x^(-1 - 3*n)*(a*x^2 + b*x^3)^(1 + n)*Hypergeometric2F1[1, 2, 2 - n, -((b*x
)/a)])/(a*(1 - n))
```

---

**Rubi [A]** time = 0.0371099, antiderivative size = 61, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2032, 66, 64}

$$\frac{x^{1-3n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(1-n,-n;2-n;-\frac{bx}{a}\right)}{1-n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^n/x^(3*n),x]
```

```
[Out] (x^(1 - 3*n)*(a*x^2 + b*x^3)^n*Hypergeometric2F1[1 - n, -n, 2 - n, -((b*x)/
a)])/((1 - n)*(1 + (b*x)/a)^n)
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] &&  !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int x^{-3n}\left(ax^2+bx^3\right)^n\,dx &= \left(x^{-2n}(a+bx)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{-n}(a+bx)^n\,dx\\ &= \left(x^{-2n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{-n}\left(1+\frac{bx}{a}\right)^n\,dx\\ &= \frac{x^{1-3n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(1-n,-n;2-n;-\frac{bx}{a}\right)}{1-n}\end{aligned}$$

**Mathematica [A]** time = 0.012427, size = 59, normalized size = 1.23

$$\frac{x^{1-3n}\left(x^2(a+bx)\right)^n\left(\frac{bx}{a}+1\right)^{-n}\,{}_2F_1\left(1-n,-n;2-n;-\frac{bx}{a}\right)}{1-n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^n/x^(3*n),x]
```

```
[Out] (x^(1 - 3*n)*(x^2*(a + b*x))^n*Hypergeometric2F1[1 - n, -n, 2 - n, -((b*x)/
a)])/((1 - n)*(1 + (b*x)/a)^n)
```

---

**Maple [F]** time = 0.252, size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax^2\right)^n}{x^{3n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^n/(x^(3*n)),x)
```

```
[Out] int((b*x^3+a*x^2)^n/(x^(3*n)),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+ax^2\right)^n}{x^{3n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n/x^(3*n), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^3+ax^2\right)^n}{x^{3n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a*x^2)^n/x^(3*n), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{-3n}\left(x^{2}\left(a+bx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**n/(x**(3*n)),x)
```

```
[Out] Integral(x**(-3*n)*(x**2*(a + b*x))**n, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{3}+ax^{2}\right)^{n}}{x^{3\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n/x^(3*n), x)
```

## 3.279 $\int x^{-1-3n}\left(ax^2+bx^3\right)^n\,dx$

**Optimal**. Leaf size=54

$$-\frac{x^{-3n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(-n,-n;1-n;-\frac{bx}{a}\right)}{n}$$

```
[Out] -(((a*x^2 + b*x^3)^n*Hypergeometric2F1[-n, -n, 1 - n, -((b*x)/a)])/(n*x^(3*
n)*(1 + (b*x)/a)^n))
```

---

**Rubi [A]** time = 0.0382429, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 66, 64}

$$-\frac{x^{-3n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n\,{}_2F_1\left(-n,-n;1-n;-\frac{bx}{a}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -(((a*x^2 + b*x^3)^n*Hypergeometric2F1[-n, -n, 1 - n, -((b*x)/a)])/(n*x^(3*
n)*(1 + (b*x)/a)^n))
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\begin{aligned}\int x^{-1-3n}\left(ax^2+bx^3\right)^n dx &= \left(x^{-2n}(a+bx)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{-1-n}(a+bx)^n dx\\ &= \left(x^{-2n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{-1-n}\left(1+\frac{bx}{a}\right)^n dx\\ &= -\frac{x^{-3n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n {}_2F_1\left(-n,-n;1-n;-\frac{bx}{a}\right)}{n}\end{aligned}$$

**Mathematica [A]** time = 0.00813, size = 52, normalized size = 0.96

$$-\frac{x^{-3n}\left(x^2(a+bx)\right)^n\left(\frac{bx}{a}+1\right)^{-n} {}_2F_1\left(-n,-n;1-n;-\frac{bx}{a}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -(((x^2*(a + b*x))^n*Hypergeometric2F1[-n, -n, 1 - n, -((b*x)/a)])/(n*x^(3*
n)*(1 + (b*x)/a)^n))
```

---

**Maple [F]** time = 0.241, size = 0, normalized size = 0.

$$\int x^{-1-3\,n}\left(bx^3+ax^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)*(b*x^3+a*x^2)^n,x)
```

```
[Out] int(x^(-1-3*n)*(b*x^3+a*x^2)^n,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(bx^3+ax^2\right)^n x^{-3\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^3+ax^2\right)^n x^{-3\,n-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{-3n-1} \left(x^2 (a + bx)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Integral(x**(-3*n - 1)*(x**2*(a + b*x))**n, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + ax^2\right)^n x^{-3n-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

## 3.280 $\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx$

**Optimal**. Leaf size=32

$$-\frac{x^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a(n+1)}$$

```
[Out] -((a*x^2 + b*x^3)^(1 + n)/(a*(1 + n)*x^(3*(1 + n))))
```

---

**Rubi [A]** time = 0.0251077, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2014}

$$-\frac{x^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((a*x^2 + b*x^3)^(1 + n)/(a*(1 + n)*x^(3*(1 + n))))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx=-\frac{x^{-3(1+n)}\left(ax^2+bx^3\right)^{1+n}}{a(1+n)}$$

**Mathematica [A]** time = 0.014193, size = 30, normalized size = 0.94

$$-\frac{x^{-3(n+1)}\left(x^2(a+bx)\right)^{n+1}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^2*(a + b*x))^(1 + n)/(a*(1 + n)*x^(3*(1 + n))))
```

---

**Maple [A]** time = 0.003, size = 36, normalized size = 1.1

$$-\frac{x^{-1-3\,n}\left(bx+a\right)\left(bx^3+ax^2\right)^n}{a\left(1+n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2-3*n)*(b*x^3+a*x^2)^n,x)
```

```
[Out] -x^(-1-3*n)*(b*x+a)/a/(1+n)*(b*x^3+a*x^2)^n
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+ax^2\right)^n x^{-3\,n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 2), x)
```

---

**Fricas [A]** time = 0.946151, size = 77, normalized size = 2.41

$$-\frac{\left(bx^2+ax\right)\left(bx^3+ax^2\right)^n x^{-3\,n-2}}{an+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] -(b*x^2 + a*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 2)/(a*n + a)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+ax^2\right)^n x^{-3\,n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 2), x)
```

## 3.281 $\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx$

**Optimal**. Leaf size=70

$$\frac{bx^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a(n+2)}$$

```
[Out] -((x^(-4 - 3*n)*(a*x^2 + b*x^3)^(1 + n))/(a*(2 + n))) + (b*(a*x^2 + b*x^3)^
(1 + n))/(a^2*(1 + n)*(2 + n)*x^(3*(1 + n)))
```

---

**Rubi [A]** time = 0.0543414, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{bx^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^(-4 - 3*n)*(a*x^2 + b*x^3)^(1 + n))/(a*(2 + n))) + (b*(a*x^2 + b*x^3)^
(1 + n))/(a^2*(1 + n)*(2 + n)*x^(3*(1 + n)))
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx&=-\frac{x^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a(2+n)}-\frac{b\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx}{a(2+n)}\\&=-\frac{x^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a(2+n)}+\frac{bx^{-3(1+n)}\left(ax^2+bx^3\right)^{1+n}}{a^2(1+n)(2+n)}\end{aligned}$$

**Mathematica [A]** time = 0.0208736, size = 44, normalized size = 0.63

$$-\frac{x^{-3n-4}(an+a-bx)\left(x^2(a+bx)\right)^{n+1}}{a^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^(-4 - 3*n)*(a + a*n - b*x)*(x^2*(a + b*x))^(1 + n))/(a^2*(1 + n)*(2 +
n)))
```

---

**Maple [A]** time = 0.005, size = 50, normalized size = 0.7

$$-\frac{x^{-2-3n}\left(bx^3+ax^2\right)^n(an-bx+a)(bx+a)}{(2+n)(1+n)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-3*n)*(b*x^3+a*x^2)^n,x)
```

```
[Out] -(b*x^3+a*x^2)^n*x^(-2-3*n)*(a*n-b*x+a)*(b*x+a)/(2+n)/(1+n)/a^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(bx^3+ax^2\right)^n x^{-3n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 3), x)
```

---

**Fricas [A]** time = 0.811785, size = 136, normalized size = 1.94

$$-\frac{\left(abnx^2-b^2x^3+\left(a^2n+a^2\right)x\right)\left(bx^3+ax^2\right)^n x^{-3n-3}}{a^2n^2+3a^2n+2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] -(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 3)/(a^
2*n^2 + 3*a^2*n + 2*a^2)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + ax^2\right)^n x^{-3n-3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 3), x)
```

## 3.282 $\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx$

**Optimal**. Leaf size=116

$$-\frac{2b^2x^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a^3(n+1)(n+2)(n+3)}+\frac{2bx^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a^2(n+2)(n+3)}-\frac{x^{-3n-5}\left(ax^2+bx^3\right)^{n+1}}{a(n+3)}$$

```
[Out] -((x^(-5 - 3*n)*(a*x^2 + b*x^3)^(1 + n))/(a*(3 + n))) + (2*b*x^(-4 - 3*n)*(
a*x^2 + b*x^3)^(1 + n))/(a^2*(2 + n)*(3 + n)) - (2*b^2*(a*x^2 + b*x^3)^(1 +
 n))/(a^3*(1 + n)*(2 + n)*(3 + n)*x^(3*(1 + n)))
```

---

**Rubi [A]** time = 0.0914168, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$-\frac{2b^2x^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a^3(n+1)(n+2)(n+3)}+\frac{2bx^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a^2(n+2)(n+3)}-\frac{x^{-3n-5}\left(ax^2+bx^3\right)^{n+1}}{a(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^(-5 - 3*n)*(a*x^2 + b*x^3)^(1 + n))/(a*(3 + n))) + (2*b*x^(-4 - 3*n)*(
a*x^2 + b*x^3)^(1 + n))/(a^2*(2 + n)*(3 + n)) - (2*b^2*(a*x^2 + b*x^3)^(1 +
 n))/(a^3*(1 + n)*(2 + n)*(3 + n)*x^(3*(1 + n)))
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx&=-\frac{x^{-5-3n}\left(ax^2+bx^3\right)^{1+n}}{a(3+n)}-\frac{(2b)\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx}{a(3+n)}\\&=-\frac{x^{-5-3n}\left(ax^2+bx^3\right)^{1+n}}{a(3+n)}+\frac{2bx^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a^2(2+n)(3+n)}+\frac{\left(2b^2\right)\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx}{a^2(2+n)(3+n)}\\&=-\frac{x^{-5-3n}\left(ax^2+bx^3\right)^{1+n}}{a(3+n)}+\frac{2bx^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a^2(2+n)(3+n)}-\frac{2b^2x^{-3(1+n)}\left(ax^2+bx^3\right)^{1+n}}{a^3(1+n)(2+n)(3+n)}\end{aligned}$$

**Mathematica [A]** time = 0.0286695, size = 72, normalized size = 0.62

$$-\frac{x^{-3(n+1)}(a+bx)\left(x^2(a+bx)\right)^n\left(a^2\left(n^2+3n+2\right)-2ab(n+1)x+2b^2x^2\right)}{a^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -(((a + b*x)*(x^2*(a + b*x))^n*(a^2*(2 + 3*n + n^2) - 2*a*b*(1 + n)*x + 2*b
^2*x^2))/(a^3*(1 + n)*(2 + n)*(3 + n)*x^(3*(1 + n))))
```

---

**Maple [A]** time = 0.005, size = 84, normalized size = 0.7

$$-\frac{(bx+a)\,x^{-3-3\,n}\left(a^2n^2-2\,abnx+2\,b^2x^2+3\,a^2n-2\,abx+2\,a^2\right)\left(bx^3+ax^2\right)^n}{(3+n)\,(2+n)\,(1+n)\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-4-3*n)*(b*x^3+a*x^2)^n,x)
```

```
[Out] -(b*x+a)*x^(-3-3*n)*(a^2*n^2-2*a*b*n*x+2*b^2*x^2+3*a^2*n-2*a*b*x+2*a^2)*(b*
x^3+a*x^2)^n/(3+n)/(2+n)/(1+n)/a^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+ax^2\right)^n x^{-3\,n-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 4), x)
```

---

**Fricas [A]** time = 0.91543, size = 217, normalized size = 1.87

$$\frac{\left(2\,ab^2nx^3-2\,b^3x^4-\left(a^2bn^2+a^2bn\right)x^2-\left(a^3n^2+3\,a^3n+2\,a^3\right)x\right)\left(bx^3+ax^2\right)^n x^{-3\,n-4}}{a^3n^3+6\,a^3n^2+11\,a^3n+6\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] (2*a*b^2*n*x^3 - 2*b^3*x^4 - (a^2*b*n^2 + a^2*b*n)*x^2 - (a^3*n^2 + 3*a^3*n
 + 2*a^3)*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 4)/(a^3*n^3 + 6*a^3*n^2 + 11*a^3*n
 + 6*a^3)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-4-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + ax^2\right)^n x^{-3\,n-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 4), x)
```

## 3.283 $\int \frac{x^{11}}{\left(ax^2+bx^5\right)^3}\,dx$

**Optimal**. Leaf size=19

$$\frac{x^6}{6a\left(a+bx^3\right)^2}$$

```
[Out] x^6/(6*a*(a + b*x^3)^2)
```

---

**Rubi [A]** time = 0.0100698, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 264}

$$\frac{x^6}{6a\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a*x^2 + b*x^5)^3,x]
```

```
[Out] x^6/(6*a*(a + b*x^3)^2)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(ax^2+bx^5\right)^3}\,dx &= \int \frac{x^5}{\left(a+bx^3\right)^3}\,dx\\ &= \frac{x^6}{6a\left(a+bx^3\right)^2}\end{aligned}$$

**Mathematica [A]** time = 0.00836, size = 24, normalized size = 1.26

$$-\frac{a+2bx^3}{6b^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a*x^2 + b*x^5)^3,x]
```

```
[Out] -(a + 2*b*x^3)/(6*b^2*(a + b*x^3)^2)
```

---

**Maple [A]** time = 0.007, size = 31, normalized size = 1.6

$$\frac{a}{6\,b^{2}\left(b x^{3}+a\right)^{2}}-\frac{1}{3\,b^{2}\left(b x^{3}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^5+a*x^2)^3,x)
```

```
[Out] 1/6/b^2*a/(b*x^3+a)^2-1/3/b^2/(b*x^3+a)
```

---

**Maxima [B]** time = 1.17217, size = 49, normalized size = 2.58

$$-\frac{2\,b x^{3}+a}{6\left(b^{4} x^{6}+2\,a b^{3} x^{3}+a^{2} b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

---

**Fricas [B]** time = 0.71859, size = 73, normalized size = 3.84

$$-\frac{2\,b x^{3}+a}{6\left(b^{4} x^{6}+2\,a b^{3} x^{3}+a^{2} b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

---

**Sympy [B]** time = 1.50915, size = 36, normalized size = 1.89

$$-\frac{a+2bx^{3}}{6a^{2}b^{2}+12ab^{3}x^{3}+6b^{4}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**5+a*x**2)**3,x)
```

```
[Out] -(a + 2*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

**Giac [A]** time = 1.16428, size = 30, normalized size = 1.58

$$-\frac{2\,bx^3+a}{6\left(bx^3+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/6*(2*b*x^3 + a)/((b*x^3 + a)^2*b^2)
```

## 3.284 $\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=80

$$\frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x}-\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}$$

```
[Out] (16*a^2*Sqrt[a*x^2 + b*x^5])/(45*b^3*x) - (8*a*x^2*Sqrt[a*x^2 + b*x^5])/(45
*b^2) + (2*x^5*Sqrt[a*x^2 + b*x^5])/(15*b)
```

---

**Rubi [A]** time = 0.114813, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$\frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x}-\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (16*a^2*Sqrt[a*x^2 + b*x^5])/(45*b^3*x) - (8*a*x^2*Sqrt[a*x^2 + b*x^5])/(45
*b^2) + (2*x^5*Sqrt[a*x^2 + b*x^5])/(15*b)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx &= \frac{2x^5\sqrt{ax^2+bx^5}}{15b}-\frac{(4a)\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx}{5b}\\ &= -\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}+\frac{\left(8a^2\right)\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx}{15b^2}\\ &= \frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x}-\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}\end{aligned}$$

**Mathematica [A]** time = 0.0246324, size = 46, normalized size = 0.57

$$\frac{2\sqrt{x^2\left(a+bx^3\right)}\left(8a^2-4abx^3+3b^2x^6\right)}{45b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x^3)]*(8*a^2 - 4*a*b*x^3 + 3*b^2*x^6))/(45*b^3*x)
```

---

**Maple [A]** time = 0.006, size = 48, normalized size = 0.6

$$\frac{\left(2\,bx^3+2\,a\right)\left(3\,b^2x^6-4\,abx^3+8\,a^2\right)x}{45\,b^3}\frac{1}{\sqrt{bx^5+ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 2/45*(b*x^3+a)*(3*b^2*x^6-4*a*b*x^3+8*a^2)*x/b^3/(b*x^5+a*x^2)^(1/2)
```

---

**Maxima [A]** time = 1.0347, size = 62, normalized size = 0.78

$$\frac{2\left(3\,b^3x^9-ab^2x^6+4\,a^2bx^3+8\,a^3\right)}{45\,\sqrt{bx^3+a}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*(3*b^3*x^9 - a*b^2*x^6 + 4*a^2*b*x^3 + 8*a^3)/(sqrt(b*x^3 + a)*b^3)
```

---

**Fricas [A]** time = 0.65072, size = 89, normalized size = 1.11

$$\frac{2\left(3\,b^2x^6-4\,abx^3+8\,a^2\right)\sqrt{bx^5+ax^2}}{45\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*b^2*x^6 - 4*a*b*x^3 + 8*a^2)*sqrt(b*x^5 + a*x^2)/(b^3*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{x^9}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**9/sqrt(x**2*(a + b*x**3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^9/sqrt(b*x^5 + a*x^2), x)
```

## 3.285 $\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=52

$$\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}$$

```
[Out] (-4*a*Sqrt[a*x^2 + b*x^5])/(9*b^2*x) + (2*x^2*Sqrt[a*x^2 + b*x^5])/(9*b)
```

---

**Rubi [A]** time = 0.0649265, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-4*a*Sqrt[a*x^2 + b*x^5])/(9*b^2*x) + (2*x^2*Sqrt[a*x^2 + b*x^5])/(9*b)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx &= \frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{(2a)\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx}{3b}\\ &= -\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}+\frac{2x^2\sqrt{ax^2+bx^5}}{9b}\end{aligned}$$

**Mathematica [A]** time = 0.0171806, size = 34, normalized size = 0.65

$$\frac{2\left(bx^3-2a\right)\sqrt{x^2\left(a+bx^3\right)}}{9b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*(-2*a + b*x^3)*Sqrt[x^2*(a + b*x^3)])/(9*b^2*x)
```

---

**Maple [A]** time = 0.005, size = 37, normalized size = 0.7

$$-\frac{\left(2\,b{x}^{3}+2\,a\right)\left(-b{x}^{3}+2\,a\right)x}{9\,{b}^{2}}\frac{1}{\sqrt{b{x}^{5}+a{x}^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2/9*(b*x^3+a)*(-b*x^3+2*a)*x/b^2/(b*x^5+a*x^2)^(1/2)
```

---

**Maxima [A]** time = 1.15932, size = 46, normalized size = 0.88

$$\frac{2\left(b^{2}x^{6}-abx^{3}-2\,a^{2}\right)}{9\,\sqrt{bx^{3}+a}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b^2*x^6 - a*b*x^3 - 2*a^2)/(sqrt(b*x^3 + a)*b^2)
```

---

**Fricas [A]** time = 0.809841, size = 63, normalized size = 1.21

$$\frac{2\,\sqrt{bx^{5}+ax^{2}}\left(bx^{3}-2\,a\right)}{9\,b^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 - 2*a)/(b^2*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{6}}{\sqrt{x^{2}\left(a+bx^{3}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**6/sqrt(x**2*(a + b*x**3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/sqrt(b*x^5 + a*x^2), x)
```

## 3.286 $\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=25

$$\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

```
[Out] (2*Sqrt[a*x^2 + b*x^5])/(3*b*x)
```

---

**Rubi [A]** time = 0.016886, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^5])/(3*b*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx=\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

**Mathematica [A]** time = 0.0077292, size = 25, normalized size = 1.

$$\frac{2\sqrt{x^2\left(a+bx^3\right)}}{3bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x^3)])/(3*b*x)
```

---

**Maple [A]** time = 0.004, size = 27, normalized size = 1.1

$$\frac{\left(2\,bx^3+2\,a\right)x}{3\,b}\frac{1}{\sqrt{bx^5+ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 2/3*(b*x^3+a)*x/b/(b*x^5+a*x^2)^(1/2)
```

---

**Maxima [A]** time = 1.1064, size = 19, normalized size = 0.76

$$\frac{2\sqrt{bx^3+a}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(b*x^3 + a)/b
```

---

**Fricas [A]** time = 0.796274, size = 42, normalized size = 1.68

$$\frac{2\sqrt{bx^5+ax^2}}{3bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^5 + a*x^2)/(b*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(a + b*x**3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(b*x^5 + a*x^2), x)
```

# 3.287 $\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=32

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

```
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[a])
```

---

**Rubi [A]** time = 0.0109513, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2008, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[a])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx &= -\left(\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^5}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}\end{aligned}$$

**Mathematica [A]** time = 0.0103286, size = 54, normalized size = 1.69

$$-\frac{2x\sqrt{a+bx^3}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-2*x*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*Sqrt[x^2
*(a + b*x^3)])
```

---

**Maple [A]** time = 0.005, size = 43, normalized size = 1.3

$$-\frac{2x}{3}\sqrt{bx^3+a}\,\mathrm{Artanh}\left(\sqrt{bx^3+a}\,\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{bx^5+ax^2}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2/3/(b*x^5+a*x^2)^(1/2)*x*(b*x^3+a)^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))
/a^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^5 + a*x^2), x)
```

---

**Fricas [A]** time = 0.875145, size = 180, normalized size = 5.62

$$\left[\frac{\log\left(\frac{bx^4+2\,ax-2\,\sqrt{bx^5+ax^2}\sqrt{a}}{x^4}\right)}{3\,\sqrt{a}},\ \frac{2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^5+ax^2}\sqrt{-a}}{ax}\right)}{3\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*log((b*x^4 + 2*a*x - 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^4)/sqrt(a), 2/3*
sqrt(-a)*arctan(sqrt(b*x^5 + a*x^2)*sqrt(-a)/(a*x))/a]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**5), x)
```

---

**Giac [A]** time = 1.16395, size = 63, normalized size = 1.97

$$-\frac{2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{3\sqrt{-a}}+\frac{2 \arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*arctan(sqrt(b*x^3 + a)/
sqrt(-a))/(sqrt(-a)*sgn(x))
```

## 3.288 $\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=59

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}-\frac{\sqrt{ax^2+bx^5}}{3ax^4}$$

```
[Out] -Sqrt[a*x^2 + b*x^5]/(3*a*x^4) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^5]
])/(3*a^(3/2))
```

---

**Rubi [A]** time = 0.0545933, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}-\frac{\sqrt{ax^2+bx^5}}{3ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] -Sqrt[a*x^2 + b*x^5]/(3*a*x^4) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^5]
])/(3*a^(3/2))
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{ax^2+bx^5}}{3ax^4} - \frac{b\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx}{2a}$$
$$= -\frac{\sqrt{ax^2+bx^5}}{3ax^4} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^5}}\right)}{3a}$$
$$= -\frac{\sqrt{ax^2+bx^5}}{3ax^4} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}$$

**Mathematica [A]** time = 0.061394, size = 71, normalized size = 1.2

$$\frac{2b\sqrt{x^2\left(a+bx^3\right)}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)}{2\sqrt{\frac{bx^3}{a}+1}} - \frac{a}{2bx^3}\right)}{3a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (2*b*Sqrt[x^2*(a + b*x^3)]*(-a/(2*b*x^3) + ArcTanh[Sqrt[1 + (b*x^3)/a]]/(2*
Sqrt[1 + (b*x^3)/a])))/(3*a^2*x)
```

---

**Maple [A]** time = 0.006, size = 66, normalized size = 1.1

$$-\frac{1}{3x^2}\sqrt{bx^3+a}\left(-b\mathrm{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)ax^3+\sqrt{bx^3+a}a^{\frac{3}{2}}\right)\frac{1}{\sqrt{bx^5+ax^2}}a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -1/3/x^2*(b*x^3+a)^(1/2)*(-b*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a*x^3+(b*x^3+
a)^(1/2)*a^(3/2))/(b*x^5+a*x^2)^(1/2)/a^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5+ax^2}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^3), x)
```

---

**Fricas [A]** time = 0.809663, size = 294, normalized size = 4.98

$$\left[\frac{\sqrt{a}bx^4\log\left(\frac{bx^4+2\,ax+2\,\sqrt{bx^5+ax^2}\sqrt{a}}{x^4}\right)-2\,\sqrt{bx^5+ax^2}a}{6\,a^2x^4}, -\frac{\sqrt{-a}bx^4\arctan\left(\frac{\sqrt{bx^5+ax^2}\sqrt{-a}}{ax}\right)+\sqrt{bx^5+ax^2}a}{3\,a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*b*x^4*log((b*x^4 + 2*a*x + 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^4
) - 2*sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4), -1/3*(sqrt(-a)*b*x^4*arctan(sqrt(b*
x^5 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(a + b*x**3))), x)
```

---

**Giac [A]** time = 1.16582, size = 77, normalized size = 1.31

$$-\frac{b\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a\mathrm{sgn}\left(x\right)}-\frac{\sqrt{\frac{b}{x}+\frac{a}{x^4}}}{3\,ax\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(x)) - 1/3*sqrt(b/x
+ a/x^4)/(a*x*sgn(x))
```

## 3.289 $\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=238

$$\frac{2\sqrt{ax^2+bx^5}}{5b}-\frac{4\sqrt{2+\sqrt{3}}ax\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] (2*Sqrt[a*x^2 + b*x^5])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*x*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a*x^2 + b*x^5])
```

---

**Rubi [A]** time = 0.141568, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2024, 2032, 218}

$$\frac{2\sqrt{ax^2+bx^5}}{5b}-\frac{4\sqrt{2+\sqrt{3}}ax\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{bx}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^5])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*x*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a*x^2 + b*x^5])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx &= \frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{(2a)\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx}{5b}\\
&= \frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{\left(2ax\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{5b\sqrt{ax^2+bx^5}}\\
&= \frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{4\sqrt{2+\sqrt{3}}ax\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0243369, size = 68, normalized size = 0.29

$$\frac{2x^2\left(-a\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+a+bx^3\right)}{5b\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x^2*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3,
-((b*x^3)/a)]))/(5*b*Sqrt[x^2*(a + b*x^3)])
```

---

**Maple [A]** time = 0.297, size = 248, normalized size = 1.

$$\frac{2x}{15b^2}\left(ia\sqrt{3}\sqrt[3]{-b^2a}\sqrt{-i\sqrt{3}\left(i\sqrt{3}\sqrt[3]{-b^2a}-2bx-\sqrt[3]{-b^2a}\right)\frac{1}{\sqrt[3]{-b^2a}}}\sqrt{-2\frac{-bx+\sqrt[3]{-b^2a}}{\sqrt[3]{-b^2a}\left(i\sqrt{3}-3\right)}}\sqrt{-i\sqrt{3}\left(i\sqrt{3}\sqrt[3]{-b^2a}+2bx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 2/15*x*(I*a*3^(1/2)*(-b^2*a)^(1/3)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^
2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*(-2*(-b*x+(-b^2*a)^(1/3))/(-b^2*a
)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(
1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^
(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2),2^
(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))+3*x^4*b^2+3*a*b*x)/(b*x^5+a*x^2)^(1/
2)/b^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(b*x^5 + a*x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^5 + ax^2}x^2}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*x^2/(b*x^3 + a), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{x^2\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**2*(a + b*x**3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(b*x^5 + a*x^2), x)
```

## 3.290 $\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=212

$$\frac{2\sqrt{2+\sqrt{3}}x\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] (2*Sqrt[2 + Sqrt[3]]*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

---

**Rubi [A]** time = 0.0667158, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2032, 218}

$$\frac{2\sqrt{2+\sqrt{3}}x\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx}+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{bx}+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{bx}+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}}\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}}$$
$$= \frac{2\sqrt{2+\sqrt{3}}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

**Mathematica [C]** time = 0.0094612, size = 52, normalized size = 0.25

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/Sq
rt[x^2*(a + b*x^3)]
```

---

**Maple [A]** time = 0.005, size = 231, normalized size = 1.1

$$\frac{-\frac{i}{3}x\sqrt{3}}{b}\sqrt[3]{-b^2a}\sqrt{-i\sqrt{3}\left(i\sqrt{3}\sqrt[3]{-b^2a}-2\,bx-\sqrt[3]{-b^2a}\right)\frac{1}{\sqrt[3]{-b^2a}}}\sqrt{-2\,\frac{-bx+\sqrt[3]{-b^2a}}{\sqrt[3]{-b^2a}\left(i\sqrt{3}-3\right)}}\sqrt{-i\sqrt{3}\left(i\sqrt{3}\sqrt[3]{-b^2a}+2\,bx+\sqrt[3]{-b^2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -1/3*I/(b*x^5+a*x^2)^(1/2)*x*3^(1/2)/b*(-b^2*a)^(1/3)*(-I*(I*3^(1/2)*(-b^2*
a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*(-2*(-b*x+(-b^
2*a)^(1/3))/(-b^2*a)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/
3)+2*b*x+(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2
)*2^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2
*a)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*x^5 + a*x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^5+ax^2}}{bx^4+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)/(b*x^4 + a*x), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(a + b*x**3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*x^5 + a*x^2), x)
```

## 3.291 $\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=243

$$-\frac{\sqrt{2+\sqrt{3}}b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{2ax^3}$$

```
[Out] -Sqrt[a*x^2 + b*x^5]/(2*a*x^3) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*x*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
*x^2 + b*x^5])
```

---

**Rubi [A]** time = 0.128627, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2032, 218}

$$-\frac{\sqrt{2+\sqrt{3}}b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] -Sqrt[a*x^2 + b*x^5]/(2*a*x^3) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*x*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
*x^2 + b*x^5])
```

### Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx &= -\frac{\sqrt{ax^2+bx^5}}{2ax^3}-\frac{b\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx}{4a}\\
&= -\frac{\sqrt{ax^2+bx^5}}{2ax^3}-\frac{\left(bx\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{4a\sqrt{ax^2+bx^5}}\\
&= -\frac{\sqrt{ax^2+bx^5}}{2ax^3}-\frac{\sqrt{2+\sqrt{3}}b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7-\right.}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0130886, size = 55, normalized size = 0.23

$$
-\frac{\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a}\right)}{2x\sqrt{x^2\left(a+bx^3\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] -(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])/(2*x
*Sqrt[x^2*(a + b*x^3)])
```

---

**Maple [A]** time = 0.005, size = 248, normalized size = 1.

$$
\frac{1}{12\,ax}\left(i\sqrt{3}\sqrt[3]{-b^2a}\sqrt{-i\sqrt{3}\left(i\sqrt{3}\sqrt[3]{-b^2a}-2\,bx-\sqrt[3]{-b^2a}\right)\frac{1}{\sqrt[3]{-b^2a}}}\sqrt{-2\,\frac{-bx+\sqrt[3]{-b^2a}}{\sqrt[3]{-b^2a}\left(i\sqrt{3}-3\right)}}\sqrt{-i\sqrt{3}\left(i\sqrt{3}\sqrt[3]{-b^2a}+2\,bx+\right.}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 1/12*(I*3^(1/2)*(-b^2*a)^(1/3)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)
^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*(-2*(-b*x+(-b^2*a)^(1/3))/(-b^2*a)^(1
/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3)
)*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1/2
)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2),2^(1/2
)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*x^2-6*b*x^3-6*a)/x/(b*x^5+a*x^2)^(1/2)/a
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}}{bx^7 + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)/(b*x^7 + a*x^4), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{x^2\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x**3))), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^2), x)
```

## 3.292 $\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=514

$$-\frac{8\sqrt{2}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}x\left(\sqrt[3]{a}+\right.}{}$$

```
[Out] (-8*a*x*(a + b*x^3))/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*
x^2 + b*x^5]) + (2*x*Sqrt[a*x^2 + b*x^5])/(7*b) + (4*3^(1/4)*Sqrt[2 - Sqrt[
3]]*a^(4/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) - (8*Sqrt[2]*a^(4/3)*x*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*b^(5/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a*x^2 + b*x^5])
```

---

**Rubi [A]** time = 0.308019, antiderivative size = 514, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2024, 2032, 303, 218, 1877}

$$-\frac{8\sqrt{2}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{7b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-8*a*x*(a + b*x^3))/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*
x^2 + b*x^5]) + (2*x*Sqrt[a*x^2 + b*x^5])/(7*b) + (4*3^(1/4)*Sqrt[2 - Sqrt[
3]]*a^(4/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) - (8*Sqrt[2]*a^(4/3)*x*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*b^(5/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a*x^2 + b*x^5])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
```

```
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

### Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

### Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

### Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx &= \frac{2x\sqrt{ax^2+bx^5}}{7b} - \frac{(4a)\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx}{7b}\\
&= \frac{2x\sqrt{ax^2+bx^5}}{7b} - \frac{\left(4ax\sqrt{a+bx^3}\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{7b\sqrt{ax^2+bx^5}}\\
&= \frac{2x\sqrt{ax^2+bx^5}}{7b} - \frac{\left(4ax\sqrt{a+bx^3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}\sqrt{ax^2+bx^5}} - \frac{\left(4\sqrt{2\left(2-\sqrt{3}\right)}a^{4/3}x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}\sqrt{ax^2+bx^5}}\\
&= -\frac{8ax\left(a+bx^3\right)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} + \frac{2x\sqrt{ax^2+bx^5}}{7b} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-}{((1+}}}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}}{((1+}}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0233591, size = 68, normalized size = 0.13

$$\frac{2x^3\left(-a\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+a+bx^3\right)}{7b\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x^3*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3,
-((b*x^3)/a)]))/(7*b*Sqrt[x^2*(a + b*x^3)])
```

---

**Maple [A]** time = 0.008, size = 676, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2/21*x*(3*I*(-2*(-b*x+(-b^2*a)^(1/3))/(-b^2*a)^(1/3)/(I*3^(1/2)-3))^(1/2)*
(-I*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))
^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-
b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3
))^(1/2))*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2
*a)^(1/3))^(1/2)*(-b^2*a)^(2/3)*3^(1/2)*a-2*I*(-2*(-b*x+(-b^2*a)^(1/3))/(-b
^2*a)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*
a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(
I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2
),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*
b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*(-b^2*a)^(2/3)*3^(1/2)*a-
3*x^5*b^3+3*(-2*(-b*x+(-b^2*a)^(1/3))/(-b^2*a)^(1/3)/(I*3^(1/2)-3))^(1/2)*(
-I*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^
(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b
^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3)
)^(1/2))*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*
a)^(1/3))^(1/2)*(-b^2*a)^(2/3)*a-3*a*b^2*x^2)/(b*x^5+a*x^2)^(1/2)/b^3
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/sqrt(b*x^5 + a*x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}x^3}{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*x^3/(b*x^3 + a), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(x**2*(a + b*x**3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/sqrt(b*x^5 + a*x^2), x)
```

## 3.293 $\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=484

$$\frac{2\sqrt{2}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/}}$$

```
[Out] (2*x*(a + b*x^3))/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*x^2 +
 b*x^5]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (
2*Sqrt[2]*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

---

**Rubi [A]** time = 0.189697, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2032, 303, 218, 1877}

$$\frac{2\sqrt{2}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-}{((1+}}}{b^{2/3}\sqrt{\frac{\sqrt[3]{}}{((1+}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x*(a + b*x^3))/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*x^2 +
 b*x^5]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (
2*Sqrt[2]*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx &= \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}}\\
&= \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}\sqrt{ax^2+bx^5}} + \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\sqrt[3]{a}x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}\sqrt{ax^2+bx^5}}\\
&= \frac{2x\left(a+bx^3\right)}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\Bigg(\sin^{-1}\Bigg(}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0110842, size = 55, normalized size = 0.11

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(2
*Sqrt[x^2*(a + b*x^3)])
```

---

**Maple [A]** time = 0.005, size = 394, normalized size = 0.8

$$\frac{-\frac{i}{6}x\sqrt{3}}{b^2}\left(-b^2a\right)^{\frac{2}{3}}\sqrt{-i\sqrt{3}\left(i\sqrt{3}\sqrt[3]{-b^2a}-2\,bx-\sqrt[3]{-b^2a}\right)\frac{1}{\sqrt[3]{-b^2a}}}\sqrt{-2\,\frac{-bx+\sqrt[3]{-b^2a}}{\sqrt[3]{-b^2a}\left(i\sqrt{3}-3\right)}}\sqrt{-i\sqrt{3}\Bigl(i\sqrt{3}\sqrt[3]{-b^2a}+2\,bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -1/6*I*x*3^(1/2)*(-b^2*a)^(2/3)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a
)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*(-2*(-b*x+(-b^2*a)^(1/3))/(-b^2*a)^(
1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3
))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*(I*EllipticE(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^
(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2),2^
(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*3^(1/2)-3*EllipticE(1/6*3^(1/2)*2^(1
/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1
/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))+2*EllipticF(1/6*3^(1/2)
*2^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*
a)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2)))/(b*x^5+a*x^2)^(1/
2)/b^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*x^5 + a*x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^5+ax^2}}{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)/(b*x^3 + a), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(a + b*x**3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*x^5 + a*x^2), x)
```

## 3.294 $\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=510

$$\frac{\sqrt{2}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}}$$

```
[Out] (b^(1/3)*x*(a + b*x^3))/(a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*x^2 +
 b*x^5]) - Sqrt[a*x^2 + b*x^5]/(a*x^2) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)
*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a
^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (Sqrt[2]*b^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b
*x^5])
```

---

**Rubi [A]** time = 0.273408, antiderivative size = 510, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2025, 2032, 303, 218, 1877}

$$\frac{\sqrt{2}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (b^(1/3)*x*(a + b*x^3))/(a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*x^2 +
 b*x^5]) - Sqrt[a*x^2 + b*x^5]/(a*x^2) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)
*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a
^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (Sqrt[2]*b^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b
*x^5])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx &= -\frac{\sqrt{ax^2+bx^5}}{ax^2}+\frac{b\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx}{2a}\\
&= -\frac{\sqrt{ax^2+bx^5}}{ax^2}+\frac{\left(bx\sqrt{a+bx^3}\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{2a\sqrt{ax^2+bx^5}}\\
&= -\frac{\sqrt{ax^2+bx^5}}{ax^2}+\frac{\left(b^{2/3}x\sqrt{a+bx^3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{2a\sqrt{ax^2+bx^5}}+\frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}b^{2/3}x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{a^{2/3}\sqrt{ax^2+bx^5}}\\
&= \frac{\sqrt[3]{b}x\left(a+bx^3\right)}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{ax^2}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}}}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}\right.}}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0107877, size = 50, normalized size = 0.1

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] -((Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/Sqr
t[x^2*(a + b*x^3)])
```

---

**Maple [A]** time = 0.009, size = 673, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 1/12*(3*I*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2
*a)^(1/3))^(1/2)*(-2*(-b*x+(-b^2*a)^(1/3))/(-b^2*a)^(1/3)/(I*3^(1/2)-3))^(1
/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1
/3))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*
x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/
2)-3))^(1/2))*3^(1/2)*(-b^2*a)^(2/3)*x-2*I*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*
b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*(-2*(-b*x+(-b^2*a)^(1/3))
/(-b^2*a)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-
b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(
-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^
(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*3^(1/2)*(-b^2*a)^(2/3)*x+3*(
-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^
(1/2)*(-2*(-b*x+(-b^2*a)^(1/3))/(-b^2*a)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*
3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2)*
EllipticE(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^
(1/3))*3^(1/2)/(-b^2*a)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2
))*(-b^2*a)^(2/3)*x-12*b^2*x^3-12*a*b)/(b*x^5+a*x^2)^(1/2)/a/b
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5+ax^2}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{bx^5+ax^2}}{bx^6+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)/(b*x^6 + a*x^3), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(a + b*x**3))), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5+ax^2}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x), x)
```

## 3.295 $\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=265

$$\frac{7a^{5/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}}+\frac{x^{5/2}\sqrt{ax^2}}{5b}$$

```
[Out] (-7*a*Sqrt[a*x^2 + b*x^5])/(20*b^2*Sqrt[x]) + (x^(5/2)*Sqrt[a*x^2 + b*x^5])
/(5*b) + (7*a^(5/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(40*3^(1/4)*b^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x)
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

---

**Rubi [A]** time = 0.281867, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2024, 2032, 329, 225}

$$\frac{7a^{5/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}}+\frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-7*a*Sqrt[a*x^2 + b*x^5])/(20*b^2*Sqrt[x]) + (x^(5/2)*Sqrt[a*x^2 + b*x^5])
/(5*b) + (7*a^(5/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(40*3^(1/4)*b^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x)
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

### Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx &= \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} - \frac{(7a)\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx}{10b}\\
&= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{\left(7a^2\right)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{40b^2}\\
&= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{\left(7a^2x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{40b^2\sqrt{ax^2+bx^5}}\\
&= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{\left(7a^2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{20b^2\sqrt{ax^2+bx^5}}\\
&= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{7a^{5/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}}\right.\right.}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0323308, size = 86, normalized size = 0.32

$$\frac{x^{3/2}\left(7a^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)-7a^2-3abx^3+4b^2x^6\right)}{20b^2\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(3/2)*(-7*a^2 - 3*a*b*x^3 + 4*b^2*x^6 + 7*a^2*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(20*b^2*Sqrt[x^2*(a + b*x^3)])
```

---

**Maple [C]** time = 0.533, size = 2017, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -1/20/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^3/(-b^2*a)^(1/3)*(14*I*(-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2
*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*
((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2
*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*
a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*3^(1/2)*x^2*a^2*b^2-28*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(
-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I
*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b
^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(
1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+
I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*3^(1/2)*x*a^2
*b+14*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I
*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^
(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2
))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)
)/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(2/3)*3^(1/2)*a^2-14*(-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(
-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2
*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)
*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b
^2-4*I*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2
*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*((b*x^3+a
)*x)^(1/2)*(-b^2*a)^(1/3)*3^(1/2)*x^3*b^2+28*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1
/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)
^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/
3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*Ellipt
icF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(
1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*x
*a^2*b-14*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*(
(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a
)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1
/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/
2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2)
)/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(2/3)*a^2+12*x^3*((b*x^3+a)*x)^(1/2)*b^2*(
-b^2*a)^(1/3)*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*
x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)+7*
I*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(
1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*((b*x^3+a)*x)^
(1/2)*(-b^2*a)^(1/3)*3^(1/2)*a*b-21*a*((b*x^3+a)*x)^(1/2)*b*(-b^2*a)^(1/3)*
(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/
3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2))/((b*x^3+a)*x)^(
1/2)/(I*3^(1/2)-3)/(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)
+2*b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/
2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(13/2)/sqrt(b*x^5 + a*x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}x^{\frac{9}{2}}}{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*x^(9/2)/(b*x^3 + a), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(13/2)/sqrt(b*x^5 + a*x^2), x)
```

## 3.296 $\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=525

$$\frac{5\left(1-\sqrt{3}\right)a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{5\sqrt[4]{3}a^{4/3}x^{3/2}\left(\sqrt[3]{a}\right.}{}$$

```
[Out] (-5*(1 + Sqrt[3])*a*x^(3/2)*(a + b*x^3))/(8*b^(5/3)*(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) + (x^(3/2)*Sqrt[a*x^2 + b*x^5])/(4*b) + (
5*3^(1/4)*a^(4/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcC
os[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)]
, (2 + Sqrt[3])/4])/(8*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (5*(1 - Sqrt[3])*a
^(4/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3)
 + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt
[3])/4])/(16*3^(1/4)*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

---

**Rubi [A]** time = 0.500006, antiderivative size = 525, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2024, 2032, 329, 308, 225, 1881}

$$\frac{5\left(1-\sqrt{3}\right)a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{5\sqrt[4]{3}a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{8b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-5*(1 + Sqrt[3])*a*x^(3/2)*(a + b*x^3))/(8*b^(5/3)*(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) + (x^(3/2)*Sqrt[a*x^2 + b*x^5])/(4*b) + (
5*3^(1/4)*a^(4/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcC
os[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)]
, (2 + Sqrt[3])/4])/(8*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (5*(1 - Sqrt[3])*a
^(4/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3)
 + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt
[3])/4])/(16*3^(1/4)*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
```

```
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

### Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

### Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

### Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx &= \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-\frac{(5a)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx}{8b}\\
&= \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-\frac{\left(5ax\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{8b\sqrt{ax^2+bx^5}}\\
&= \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}-\frac{\left(5ax\sqrt{a+bx^3}\right)\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{4b\sqrt{ax^2+bx^5}}\\
&= \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}+\frac{\left(5ax\sqrt{a+bx^3}\right)\mathrm{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{8b^{5/3}\sqrt{ax^2+bx^5}}+\frac{\left(5\left(1-\sqrt{3}\right)a^{5/3}x\sqrt{a+\right.}{8b}\\
&= -\frac{5\left(1+\sqrt{3}\right)ax^{3/2}\left(a+bx^3\right)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}+\frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b}+\frac{5\sqrt[4]{3}a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}}{\left(\sqrt[3]{a}+(1\right.}}}{8b^{5/3}\sqrt{\frac{\sqrt[3]{}}{\left(\sqrt[3]{a}\right.}}}
\end{aligned}$$

**Mathematica [C]** time = 0.0257397, size = 70, normalized size = 0.13

$$\frac{x^{7/2}\left(-a\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)+a+bx^3\right)}{4b\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(7/2)*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/
6, -((b*x^3)/a)]))/(4*b*Sqrt[x^2*(a + b*x^3)])
```

---

**Maple [C]** time = 0.052, size = 2586, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 1/4*x^(3/2)*(b*x^3+a)*(-5*I*(-b^2*a)^(2/3)*3^(1/2)*x*a-5*I*3^(1/2)*x^3*a*b^
2-10*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^
(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/
3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/
(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*
3^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*x^2*a*b+15*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1
/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)
^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/
3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*Ellipt
icE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(
1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*x
^2*a*b+5*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*
```

```
((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*
a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(
1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1
/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2
))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*a^2*b-5*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2
))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(
1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)
-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*Elliptic
E((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/
2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*3^(
1/2)*x^2*a*b+20*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(
1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+
I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I
*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(2/3)*x*a-30*(-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(
-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2
*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)
*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^
(2/3)*x*a+10*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1
/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-
b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I
*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*
3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(2/3)*3^(1/2)*x*a+10*(-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+
2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2
)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3))
)^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))
^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^
2*b-15*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*
3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(
1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2)
)/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))
/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(
I*3^(1/2)-3))^(1/2))*a^2*b+I*((b*x^3+a)*x)^(1/2)*(1/b^2*x*(-b*x+(-b^2*a)^(1
/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1
/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*3^(1/2)*x^2*b^2-3*((b*x^3+a)*x)^(1/2)*(1/b
^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*
(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*x^2*b^2-5*I*(-b^2*a)
^(1/3)*3^(1/2)*x^2*a*b+15*a*x^3*b^2+15*(-b^2*a)^(1/3)*x^2*a*b+15*(-b^2*a)^(
2/3)*x*a)/(b*x^5+a*x^2)^(1/2)/b^3/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/(1/b^2*
x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I*
3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{11}{2}}}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(11/2)/sqrt(b*x^5 + a*x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}x^{\frac{7}{2}}}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*x^(7/2)/(b*x^3 + a), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{11}{2}}}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(11/2)/sqrt(b*x^5 + a*x^2), x)
```

## 3.297 $\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=65

$$\frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

```
[Out] (Sqrt[x]*Sqrt[a*x^2 + b*x^5])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x
^2 + b*x^5]])/(3*b^(3/2))
```

---

**Rubi [A]** time = 0.0896446, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2024, 2029, 206}

$$\frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (Sqrt[x]*Sqrt[a*x^2 + b*x^5])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x
^2 + b*x^5]])/(3*b^(3/2))
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx &= \frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b} - \frac{a\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx}{2b} \\
&= \frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b} \\
&= \frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.0373513, size = 81, normalized size = 1.25

$$
\frac{\sqrt{b}x^{5/2}\left(a+bx^3\right) - ax\sqrt{a+bx^3}\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)}{3b^{3/2}\sqrt{x^2\left(a+bx^3\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (Sqrt[b]*x^(5/2)*(a + b*x^3) - a*x*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2)
)/Sqrt[a + b*x^3]])/(3*b^(3/2)*Sqrt[x^2*(a + b*x^3)])
```

---

**Maple [C]** time = 0.051, size = 3347, normalized size = 51.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 1/3/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^3*(6*I*(-(I*3^(1/2)-3)*x*b/(-1+
I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-
b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*
a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*
EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),(
(I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^
2*a*b^2-6*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)
*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2
*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^
(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^
(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2
)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a*b^2-1
2*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(
1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3
)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(
-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-
b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3
^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*3^(1/2)*x*a*b+12*I*(-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(
-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2
*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)
*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)
,(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(
```

```
I*3^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*3^(1/2)*x*a*b+6*I*(-(I*3^(1/2)-3)*x*b/(
-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x
+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b
^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/
2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2
),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a
)^(2/3)*3^(1/2)*a-6*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)
))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b
*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/
(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1
+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a*b^2-6*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3
^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2
*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^
(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*Ell
ipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),(-1
+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^
(1/2)-3))^(1/2))*(-b^2*a)^(2/3)*3^(1/2)*a+6*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/
2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^
(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3
)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*Ellipti
cPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),(-1+I*3
^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*x^2*a*b^2+12*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)
^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2
))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(
1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)
*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/
2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*x*a*b-12*(-(I*3^(1/2
)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(
1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3
^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(
1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(
1/3)))^(1/2),(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+
I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*x*a*b+I*((b*x^3+a)*x)^(1/2)
*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1
/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*3^(1/2)*x*b^2-6
*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2
)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))
^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*
x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x
+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1
/2)-3))^(1/2))*(-b^2*a)^(2/3)*a+6*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+
I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-
b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticPi((-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),(-1+I*3^(1/2))/(I
*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2
))*(-b^2*a)^(2/3)*a-3*x*((b*x^3+a)*x)^(1/2)*b^2*(1/b^2*x*(-b*x+(-b^2*a)^(1/
3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/
3)-2*b*x-(-b^2*a)^(1/3)))^(1/2))/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/(1/b^2*x
*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I*3
^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{9}{2}}}{\sqrt{bx^5 + ax^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(9/2)/sqrt(b*x^5 + a*x^2), x)
```

---

**Fricas [A]** time = 1.13319, size = 358, normalized size = 5.51

$$\left[\frac{a\sqrt{b}\log\left(-8\,b^2x^6 - 8\,abx^3 + 4\,\sqrt{bx^5 + ax^2}\left(2\,bx^3 + a\right)\sqrt{b}\sqrt{x} - a^2\right) + 4\,\sqrt{bx^5 + ax^2}b\sqrt{x}}{12\,b^2}, \frac{a\sqrt{-b}\arctan\left(\frac{2\,\sqrt{bx^5+ax^2}\sqrt{-b}\sqrt{x}}{2\,bx^3+a}\right.}{6\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 + 4*sqrt(b*x^5 + a*x^2)*(2*b*x^
3 + a)*sqrt(b)*sqrt(x) - a^2) + 4*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2, 1/6*(
a*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqrt(-b)*sqrt(x)/(2*b*x^3 + a)) + 2
*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.39752, size = 59, normalized size = 0.91

$$\frac{\sqrt{bx^3 + ax^2}\,x^{\frac{3}{2}}}{3\,b} + \frac{a\log\left(\left|-\sqrt{b}x^{\frac{3}{2}} + \sqrt{bx^3 + a}\right|\right)}{3\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*x^3 + a)*x^(3/2)/b + 1/3*a*log(abs(-sqrt(b)*x^(3/2) + sqrt(b*x^3
 + a)))/b^(3/2)
```

## 3.298 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=237

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] Sqrt[a*x^2 + b*x^5]/(2*b*Sqrt[x]) - (a^(2/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
+ (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x
*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 +
 b*x^5])
```

---

**Rubi [A]** time = 0.225232, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2024, 2032, 329, 225}

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] Sqrt[a*x^2 + b*x^5]/(2*b*Sqrt[x]) - (a^(2/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
+ (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x
*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 +
 b*x^5])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx &= \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{a\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{4b}\\
&= \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{\left(ax\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{4b\sqrt{ax^2+bx^5}}\\
&= \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{\left(ax\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{2b\sqrt{ax^2+bx^5}}\\
&= \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0249058, size = 70, normalized size = 0.3

$$\frac{x^{3/2}\left(-a\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)+a+bx^3\right)}{2b\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(3/2)*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6
, -((b*x^3)/a)]))/(2*b*Sqrt[x^2*(a + b*x^3)])
```

---

**Maple [C]** time = 0.032, size = 1793, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 1/2/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^2/(-b^2*a)^(1/3)*(2*I*(-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)
^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I
*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*3^(1/2)*x^2*a*b^2-4*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a
)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/
2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^
(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1
/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*3^(1/2)*x*a*b+2*I*(
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*
(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(
1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+
(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(
-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*(-b^2*a)^(2/3)*3^(1/2)*a-2*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/
(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3
))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*
b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+
3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a*b^2+4*(-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)
^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I
*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*(-b^2*a)^(1/3)*x*a*b-2*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*
a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1
/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)
^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(
1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(2/3)*a+I*((b*x^3+a)*x)^
(1/2)*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*
a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*(-b^2*a)^(
1/3)*3^(1/2)*b-3*((b*x^3+a)*x)^(1/2)*b*(-b^2*a)^(1/3)*(1/b^2*x*(-b*x+(-b^2*
a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*
a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2))/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/(1
/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3)
)*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/sqrt(b*x^5 + a*x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}x^{\frac{3}{2}}}{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*x^(3/2)/(b*x^3 + a), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(7/2)/sqrt(x**2*(a + b*x**3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(7/2)/sqrt(b*x^5 + a*x^2), x)
```

## 3.299 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=492

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt[3]{a}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{}$$

```
[Out] ((1 + Sqrt[3])*x^(3/2)*(a + b*x^3))/(b^(2/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/
3)*x)*Sqrt[a*x^2 + b*x^5]) - (3^(1/4)*a^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
 + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(b^(2/3)*Sqrt[(b^(1/3)*x*(a
^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*
x^5]) - ((1 - Sqrt[3])*a^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*E
llipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3
) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

---

**Rubi [A]** time = 0.398378, antiderivative size = 492, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2032, 329, 308, 225, 1881}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt[3]{a}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-}{\left(\sqrt[3]{a}+\right.}}}{b^{2/3}\sqrt{\frac{}{(}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] ((1 + Sqrt[3])*x^(3/2)*(a + b*x^3))/(b^(2/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/
3)*x)*Sqrt[a*x^2 + b*x^5]) - (3^(1/4)*a^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
 + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(b^(2/3)*Sqrt[(b^(1/3)*x*(a
^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*
x^5]) - ((1 - Sqrt[3])*a^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*E
llipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3
) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
```

```
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx &= \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}}\\
&= \frac{\left(2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax^2+bx^5}}\\
&= -\frac{\left(x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{b^{2/3}\sqrt{ax^2+bx^5}} - \frac{\left(\left(1-\sqrt{3}\right)a^{2/3}x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}}\right.}{b^{2/3}\sqrt{ax^2+bx^5}}\\
&= \frac{\left(1+\sqrt{3}\right)x^{3/2}\left(a+bx^3\right)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} - \frac{\sqrt[4]{3}\sqrt[3]{a}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+}{\sqrt[3]{a}+}\right.\right.}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx}}
\end{aligned}$$

**Mathematica [C]** time = 0.0131643, size = 57, normalized size = 0.12

$$\frac{2x^{7/2}\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)}{5\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/
a)])/(5*Sqrt[x^2*(a + b*x^3)])
```

---

**Maple [C]** time = 0.034, size = 2374, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2*x^(3/2)*(b*x^3+a)*(-I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^
(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2)
)/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1
/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2
))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*3^(1/2)*x^2*b+2*I*(-(
I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-
b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/
2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-
b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b
^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-
3))^(1/2))*(-b^2*a)^(2/3)*3^(1/2)*x-2*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-
b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))
/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*
x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(
I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)
*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*x^2*b+3*
(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)
*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^
(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x
+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/
2)-3))^(1/2))*(-b^2*a)^(1/3)*x^2*b+I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b
*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/
(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x
-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I
*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*
(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*a*b+4*(-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^
(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*
3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/
2))*(-b^2*a)^(2/3)*x-6*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1
/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/
```

```
(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3
))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*
b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(2/3)*x-I*3^(1/2)*x^3*b^2+2*(
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*
(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(
1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+
(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(
-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*a*b-3*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3))
)^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*
x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(
-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-
1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+
I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b-I*(-b^2*a)^(1/3)*3^(1/2)*x^2*b-I*(-b^2
*a)^(2/3)*3^(1/2)*x+3*b^2*x^3+3*(-b^2*a)^(1/3)*x^2*b+3*(-b^2*a)^(2/3)*x)/(b
*x^5+a*x^2)^(1/2)/b^2/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/(1/b^2*x*(-b*x+(-b^
2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^
2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{bx^5 + ax^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/sqrt(b*x^5 + a*x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}\sqrt{x}}{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^3 + a), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{x^2\left(a + bx^3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)/sqrt(x**2*(a + b*x**3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/sqrt(b*x^5 + a*x^2), x)
```

## 3.300 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=36

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[b])
```

---

**Rubi [A]** time = 0.0475052, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2029, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[b])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx &= \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}\end{aligned}$$

**Mathematica [A]** time = 0.0120326, size = 59, normalized size = 1.64

$$\frac{2x\sqrt{a+bx^3}\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)}{3\sqrt{b}\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/(3*Sqrt[b]
*Sqrt[x^2*(a + b*x^3)])
```

---

**Maple [C]** time = 0.118, size = 480, normalized size = 13.3

$$-4\,\frac{x^{3/2}\left(bx^{3}+a\right)\left(-1+i\sqrt{3}\right)\left(-bx+\sqrt[3]{-b^{2}a}\right)^{2}}{\sqrt{bx^{5}+ax^{2}}\,b^{2}\sqrt{\left(bx^{3}+a\right)x}\,\left(i\sqrt{3}-3\right)}\sqrt{-\frac{\left(i\sqrt{3}-3\right)xb}{\left(-1+i\sqrt{3}\right)\left(-bx+\sqrt[3]{-b^{2}a}\right)}}\sqrt{\frac{i\sqrt{3}\sqrt[3]{-b^{2}a}+2\,bx+\sqrt[3]{-b^{2}a}}{\left(1+i\sqrt{3}\right)\left(-bx+\sqrt[3]{-b^{2}a}\right)}}\sqrt{\frac{i\sqrt{3}\sqrt[3]{-b^{2}}}{(-1+i\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -4*x^(3/2)*(b*x^3+a)*(-1+I*3^(1/2))*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*
x+(-b^2*a)^(1/3)))^(1/2)*(-b*x+(-b^2*a)^(1/3))^2*((I*3^(1/2)*(-b^2*a)^(1/3)
+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/
2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)
))^(1/2)*(EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)
))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))-
EllipticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),
(-1+I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2)))/(b*x^5+a*x^2)^(1/2)/b^2/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)
-3)/(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)
^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{bx^{5}+ax^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(b*x^5 + a*x^2), x)
```

---

**Fricas [A]** time = 1.33068, size = 247, normalized size = 6.86

$$\left[\frac{\log\left(-8\,b^{2}x^{6}-8\,abx^{3}-4\,\sqrt{bx^{5}+ax^{2}}\left(2\,bx^{3}+a\right)\sqrt{b}\sqrt{x}-a^{2}\right)}{6\,\sqrt{b}},\,-\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{bx^{5}+ax^{2}}\sqrt{-b}\sqrt{x}}{2\,bx^{3}+a}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*sqrt(b*x^5 + a*x^2)*(2*b*x^3 + a)*sqrt(
b)*sqrt(x) - a^2)/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqrt(
-b)*sqrt(x)/(2*b*x^3 + a))/b]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x^2 \left(a + bx^3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x**2*(a + b*x**3)), x)
```

---

**Giac [A]** time = 1.28855, size = 55, normalized size = 1.53

$$-\frac{2 \arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3 \sqrt{-b}} + \frac{2 \arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)}{3 \sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b) + 2/3*arctan(sqrt(b)/sqrt(-b
))/sqrt(-b)
```

## 3.301 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=203

$$\frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] (x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1
- Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4
])/(3^(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

---

**Rubi [A]** time = 0.165711, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 329, 225}

$$\frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1
- Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4
])/(3^(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx &= \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}}\\
&= \frac{\left(2x\sqrt{a+bx^3}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax^2+bx^5}}\\
&= \frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

**Mathematica [C]**  time = 0.0135442, size = 55, normalized size = 0.27

$$
\frac{2x^{3/2}\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x^(3/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a
)])/Sqrt[x^2*(a + b*x^3)]
```

---

**Maple [C]**  time = 0.126, size = 437, normalized size = 2.2

$$
-4\,\frac{x^{3/2}\left(bx^3+a\right)\left(i\sqrt{3}x^2b^2-2\,i\sqrt[3]{-b^2a}\sqrt{3}xb+i\left(-b^2a\right)^{2/3}\sqrt{3}-b^2x^2+2\,\sqrt[3]{-b^2a}xb-\left(-b^2a\right)^{2/3}\right)}{\sqrt{bx^5+ax^2}\sqrt[3]{-b^2a}b\sqrt{\left(bx^3+a\right)x}\left(i\sqrt{3}-3\right)}\sqrt{-\frac{\left(i\sqrt{3}-3\right)x}{\left(-1+i\sqrt{3}\right)\left(-bx+\right.}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -4/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/(-b^2*a)^(1/3)/b*(-(I*3^(1/2)-3)*x
*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2
*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)
*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))
^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^
(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(I*
3^(1/2)*x^2*b^2-2*I*(-b^2*a)^(1/3)*3^(1/2)*x*b+I*(-b^2*a)^(2/3)*3^(1/2)-b^2
*x^2+2*(-b^2*a)^(1/3)*x*b-(-b^2*a)^(2/3))/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)
/(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1
```

```
/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(b*x^5 + a*x^2), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)/sqrt(b*x^5 + a*x^2), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(x**2*(a + b*x**3)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/sqrt(b*x^5 + a*x^2), x)
```

## 3.302 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=519

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{2\sqrt[4]{3}\sqrt[3]{b}x^{3/2}\left(\sqrt[3]{a}+\right.}{}$$

```
[Out] (2*(1 + Sqrt[3])*b^(1/3)*x^(3/2)*(a + b*x^3))/(a*(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (2*Sqrt[a*x^2 + b*x^5])/(a*x^(3/2)) - (2*3
^(1/4)*b^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[
(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (
2 + Sqrt[3])/4])/(a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) - ((1 - Sqrt[3])*b^(1/3)*
x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 -
 Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4]
)/(3^(1/4)*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

---

**Rubi [A]** time = 0.486824, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2025, 2032, 329, 308, 225, 1881}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{2\sqrt[4]{3}\sqrt[3]{b}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (2*(1 + Sqrt[3])*b^(1/3)*x^(3/2)*(a + b*x^3))/(a*(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (2*Sqrt[a*x^2 + b*x^5])/(a*x^(3/2)) - (2*3
^(1/4)*b^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[
(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (
2 + Sqrt[3])/4])/(a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) - ((1 - Sqrt[3])*b^(1/3)*
x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 -
 Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4]
)/(3^(1/4)*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx &= -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}+\frac{(2b)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx}{a}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}+\frac{\left(2bx\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{a\sqrt{ax^2+bx^5}}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}+\frac{\left(4bx\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax^2+bx^5}}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}-\frac{\left(2\sqrt[3]{b}x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax^2+bx^5}}-\frac{\left(2\left(1-\sqrt{3}\right)\sqrt[3]{b}x\sqrt{}\right.}{}\\
&= \frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}x^{3/2}\left(a+bx^3\right)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}}-\frac{2\sqrt[4]{3}\sqrt[3]{b}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}}}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\right.}{\left(\sqrt[3]{a}+\left(1+\sqrt{}\right.\right.}}}\end{aligned}$$

**Mathematica [C]** time = 0.0152446, size = 55, normalized size = 0.11

$$-\frac{2\sqrt{x}\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/6, 1/2, 5/6, -((b*x^3)
/a)])/Sqrt[x^2*(a + b*x^3)]
```

---

**Maple [C]** time = 0.041, size = 2860, normalized size = 5.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2*x^(1/2)*(-2*I*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(2/3)*3^(1/2)*x+4*I*(-(I*3^(1
/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)
^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I
*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)
^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(2/3)*3^(1/2)*x-4*(-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(
-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2
*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)
*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a
)*x)^(1/2)*(-b^2*a)^(1/3)*x^2*b+6*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
```

```
(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+
I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-
b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^
(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1
+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*(-b^2*a
)^(1/3)*x^2*b-2*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))
^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x
+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-
1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1
+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(1/3)*3^(1/2)*
x^2*b+8*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I
*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^
(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2
))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2)
)/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(2/3)*x-12*(-(I*3^(1/2)-
3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/
3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(
1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/
3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3
)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(2/3)*x+I*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3
^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x
-(-b^2*a)^(1/3)))^(1/2)*3^(1/2)*a*b+I*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(
1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(
-b^2*a)^(1/3)))^(1/2)*3^(1/2)*x^3*b^2+4*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/
(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3
))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*
b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+
3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*a
*b-6*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^
(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/
3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/
(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(
-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*
3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*a*b-2*I*((b*x^3+a)*x)^(1/2)*3^(1/2)*
x^3*b^2-3*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-
b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*x^3*b^
2+2*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3
^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1
/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))
/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/
(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*3^(1/2)*a*b+6*((b*x^3+a)*x)^(1/2)*x
^3*b^2-2*I*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(1/3)*3^(1/2)*x^2*b+6*((b*x^3+a)*x)
^(1/2)*(-b^2*a)^(1/3)*x^2*b+6*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(2/3)*x-3*(1/b^2
*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I
*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*a*b)/(b*x^5+a*x^2)^(1/
2)/b/a/(I*3^(1/2)-3)/(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/
3)+2*b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(
1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*sqrt(x)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}\sqrt{x}}{bx^6 + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^6 + a*x^3), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\sqrt{x^2\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(x**2*(a + b*x**3))), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*sqrt(x)), x)
```

## 3.303 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=27

$$-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(3*a*x^(5/2))
```

---

**Rubi [A]** time = 0.0400189, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2014}

$$-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]

[Out] (-2*Sqrt[a*x^2 + b*x^5])/(3*a*x^(5/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

**Mathematica [A]** time = 0.0108333, size = 27, normalized size = 1.

$$-\frac{2\sqrt{x^2\left(a+bx^3\right)}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]

[Out] (-2*Sqrt[x^2*(a + b*x^3)])/(3*a*x^(5/2))
```

---

**Maple [A]** time = 0.003, size = 29, normalized size = 1.1

$$-\frac{2\,bx^3+2\,a}{3\,a}\frac{1}{\sqrt{x}}\frac{1}{\sqrt{bx^5+ax^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2/3*(b*x^3+a)/x^(1/2)/a/(b*x^5+a*x^2)^(1/2)
```

---

**Maxima [A]** time = 1.11984, size = 35, normalized size = 1.3

$$-\frac{2\left(bx^4+ax\right)}{3\sqrt{bx^3+a}ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))
```

---

**Fricas [A]** time = 0.920576, size = 51, normalized size = 1.89

$$-\frac{2\sqrt{bx^5+ax^2}}{3ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x^5 + a*x^2)/(a*x^(5/2))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x**3))), x)
```

---

**Giac [A]** time = 1.4494, size = 31, normalized size = 1.15

$$-\frac{2\sqrt{b+\frac{a}{x^3}}}{3a}+\frac{2\sqrt{b}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b + a/x^3)/a + 2/3*sqrt(b)/a
```

## 3.304 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=235

$$-\frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}$$

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(5*a*x^(7/2)) - (2*b*x^(3/2)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
 + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt[(b^
(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[
a*x^2 + b*x^5])
```

---

**Rubi [A]** time = 0.221022, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2025, 2032, 329, 225}

$$-\frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(5*a*x^(7/2)) - (2*b*x^(3/2)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
 + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt[(b^
(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[
a*x^2 + b*x^5])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx &= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}-\frac{(2b)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{5a}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}-\frac{\left(2bx\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{5a\sqrt{ax^2+bx^5}}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}-\frac{\left(4bx\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{5a\sqrt{ax^2+bx^5}}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}-\frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0134694, size = 57, normalized size = 0.24

$$
-\frac{2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};-\frac{bx^3}{a}\right)}{5x^{3/2}\sqrt{x^2\left(a+bx^3\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((b*x^3)/a)])/(5
*x^(3/2)*Sqrt[x^2*(a + b*x^3)])
```

---

**Maple [C]** time = 0.037, size = 1795, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2/5*(b*x^3+a)*(-4*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3
)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-
b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))
/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^5*b^2+8*I*(-(I*3^(1/2)-3)*x*b/
(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*
x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-
b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/
2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*
a)^(1/3)*3^(1/2)*x^4*b-4*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a
)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/
2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^
(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1
/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(2/3)*3^(1/2)*x^3+4*(-(I*
3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^
2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)
*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^
2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2
*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*x^5*b^2-8*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3))
)^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*
x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(
-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-
1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+
I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*x^4*b+4*(-(I*3^(1/2)-3)*x*b
/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b
*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(
-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(
1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1
/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2
*a)^(2/3)*x^3+I*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*
b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*
(-b^2*a)^(1/3)*3^(1/2)*((b*x^3+a)*x)^(1/2)-3*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(
1/3)*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a
)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2))/(b*x^5+a*x
^2)^(1/2)/x^(3/2)/(-b^2*a)^(1/3)/a/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/(1/b^2
*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I
*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}\,x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(5/2)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}\sqrt{x}}{bx^8+ax^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^8 + a*x^5), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^{\frac{5}{2}}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(x**2*(a + b*x**3))), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{bx^5+ax^2}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(5/2)), x)
```

## 3.305 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=555

$$\frac{4\left(1-\sqrt{3}\right)b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{8\left(1+\sqrt{3}\right)b^{4/3}x^{3}}{7a^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)}$$

```
[Out] (-8*(1 + Sqrt[3])*b^(4/3)*x^(3/2)*(a + b*x^3))/(7*a^2*(a^(1/3) + (1 + Sqrt[
3])*b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (2*Sqrt[a*x^2 + b*x^5])/(7*a*x^(9/2))
 + (8*b*Sqrt[a*x^2 + b*x^5])/(7*a^2*x^(3/2)) + (8*3^(1/4)*b^(4/3)*x^(3/2)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])
*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(7*a^(5
/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*Sqrt[a*x^2 + b*x^5]) + (4*(1 - Sqrt[3])*b^(4/3)*x^(3/2)*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(7*3^(1/4)*a^(5/3)
*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*Sqrt[a*x^2 + b*x^5])
```

---

**Rubi [A]** time = 0.573181, antiderivative size = 555, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2025, 2032, 329, 308, 225, 1881}

$$-\frac{8\left(1+\sqrt{3}\right)b^{4/3}x^{3/2}\left(a+bx^3\right)}{7a^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}+\frac{4\left(1-\sqrt{3}\right)b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(\right.\right.}{7\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-8*(1 + Sqrt[3])*b^(4/3)*x^(3/2)*(a + b*x^3))/(7*a^2*(a^(1/3) + (1 + Sqrt[
3])*b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (2*Sqrt[a*x^2 + b*x^5])/(7*a*x^(9/2))
 + (8*b*Sqrt[a*x^2 + b*x^5])/(7*a^2*x^(3/2)) + (8*3^(1/4)*b^(4/3)*x^(3/2)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])
*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(7*a^(5
/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*Sqrt[a*x^2 + b*x^5]) + (4*(1 - Sqrt[3])*b^(4/3)*x^(3/2)*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(7*3^(1/4)*a^(5/3)
*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} - \frac{(4b)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx}{7a}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} - \frac{\left(8b^2\right)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx}{7a^2}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} - \frac{\left(8b^2x\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{7a^2\sqrt{ax^2+bx^5}}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} - \frac{\left(16b^2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{7a^2\sqrt{ax^2+bx^5}}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} + \frac{\left(8b^{4/3}x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{7a^2\sqrt{ax^2+bx^5}} + \frac{\left(8\right.}{}$$

$$= -\frac{8\left(1+\sqrt{3}\right)b^{4/3}x^{3/2}\left(a+bx^3\right)}{7a^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}} - \frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} + \frac{8\sqrt[4]{3}b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\right.}{}$$

**Mathematica [C]** time = 0.0141385, size = 57, normalized size = 0.1

$$-\frac{2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};-\frac{bx^3}{a}\right)}{7x^{5/2}\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-7/6, 1/2, -1/6, -((b*x^3)/a)])/(
7*x^(5/2)*Sqrt[x^2*(a + b*x^3)])
```

---

**Maple [C]** time = 0.043, size = 3048, normalized size = 5.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 2/7*(-8*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*(
(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a
)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1
/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/
2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2)
)/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(1/3)*3^(1/2)*x^5*b-I*
(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/
3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*3^(1/2)*a^2-16*(
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*
(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(
```

```
1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+
(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(
-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(1/3)*x^5*b+24*(-(I*3^(1/2)-3)*x*
b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*
b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*
(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^
(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(
1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*
x^3+a)*x)^(1/2)*(-b^2*a)^(1/3)*x^5*b+3*I*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*
3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*
x-(-b^2*a)^(1/3)))^(1/2)*3^(1/2)*x^3*a*b+32*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/
2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^
(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3
)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*Ellipti
cF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1
/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/
2)*(-b^2*a)^(2/3)*x^4-48*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^
(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2)
)/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1
/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*
x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2
))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(2/3)*x
^4-8*I*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(1/3)*3^(1/2)*x^5*b-8*I*((b*x^3+a)*x)^(
1/2)*3^(1/2)*x^6*b^2+16*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(
1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))
/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/
3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x
*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2)
)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*x^3*a*b-24*(-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2
*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*
((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2
*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*
a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*((b*x^3+a)*x)^(1/2)*x^3*a*b-8*I*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(2/3)*
3^(1/2)*x^4-12*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b
*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*x
^6*b^2+8*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*
((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*
a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(
1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1
/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2
))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*3^(1/2)*x^3*a*b+24*((b*x^3+a)*
x)^(1/2)*x^6*b^2+4*I*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/
3)+2*b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(
1/2)*3^(1/2)*x^6*b^2+24*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(1/3)*x^5*b+24*((b*x^3
+a)*x)^(1/2)*(-b^2*a)^(2/3)*x^4-9*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)
*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2
*a)^(1/3)))^(1/2)*x^3*a*b+16*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b
^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3
^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2
*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/
2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*
3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*(-b^2*a)^(
2/3)*3^(1/2)*x^4+3*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)
+2*b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/
2)*a^2)/(b*x^5+a*x^2)^(1/2)/x^(5/2)/a^2/(I*3^(1/2)-3)/(1/b^2*x*(-b*x+(-b^2*
a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*
```

```
a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(7/2)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5 + ax^2}\sqrt{x}}{bx^9 + ax^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^9 + a*x^6), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{7}{2}}\sqrt{x^2\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(x**2*(a + b*x**3))), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(7/2)), x)
```

## 3.306 $\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=56

$$\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}$$

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(9*a*x^(11/2)) + (4*b*Sqrt[a*x^2 + b*x^5])/(9*a^2*
x^(5/2))
```

---

**Rubi [A]** time = 0.0830774, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(9*a*x^(11/2)) + (4*b*Sqrt[a*x^2 + b*x^5])/(9*a^2*
x^(5/2))
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx&=-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}-\frac{(2b)\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx}{3a}\\&=-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}+\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}\end{aligned}$$

**Mathematica [A]** time = 0.0173151, size = 35, normalized size = 0.62

$$-\frac{2\left(a-2bx^3\right)\sqrt{x^2\left(a+bx^3\right)}}{9a^2x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*(a - 2*b*x^3)*Sqrt[x^2*(a + b*x^3)])/(9*a^2*x^(11/2))
```

---

**Maple [A]** time = 0.006, size = 37, normalized size = 0.7

$$-\frac{\left(2\,b{x}^{3}+2\,a\right)\left(-2\,b{x}^{3}+a\right)}{9\,{a}^{2}}{x}^{-{\frac{7}{2}}}{\frac{1}{\sqrt{b{x}^{5}+a{x}^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2/9*(b*x^3+a)*(-2*b*x^3+a)/x^(7/2)/a^2/(b*x^5+a*x^2)^(1/2)
```

---

**Maxima [A]** time = 1.10407, size = 51, normalized size = 0.91

$$\frac{2\left(2\,b^{2}x^{7}+abx^{4}-a^{2}x\right)}{9\,\sqrt{bx^{3}+a}a^{2}x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(sqrt(b*x^3 + a)*a^2*x^(11/2))
```

---

**Fricas [A]** time = 0.822707, size = 73, normalized size = 1.3

$$\frac{2\,\sqrt{bx^{5}+ax^{2}}\left(2\,bx^{3}-a\right)}{9\,a^{2}x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(2*b*x^3 - a)/(a^2*x^(11/2))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.20074, size = 49, normalized size = 0.88

$$-\frac{4\,b^{\frac{3}{2}}}{9\,a^2}-\frac{2\left(\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}-3\,\sqrt{b+\frac{a}{x^3}}b\right)}{9\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -4/9*b^(3/2)/a^2 - 2/9*((b + a/x^3)^(3/2) - 3*sqrt(b + a/x^3)*b)/a^2
```

## 3.307 $\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx$

**Optimal**. Leaf size=265

$$\frac{16b^2x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55\sqrt[4]{3}a^{7/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}$$

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(11*a*x^(13/2)) + (16*b*Sqrt[a*x^2 + b*x^5])/(55*a
^2*x^(7/2)) + (16*b^2*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[
ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)], (2 + Sqrt[3])/4])/(55*3^(1/4)*a^(7/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1
/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

---

**Rubi [A]** time = 0.284104, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2025, 2032, 329, 225}

$$\frac{16b^2x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55\sqrt[4]{3}a^{7/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}-\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(11/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(11*a*x^(13/2)) + (16*b*Sqrt[a*x^2 + b*x^5])/(55*a
^2*x^(7/2)) + (16*b^2*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[
ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)], (2 + Sqrt[3])/4])/(55*3^(1/4)*a^(7/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1
/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx &= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}-\frac{(8b)\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx}{11a}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}+\frac{\left(16b^2\right)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{55a^2}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}+\frac{\left(16b^2x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{55a^2\sqrt{ax^2+bx^5}}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}+\frac{\left(32b^2x\sqrt{a+bx^3}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{55a^2\sqrt{ax^2+bx^5}}\\
&= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}+\frac{16b^2x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+(1}{\sqrt[3]{a}+(1}\right.\right.}{55\sqrt[4]{3}a^{7/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+b}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0139006, size = 57, normalized size = 0.22

$$
-\frac{2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{11}{6},\frac{1}{2};-\frac{5}{6};-\frac{bx^3}{a}\right)}{11x^{9/2}\sqrt{x^2\left(a+bx^3\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(11/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-11/6, 1/2, -5/6, -((b*x^3)/a)])/
(11*x^(9/2)*Sqrt[x^2*(a + b*x^3)])
```

---

**Maple [C]** time = 0.041, size = 2009, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2/55/(b*x^5+a*x^2)^(1/2)/x^(9/2)*(b*x^3+a)/(-b^2*a)^(1/3)/a^2*(32*I*(-(I*3
^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2
*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*
((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2
*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*
a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*3^(1/2)*x^8*b^3-64*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2
*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(
1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a
)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)
-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^
(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(1/3)*3^(1/2)*x^7*b^2+3
2*I*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(
1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3
)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(
-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-
b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3
^(1/2)-3))^(1/2))*(-b^2*a)^(2/3)*3^(1/2)*x^6*b-32*(-(I*3^(1/2)-3)*x*b/(-1+I
*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b
^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a
)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*E
llipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((
I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^8*b^3+64*
(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)
*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^
(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x
+(-b^2*a)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+
(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/
2)-3))^(1/2))*(-b^2*a)^(1/3)*x^7*b^2-32*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/
(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3
))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2*a)^(1/3)-2*
b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),((I*3^(1/2)+
3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-b^2*a)^(2/3)*x^6*b-
8*I*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)
^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*(-b^2*a)^(1/
3)*3^(1/2)*((b*x^3+a)*x)^(1/2)*x^3*b+24*b*((b*x^3+a)*x)^(1/2)*x^3*(-b^2*a)^
(1/3)*(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*
a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)+5*I*(1/b^2
*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I
*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)*(-b^2*a)^(1/3)*3^(1/2)
*((b*x^3+a)*x)^(1/2)*a-15*((b*x^3+a)*x)^(1/2)*a*(-b^2*a)^(1/3)*(1/b^2*x*(-b
*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*(I*3^(1/
2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2))/((b*x^3+a)*x)^(1/2)/(I*3^(1
/2)-3)/(1/b^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2
*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5 + ax^2}x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(11/2)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}\sqrt{x}}{bx^{11}+ax^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^11 + a*x^8), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(11/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^5+ax^2}x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(11/2)), x)
```

## 3.308 $\int \frac{x}{ax^3+bx^4} \, dx$

**Optimal**. Leaf size=28

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

---

**Rubi [A]** time = 0.0169484, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 44}

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^3 + b*x^4),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{ax^3+bx^4}\,dx &= \int \frac{1}{x^2(a+bx)}\,dx\\ &= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\ &= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}\end{aligned}$$

**Mathematica [A]** time = 0.0043284, size = 28, normalized size = 1.

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^3 + b*x^4),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

---

**Maple [A]** time = 0.007, size = 29, normalized size = 1.

$$-\frac{1}{ax}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{b\ln\left(bx+a\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a*x^3),x)
```

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

---

**Maxima [A]** time = 1.10428, size = 38, normalized size = 1.36

$$\frac{b\log\left(bx+a\right)}{a^{2}}-\frac{b\log\left(x\right)}{a^{2}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

---

**Fricas [A]** time = 0.73642, size = 61, normalized size = 2.18

$$\frac{bx\log\left(bx+a\right)-bx\log\left(x\right)-a}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

---

**Sympy [A]** time = 0.787333, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a*x**3),x)
```

```
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

---

**Giac [A]** time = 1.20335, size = 41, normalized size = 1.46

$$\frac{b\log\left(|bx+a|\right)}{a^{2}}-\frac{b\log\left(|x|\right)}{a^{2}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

## 3.309 $\int \frac{1}{ax^3+bx^4}\,dx$

**Optimal**. Leaf size=42

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

---

**Rubi [A]** time = 0.0191524, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 44}

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^4)^(-1),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax^3+bx^4}\,dx &= \int \frac{1}{x^3(a+bx)}\,dx\\ &= \int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}\end{aligned}$$

**Mathematica [A]** time = 0.0045704, size = 42, normalized size = 1.

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^4)^(-1),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

---

**Maple [A]** time = 0.006, size = 41, normalized size = 1.

$$-\frac{1}{2\,a x^{2}}+\frac{b}{a^{2} x}+\frac{b^{2} \ln \left(x\right)}{a^{3}}-\frac{b^{2} \ln \left(b x+a\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a*x^3),x)
```

```
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

---

**Maxima [A]** time = 1.06869, size = 54, normalized size = 1.29

$$-\frac{b^{2} \log \left(b x+a\right)}{a^{3}}+\frac{b^{2} \log \left(x\right)}{a^{3}}+\frac{2\,b x-a}{2\,a^{2} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

---

**Fricas [A]** time = 0.793161, size = 103, normalized size = 2.45

$$-\frac{2\,b^{2} x^{2} \log \left(b x+a\right)-2\,b^{2} x^{2} \log \left(x\right)-2\,a b x+a^{2}}{2\,a^{3} x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

---

**Sympy [A]** time = 0.529053, size = 31, normalized size = 0.74

$$\frac{-a+2bx}{2a^{2}x^{2}}+\frac{b^{2}\left(\log{\left(x \right)}-\log{\left(\frac{a}{b}+x \right)}\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a*x**3),x)
```

```
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

---

**Giac [A]** time = 1.30535, size = 61, normalized size = 1.45

$$-\frac{b^2\log\left(|bx+a|\right)}{a^3}+\frac{b^2\log\left(|x|\right)}{a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

## 3.310 $\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx$

**Optimal**. Leaf size=112

$$\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}-\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{x\sqrt{ax^3+bx^4}}{3b}$$

```
[Out] (-5*a*Sqrt[a*x^3 + b*x^4])/(12*b^2) + (5*a^2*Sqrt[a*x^3 + b*x^4])/(8*b^3*x)
 + (x*Sqrt[a*x^3 + b*x^4])/(3*b) - (5*a^3*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3
+ b*x^4]])/(8*b^(7/2))
```

---

**Rubi [A]** time = 0.178822, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2024, 2029, 206}

$$\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}-\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{x\sqrt{ax^3+bx^4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (-5*a*Sqrt[a*x^3 + b*x^4])/(12*b^2) + (5*a^2*Sqrt[a*x^3 + b*x^4])/(8*b^3*x)
 + (x*Sqrt[a*x^3 + b*x^4])/(3*b) - (5*a^3*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3
+ b*x^4]])/(8*b^(7/2))
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx &= \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{(5a)\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx}{6b} \\
&= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{x\sqrt{ax^3+bx^4}}{3b} + \frac{\left(5a^2\right)\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx}{8b^2} \\
&= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x} + \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{\left(5a^3\right)\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx}{16b^3} \\
&= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x} + \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{\left(5a^3\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^3} \\
&= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x} + \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.145038, size = 94, normalized size = 0.84

$$
\frac{\sqrt{x^3(a+bx)}\left(\sqrt{b}\sqrt{x}\left(15a^2-10abx+8b^2x^2\right)-\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{24b^{7/2}x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (Sqrt[x^3*(a + b*x)]*(Sqrt[b]*Sqrt[x]*(15*a^2 - 10*a*b*x + 8*b^2*x^2) - (15
*a^(5/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(24*b^(7/2
)*x^(3/2))
```

---

**Maple [A]** time = 0.004, size = 120, normalized size = 1.1

$$
\frac{x}{48}\sqrt{x\,(bx+a)}\left(16\,x^2\sqrt{bx^2+ax}b^{7/2}-20\,\sqrt{bx^2+ax}b^{5/2}xa+30\,\sqrt{bx^2+ax}b^{3/2}a^2-15\,\ln\left(1/2\,\frac{2\,\sqrt{bx^2+ax}\sqrt{b}+2\,bx+}{\sqrt{b}}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] 1/48*x*(x*(b*x+a))^(1/2)*(16*x^2*(b*x^2+a*x)^(1/2)*b^(7/2)-20*(b*x^2+a*x)^(
1/2)*b^(5/2)*x*a+30*(b*x^2+a*x)^(1/2)*b^(3/2)*a^2-15*ln(1/2*(2*(b*x^2+a*x)^
(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*a^3*b)/(b*x^4+a*x^3)^(1/2)/b^(9/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{x^4}{\sqrt{bx^4+ax^3}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(b*x^4 + a*x^3), x)
```

---

**Fricas [A]** time = 0.91539, size = 390, normalized size = 3.48

$$\left[\frac{15\,a^{3}\sqrt{b}x\log\left(\frac{2\,bx^{2}+ax-2\,\sqrt{bx^{4}+ax^{3}}\sqrt{b}}{x}\right)+2\left(8\,b^{3}x^{2}-10\,ab^{2}x+15\,a^{2}b\right)\sqrt{bx^{4}+ax^{3}}}{48\,b^{4}x},\ \frac{15\,a^{3}\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^{4}+ax^{3}}\sqrt{-b}}{bx^{2}}\right)+(8\,b^{3}}{24\,b^{4}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b))
/x) + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(b^4*x), 1
/24*(15*a^3*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2)) + (8*b^
3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(b^4*x)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\sqrt{x^{3}\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**3*(a + b*x)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\sqrt{bx^{4}+ax^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(b*x^4 + a*x^3), x)
```

## 3.311 $\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx$

**Optimal**. Leaf size=86

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{\sqrt{ax^3+bx^4}}{2b}$$

```
[Out] Sqrt[a*x^3 + b*x^4]/(2*b) - (3*a*Sqrt[a*x^3 + b*x^4])/(4*b^2*x) + (3*a^2*Ar
cTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/(4*b^(5/2))
```

---

**Rubi [A]** time = 0.127086, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2024, 2029, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{\sqrt{ax^3+bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] Sqrt[a*x^3 + b*x^4]/(2*b) - (3*a*Sqrt[a*x^3 + b*x^4])/(4*b^2*x) + (3*a^2*Ar
cTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/(4*b^(5/2))
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx &= \frac{\sqrt{ax^3+bx^4}}{2b}-\frac{(3a)\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx}{4b} \\
&= \frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{\left(3a^2\right)\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx}{8b^2} \\
&= \frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{\left(3a^2\right)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^2} \\
&= \frac{\sqrt{ax^3+bx^4}}{2b}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.0515954, size = 92, normalized size = 1.07

$$
\frac{\sqrt{b}x^2\left(-3a^2-abx+2b^2x^2\right)+3a^{5/2}x^{3/2}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{5/2}\sqrt{x^3(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (Sqrt[b]*x^2*(-3*a^2 - a*b*x + 2*b^2*x^2) + 3*a^(5/2)*x^(3/2)*Sqrt[1 + (b*x
)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(5/2)*Sqrt[x^3*(a + b*x)])
```

---

**Maple [A]** time = 0.004, size = 98, normalized size = 1.1

$$
\frac{x}{8}\sqrt{x\,(bx+a)}\left(4\,x\sqrt{bx^2+ax}b^{5/2}-6\,\sqrt{bx^2+ax}b^{3/2}a+3\,\ln\left(1/2\,\frac{2\,\sqrt{bx^2+ax}\sqrt{b}+2\,bx+a}{\sqrt{b}}\right)a^2b\right)\frac{1}{\sqrt{bx^4+ax^3}}b^{-\frac{7}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] 1/8*x*(x*(b*x+a))^(1/2)*(4*x*(b*x^2+a*x)^(1/2)*b^(5/2)-6*(b*x^2+a*x)^(1/2)*
b^(3/2)*a+3*ln(1/2*(2*(b*x^2+a*x)^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*a^2*b)/(b
*x^4+a*x^3)^(1/2)/b^(7/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{x^3}{\sqrt{bx^4+ax^3}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(b*x^4 + a*x^3), x)
```

---

**Fricas [A]** time = 0.811266, size = 338, normalized size = 3.93

$$\left[\frac{3\,a^{2}\sqrt{b}x\log\left(\frac{2\,bx^{2}+ax+2\,\sqrt{bx^{4}+ax^{3}}\sqrt{b}}{x}\right)+2\,\sqrt{bx^{4}+ax^{3}}\left(2\,b^{2}x-3\,ab\right)}{8\,b^{3}x},-\frac{3\,a^{2}\sqrt{-b}x\,\mathrm{arctan}\left(\frac{\sqrt{bx^{4}+ax^{3}}\sqrt{-b}}{bx^{2}}\right)-\sqrt{bx^{4}+ax^{3}}(2\,b}{4\,b^{3}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*x*log((2*b*x^2 + a*x + 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x
) + 2*sqrt(b*x^4 + a*x^3)*(2*b^2*x - 3*a*b))/(b^3*x), -1/4*(3*a^2*sqrt(-b)*
x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2)) - sqrt(b*x^4 + a*x^3)*(2*b^2
*x - 3*a*b))/(b^3*x)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\sqrt{x^{3}\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**3*(a + b*x)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\sqrt{bx^{4}+ax^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(b*x^4 + a*x^3), x)
```

## 3.312 $\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx$

**Optimal**. Leaf size=56

$$\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

```
[Out] Sqrt[a*x^3 + b*x^4]/(b*x) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/
b^(3/2)
```

---

**Rubi [A]** time = 0.0823131, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2024, 2029, 206}

$$\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] Sqrt[a*x^3 + b*x^4]/(b*x) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/
b^(3/2)
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx &= \frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx}{2b}\\
&= \frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^3+bx^4}}\right)}{b}\\
&= \frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.0405259, size = 75, normalized size = 1.34

$$
\frac{\sqrt{b}x^2(a+bx)-a^{3/2}x^{3/2}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{x^3(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (Sqrt[b]*x^2*(a + b*x) - a^(3/2)*x^(3/2)*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]
*Sqrt[x])/Sqrt[a]])/(b^(3/2)*Sqrt[x^3*(a + b*x)])
```

---

**Maple [A]** time = 0.005, size = 78, normalized size = 1.4

$$
\frac{x}{2}\sqrt{x\,(bx+a)}\left(2\,\sqrt{bx^2+ax}b^{3/2}-a\ln\left(\frac{1}{2}\left(2\,\sqrt{bx^2+ax}\sqrt{b}+2\,bx+a\right)\frac{1}{\sqrt{b}}\right)b\right)\frac{1}{\sqrt{bx^4+ax^3}}b^{-\frac{5}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] 1/2*x*(x*(b*x+a))^(1/2)*(2*(b*x^2+a*x)^(1/2)*b^(3/2)-a*ln(1/2*(2*(b*x^2+a*x
)^(1/2)*b^(1/2)+2*b*x+a)/b^(1/2))*b)/(b*x^4+a*x^3)^(1/2)/b^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{x^2}{\sqrt{bx^4+ax^3}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*x^4 + a*x^3), x)
```

---

**Fricas [A]** time = 0.809629, size = 277, normalized size = 4.95

$$
\left[\frac{a\sqrt{b}x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^4+ax^3}\sqrt{b}}{x}\right)+2\,\sqrt{bx^4+ax^3}b}{2\,b^2x}, \frac{a\sqrt{-b}x\arctan\left(\frac{\sqrt{bx^4+ax^3}\sqrt{-b}}{bx^2}\right)+\sqrt{bx^4+ax^3}b}{b^2x}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x) +
2*sqrt(b*x^4 + a*x^3)*b)/(b^2*x), (a*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3)*
sqrt(-b)/(b*x^2)) + sqrt(b*x^4 + a*x^3)*b)/(b^2*x)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**3*(a + b*x)), x)
```

---

**Giac [A]** time = 1.21226, size = 55, normalized size = 0.98

$$\frac{\sqrt{b+\frac{a}{x}}x}{b}+\frac{a\arctan\left(\frac{\sqrt{b+\frac{a}{x}}}{\sqrt{-b}}\right)}{\sqrt{-b}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(b + a/x)*x/b + a*arctan(sqrt(b + a/x)/sqrt(-b))/(sqrt(-b)*b)
```

## 3.313 $\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx$

**Optimal**. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/Sqrt[b]
```

---

**Rubi [A]** time = 0.0343985, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2029, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/Sqrt[b]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^3+bx^4}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}\end{aligned}$$

**Mathematica [A]** time = 0.0166366, size = 59, normalized size = 1.84

$$\frac{2\sqrt{a}x^{3/2}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x^3(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (2*Sqrt[a]*x^(3/2)*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x^3*(a + b*x)])
```

---

**Maple [B]** time = 0.003, size = 56, normalized size = 1.8

$$x\sqrt{x\,(bx+a)}\ln\left(\frac{1}{2}\left(2\sqrt{bx^{2}+ax}\sqrt{b}+2\,bx+a\right)\frac{1}{\sqrt{b}}\right)\frac{1}{\sqrt{bx^{4}+ax^{3}}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] 1/(b*x^4+a*x^3)^(1/2)*x*(x*(b*x+a))^(1/2)*ln(1/2*(2*(b*x^2+a*x)^(1/2)*b^(1/
2)+2*b*x+a)/b^(1/2))/b^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{bx^{4}+ax^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*x^4 + a*x^3), x)
```

---

**Fricas [A]** time = 0.820835, size = 173, normalized size = 5.41

$$\left[\frac{\log\left(\frac{2\,bx^{2}+ax+2\sqrt{bx^{4}+ax^{3}}\sqrt{b}}{x}\right)}{\sqrt{b}},\,-\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^{4}+ax^{3}}\sqrt{-b}}{bx^{2}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((2*b*x^2 + a*x + 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x)/sqrt(b), -2*sqrt(-b
)*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2))/b]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{x^{3}\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**3*(a + b*x)), x)
```

---

**Giac [A]** time = 1.39866, size = 31, normalized size = 0.97

$$-\frac{2\arctan\left(\frac{\sqrt{b+\frac{a}{x}}}{\sqrt{-b}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(b + a/x)/sqrt(-b))/sqrt(-b)
```

## 3.314 $\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx$

**Optimal**. Leaf size=23

$$-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(a*x^2)
```

---

**Rubi [A]** time = 0.0050791, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2000}

$$-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(a*x^2)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx=-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

**Mathematica [A]** time = 0.0071141, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{x^3(a+bx)}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (-2*Sqrt[x^3*(a + b*x)])/(a*x^2)
```

---

**Maple [A]** time = 0.002, size = 25, normalized size = 1.1

$$-2\,\frac{x\,(bx+a)}{a\sqrt{bx^4+ax^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] -2*x*(b*x+a)/a/(b*x^4+a*x^3)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4 + ax^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^4 + a*x^3), x)
```

---

**Fricas [A]** time = 0.622897, size = 43, normalized size = 1.87

$$-\frac{2\sqrt{bx^4 + ax^3}}{ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x^4 + a*x^3)/(a*x^2)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^3 + bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**3 + b*x**4), x)
```

---

**Giac [A]** time = 1.21362, size = 36, normalized size = 1.57

$$\frac{2}{\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/((sqrt(b)*x - sqrt(b*x^2 + a*x))*sgn(x))
```

## 3.315 $\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx$

**Optimal**. Leaf size=52

$$\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}$$

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(3*a*x^3) + (4*b*Sqrt[a*x^3 + b*x^4])/(3*a^2*x^2)
```

---

**Rubi [A]** time = 0.0450667, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(3*a*x^3) + (4*b*Sqrt[a*x^3 + b*x^4])/(3*a^2*x^2)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx &= -\frac{2\sqrt{ax^3+bx^4}}{3ax^3}-\frac{(2b)\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax^3+bx^4}}{3ax^3}+\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}\end{aligned}$$

**Mathematica [A]** time = 0.0110932, size = 29, normalized size = 0.56

$$-\frac{2(a-2bx)\sqrt{x^3(a+bx)}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*(a - 2*b*x)*Sqrt[x^3*(a + b*x)])/(3*a^2*x^3)
```

---

**Maple [A]** time = 0.004, size = 30, normalized size = 0.6

$$-\frac{(2\,bx+2\,a)\,(-2\,bx+a)}{3\,a^2}\frac{1}{\sqrt{bx^4+ax^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] -2/3*(b*x+a)*(-2*b*x+a)/a^2/(b*x^4+a*x^3)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4+ax^3}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x), x)
```

---

**Fricas [A]** time = 0.859516, size = 63, normalized size = 1.21

$$\frac{2\,\sqrt{bx^4+ax^3}(2\,bx-a)}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^4 + a*x^3)*(2*b*x - a)/(a^2*x^3)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**3*(a + b*x))), x)
```

---

**Giac [A]** time = 1.20618, size = 36, normalized size = 0.69

$$-\frac{2\left(\left(b+\frac{a}{x}\right)^{\frac{3}{2}}-3\sqrt{b+\frac{a}{x}}b\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*((b + a/x)^(3/2) - 3*sqrt(b + a/x)*b)/a^2
```

## 3.316 $\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx$

**Optimal**. Leaf size=80

$$-\frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}$$

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(5*a*x^4) + (8*b*Sqrt[a*x^3 + b*x^4])/(15*a^2*x^3)
 - (16*b^2*Sqrt[a*x^3 + b*x^4])/(15*a^3*x^2)
```

---

**Rubi [A]** time = 0.0860185, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$-\frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(5*a*x^4) + (8*b*Sqrt[a*x^3 + b*x^4])/(15*a^2*x^3)
 - (16*b^2*Sqrt[a*x^3 + b*x^4])/(15*a^3*x^2)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx &= -\frac{2\sqrt{ax^3+bx^4}}{5ax^4}-\frac{(4b)\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx}{5a}\\ &= -\frac{2\sqrt{ax^3+bx^4}}{5ax^4}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}+\frac{\left(8b^2\right)\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx}{15a^2}\\ &= -\frac{2\sqrt{ax^3+bx^4}}{5ax^4}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}-\frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}\end{aligned}$$

**Mathematica [A]** time = 0.0122532, size = 42, normalized size = 0.52

$$-\frac{2\sqrt{x^3(a+bx)}\left(3a^2-4abx+8b^2x^2\right)}{15a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[x^3*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^4)
```

---

**Maple [A]** time = 0.003, size = 46, normalized size = 0.6

$$-\frac{(2\,bx+2\,a)\left(8\,b^2x^2-4\,abx+3\,a^2\right)}{15\,xa^3}\frac{1}{\sqrt{bx^4+ax^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] -2/15*(b*x+a)*(8*b^2*x^2-4*a*b*x+3*a^2)/x/a^3/(b*x^4+a*x^3)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4+ax^3}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^2), x)
```

---

**Fricas [A]** time = 0.763782, size = 90, normalized size = 1.12

$$-\frac{2\,\sqrt{bx^4+ax^3}\left(8\,b^2x^2-4\,abx+3\,a^2\right)}{15\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*sqrt(b*x^4 + a*x^3)*(8*b^2*x^2 - 4*a*b*x + 3*a^2)/(a^3*x^4)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**3*(a + b*x))), x)
```

---

**Giac [A]** time = 1.21621, size = 58, normalized size = 0.72

$$-\frac{2\left(3\left(b+\frac{a}{x}\right)^{\frac{5}{2}}-10\left(b+\frac{a}{x}\right)^{\frac{3}{2}}b+15\sqrt{b+\frac{a}{x}}b^{2}\right)}{15\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(3*(b + a/x)^(5/2) - 10*(b + a/x)^(3/2)*b + 15*sqrt(b + a/x)*b^2)/a^3
```

## 3.317 $\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx$

**Optimal**. Leaf size=108

$$\frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(7*a*x^5) + (12*b*Sqrt[a*x^3 + b*x^4])/(35*a^2*x^4
) - (16*b^2*Sqrt[a*x^3 + b*x^4])/(35*a^3*x^3) + (32*b^3*Sqrt[a*x^3 + b*x^4]
)/(35*a^4*x^2)
```

---

**Rubi [A]** time = 0.134198, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$\frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(7*a*x^5) + (12*b*Sqrt[a*x^3 + b*x^4])/(35*a^2*x^4
) - (16*b^2*Sqrt[a*x^3 + b*x^4])/(35*a^3*x^3) + (32*b^3*Sqrt[a*x^3 + b*x^4]
)/(35*a^4*x^2)
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx &= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}-\frac{(6b)\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx}{7a}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}+\frac{\left(24b^2\right)\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx}{35a^2}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}-\frac{\left(16b^3\right)\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx}{35a^3}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0170488, size = 53, normalized size = 0.49

$$\frac{2\sqrt{x^3(a+bx)}\left(6a^2bx-5a^3-8ab^2x^2+16b^3x^3\right)}{35a^4x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (2*Sqrt[x^3*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35
*a^4*x^5)
```

---

**Maple [A]** time = 0.005, size = 57, normalized size = 0.5

$$-\frac{(2\,bx+2\,a)\left(-16\,b^3x^3+8\,ab^2x^2-6\,bxa^2+5\,a^3\right)}{35\,x^2a^4}\frac{1}{\sqrt{bx^4+ax^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] -2/35*(b*x+a)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/x^2/a^4/(b*x^4+a*x^
3)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4+ax^3}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^3), x)
```

---

**Fricas [A]** time = 0.790084, size = 112, normalized size = 1.04

$$\frac{2\left(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3\right)\sqrt{bx^4+ax^3}}{35\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^4 + a*x^3)/(a^
4*x^5)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**3*(a + b*x))), x)
```

---

**Giac [A]** time = 1.2341, size = 77, normalized size = 0.71

$$-\frac{2\left(5\left(b+\frac{a}{x}\right)^{\frac{7}{2}}-21\left(b+\frac{a}{x}\right)^{\frac{5}{2}}b+35\left(b+\frac{a}{x}\right)^{\frac{3}{2}}b^{2}-35\sqrt{b+\frac{a}{x}}b^{3}\right)}{35\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2/35*(5*(b + a/x)^(7/2) - 21*(b + a/x)^(5/2)*b + 35*(b + a/x)^(3/2)*b^2 -
35*sqrt(b + a/x)*b^3)/a^4
```

## 3.318 $\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx$

**Optimal**. Leaf size=136

$$-\frac{256b^4\sqrt{ax^3+bx^4}}{315a^5x^2}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{2\sqrt{ax^3+bx^4}}{9ax^6}$$

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(9*a*x^6) + (16*b*Sqrt[a*x^3 + b*x^4])/(63*a^2*x^5
) - (32*b^2*Sqrt[a*x^3 + b*x^4])/(105*a^3*x^4) + (128*b^3*Sqrt[a*x^3 + b*x^
4])/(315*a^4*x^3) - (256*b^4*Sqrt[a*x^3 + b*x^4])/(315*a^5*x^2)
```

---

**Rubi [A]** time = 0.174421, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$-\frac{256b^4\sqrt{ax^3+bx^4}}{315a^5x^2}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{2\sqrt{ax^3+bx^4}}{9ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(9*a*x^6) + (16*b*Sqrt[a*x^3 + b*x^4])/(63*a^2*x^5
) - (32*b^2*Sqrt[a*x^3 + b*x^4])/(105*a^3*x^4) + (128*b^3*Sqrt[a*x^3 + b*x^
4])/(315*a^4*x^3) - (256*b^4*Sqrt[a*x^3 + b*x^4])/(315*a^5*x^2)
```

### Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

### Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx &= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}-\frac{(8b)\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx}{9a}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}+\frac{\left(16b^2\right)\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx}{21a^2}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}-\frac{\left(64b^3\right)\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx}{105a^3}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}+\frac{\left(128b^4\right)\int \frac{1}{\sqrt{ax^3+bx^4}}}{315a^4}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}-\frac{256b^4\sqrt{ax^3+bx^4}}{315a^5x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0189052, size = 64, normalized size = 0.47

$$-\frac{2\sqrt{x^3(a+bx)}\left(48a^2b^2x^2-40a^3bx+35a^4-64ab^3x^3+128b^4x^4\right)}{315a^5x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[x^3*(a + b*x)]*(35*a^4 - 40*a^3*b*x + 48*a^2*b^2*x^2 - 64*a*b^3*x^
3 + 128*b^4*x^4))/(315*a^5*x^6)
```

---

**Maple [A]** time = 0.003, size = 68, normalized size = 0.5

$$-\frac{(2\,bx+2\,a)\left(128\,b^4x^4-64\,ab^3x^3+48\,b^2x^2a^2-40\,xa^3b+35\,a^4\right)}{315\,x^3a^5}\frac{1}{\sqrt{bx^4+ax^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] -2/315*(b*x+a)*(128*b^4*x^4-64*a*b^3*x^3+48*a^2*b^2*x^2-40*a^3*b*x+35*a^4)/
x^3/a^5/(b*x^4+a*x^3)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4+ax^3}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^4), x)
```

---

**Fricas [A]** time = 0.821276, size = 143, normalized size = 1.05

$$-\frac{2\left(128\,b^4x^4-64\,ab^3x^3+48\,a^2b^2x^2-40\,a^3bx+35\,a^4\right)\sqrt{bx^4+ax^3}}{315\,a^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(128*b^4*x^4 - 64*a*b^3*x^3 + 48*a^2*b^2*x^2 - 40*a^3*b*x + 35*a^4)*
sqrt(b*x^4 + a*x^3)/(a^5*x^6)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(x**3*(a + b*x))), x)
```

---

**Giac [A]** time = 1.27836, size = 96, normalized size = 0.71

$$-\frac{2\left(35\left(b+\frac{a}{x}\right)^{\frac{9}{2}}-180\left(b+\frac{a}{x}\right)^{\frac{7}{2}}b+378\left(b+\frac{a}{x}\right)^{\frac{5}{2}}b^{2}-420\left(b+\frac{a}{x}\right)^{\frac{3}{2}}b^{3}+315\sqrt{b+\frac{a}{x}}b^{4}\right)}{315\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2/315*(35*(b + a/x)^(9/2) - 180*(b + a/x)^(7/2)*b + 378*(b + a/x)^(5/2)*b^
2 - 420*(b + a/x)^(3/2)*b^3 + 315*sqrt(b + a/x)*b^4)/a^5
```

## 3.319 $\int \frac{1}{x^3+bx^5}\,dx$

**Optimal**. Leaf size=26

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

```
[Out] -1/(2*x^2) - b*Log[x] + (b*Log[1 + b*x^2])/2
```

---

**Rubi [A]** time = 0.0176425, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1593, 266, 44}

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3 + b*x^5)^(-1),x]
```

```
[Out] -1/(2*x^2) - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3+bx^5}\,dx &= \int \frac{1}{x^3\left(1+bx^2\right)}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+bx}\right)dx,x,x^2\right)\\ &= -\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1+bx^2\right)\end{aligned}$$

**Mathematica [A]** time = 0.00508, size = 26, normalized size = 1.

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3 + b*x^5)^(-1),x]
```

```
[Out] -1/(2*x^2) - b*Log[x] + (b*Log[1 + b*x^2])/2
```

---

**Maple [A]** time = 0.005, size = 23, normalized size = 0.9

$$-\frac{1}{2\,x^{2}}-b\ln\left(x\right)+\frac{b\ln\left(b{x}^{2}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5+x^3),x)
```

```
[Out] -1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

---

**Maxima [A]** time = 1.1516, size = 30, normalized size = 1.15

$$\frac{1}{2}\,b\log\left(bx^{2}+1\right)-b\log\left(x\right)-\frac{1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+x^3),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + 1) - b*log(x) - 1/2/x^2
```

---

**Fricas [A]** time = 0.846842, size = 72, normalized size = 2.77

$$\frac{bx^{2}\log\left(bx^{2}+1\right)-2\,bx^{2}\log\left(x\right)-1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + 1) - 2*b*x^2*log(x) - 1)/x^2
```

---

**Sympy [A]** time = 0.522277, size = 22, normalized size = 0.85

$$-b\log\left(x\right)+\frac{b\log\left(x^{2}+\frac{1}{b}\right)}{2}-\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5+x**3),x)
```

```
[Out] -b*log(x) + b*log(x**2 + 1/b)/2 - 1/(2*x**2)
```

---

**Giac [A]** time = 1.23451, size = 43, normalized size = 1.65

$$-\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2}\,b\log\left(\left|bx^2+1\right|\right)+\frac{bx^2-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+x^3),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 + 1)) + 1/2*(b*x^2 - 1)/x^2
```

## 3.320 $\int \frac{1}{-x^3+bx^5}\,dx$

**Optimal**. Leaf size=27

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

```
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

---

**Rubi [A]** time = 0.0184613, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 44}

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + b*x^5)^(-1),x]
```

```
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-x^3+bx^5}\,dx &= \int \frac{1}{x^3\left(-1+bx^2\right)}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(-1+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{-1+bx}\right)dx,x,x^2\right)\\ &= \frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1-bx^2\right)\end{aligned}$$

**Mathematica [A]** time = 0.005098, size = 27, normalized size = 1.

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + b*x^5)^(-1),x]
```

```
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

---

**Maple [A]** time = 0.004, size = 23, normalized size = 0.9

$$\frac{1}{2x^{2}}-b\ln\left(x\right)+\frac{b\ln\left(bx^{2}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5-x^3),x)
```

```
[Out] 1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2-1)
```

---

**Maxima [A]** time = 1.03757, size = 30, normalized size = 1.11

$$\frac{1}{2}\,b\log\left(bx^{2}-1\right)-b\log\left(x\right)+\frac{1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5-x^3),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 - 1) - b*log(x) + 1/2/x^2
```

---

**Fricas [A]** time = 0.764366, size = 72, normalized size = 2.67

$$\frac{bx^{2}\log\left(bx^{2}-1\right)-2\,bx^{2}\log\left(x\right)+1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5-x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 - 1) - 2*b*x^2*log(x) + 1)/x^2
```

---

**Sympy [A]** time = 0.263068, size = 22, normalized size = 0.81

$$-b\log\left(x\right)+\frac{b\log\left(x^{2}-\frac{1}{b}\right)}{2}+\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5-x**3),x)
```

```
[Out] -b*log(x) + b*log(x**2 - 1/b)/2 + 1/(2*x**2)
```

---

**Giac [A]** time = 1.2824, size = 43, normalized size = 1.59

$$-\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2}\,b\log\left(\left|bx^2-1\right|\right)+\frac{bx^2+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5-x^3),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 - 1)) + 1/2*(b*x^2 + 1)/x^2
```

## 3.321 $\int \frac{1}{ax+bx}\,dx$

**Optimal**. Leaf size=8

$$\frac{\log(x)}{a+b}$$

```
[Out] Log[x]/(a + b)
```

---

**Rubi [A]** time = 0.0031854, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6, 12, 29}

$$\frac{\log(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x)^(-1),x]
```

```
[Out] Log[x]/(a + b)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+bx}\,dx &= \int \frac{1}{(a+b)x}\,dx\\ &= \frac{\int \frac{1}{x}\,dx}{a+b}\\ &= \frac{\log(x)}{a+b}\end{aligned}$$

**Mathematica [A]** time = 0.0025627, size = 14, normalized size = 1.75

$$\frac{\log(ax+bx)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x)^(-1),x]
```

```
[Out] Log[a*x + b*x]/(a + b)
```

---

**Maple [A]** time = 0., size = 9, normalized size = 1.1

$$\frac{\ln(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x),x)
```

```
[Out] ln(x)/(a+b)
```

---

**Maxima [A]** time = 1.02556, size = 19, normalized size = 2.38

$$\frac{\log(ax+bx)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x, algorithm="maxima")
```

```
[Out] log(a*x + b*x)/(a + b)
```

---

**Fricas [A]** time = 0.84249, size = 22, normalized size = 2.75

$$\frac{\log(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x, algorithm="fricas")
```

```
[Out] log(x)/(a + b)
```

---

**Sympy [A]** time = 0.093372, size = 5, normalized size = 0.62

$$\frac{\log(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x)
```

```
[Out] log(x)/(a + b)
```

---

**Giac [A]** time = 1.18924, size = 20, normalized size = 2.5

$$\frac{\log(|ax+bx|)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x, algorithm="giac")
```

```
[Out] log(abs(a*x + b*x))/(a + b)
```

## 3.322 $\int \frac{1}{(ax+bx)^2} dx$

**Optimal**. Leaf size=10

$$-\frac{1}{x(a+b)^2}$$

```
[Out] -(1/((a + b)^2*x))
```

---

**Rubi [A]** time = 0.0032584, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6, 12, 30}

$$-\frac{1}{x(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x)^(-2),x]
```

```
[Out] -(1/((a + b)^2*x))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax+bx)^2} dx &= \int \frac{1}{(a+b)^2x^2} dx\\ &= \frac{\int \frac{1}{x^2} dx}{(a+b)^2}\\ &= -\frac{1}{(a+b)^2x}\end{aligned}$$

**Mathematica [A]** time = 0.0030123, size = 10, normalized size = 1.

$$-\frac{1}{x(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x)^(-2),x]
```

```
[Out] -(1/((a + b)^2*x))
```

---

**Maple [A]** time = 0., size = 11, normalized size = 1.1

$$-\frac{1}{(a+b)^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x)^2,x)
```

```
[Out] -1/(a+b)^2/x
```

---

**Maxima [A]** time = 1.11015, size = 22, normalized size = 2.2

$$-\frac{1}{(ax+bx)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="maxima")
```

```
[Out] -1/((a*x + b*x)*(a + b))
```

---

**Fricas [A]** time = 0.800912, size = 38, normalized size = 3.8

$$-\frac{1}{\left(a^2+2\,ab+b^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="fricas")
```

```
[Out] -1/((a^2 + 2*a*b + b^2)*x)
```

---

**Sympy [A]** time = 0.111881, size = 15, normalized size = 1.5

$$-\frac{1}{x\left(a^2+2ab+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)**2,x)
```

```
[Out] -1/(x*(a**2 + 2*a*b + b**2))
```

---

**Giac [A]** time = 1.12956, size = 22, normalized size = 2.2

$$-\frac{1}{(ax+bx)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="giac")
```

```
[Out] -1/((a*x + b*x)*(a + b))
```

## 3.323 $\int \frac{1}{(ax+bx)^3}\,dx$

**Optimal**. Leaf size=12

$$-\frac{1}{2x^2(a+b)^3}$$

```
[Out] -1/(2*(a + b)^3*x^2)
```

---

**Rubi [A]** time = 0.0030896, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6, 12, 30}

$$-\frac{1}{2x^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x)^(-3),x]
```

```
[Out] -1/(2*(a + b)^3*x^2)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax+bx)^3}\,dx &= \int \frac{1}{(a+b)^3x^3}\,dx\\ &= \frac{\int \frac{1}{x^3}\,dx}{(a+b)^3}\\ &= -\frac{1}{2(a+b)^3x^2}\end{aligned}$$

**Mathematica [A]** time = 0.0034175, size = 12, normalized size = 1.

$$-\frac{1}{2x^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x)^(-3),x]
```

```
[Out] -1/(2*(a + b)^3*x^2)
```

---

**Maple [A]** time = 0.001, size = 11, normalized size = 0.9

$$-\frac{1}{2\left(a+b\right)^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x)^3,x)
```

```
[Out] -1/2/(a+b)^3/x^2
```

---

**Maxima [A]** time = 0.995037, size = 22, normalized size = 1.83

$$-\frac{1}{2\left(ax+bx\right)^{2}(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/((a*x + b*x)^2*(a + b))
```

---

**Fricas [B]** time = 0.820803, size = 59, normalized size = 4.92

$$-\frac{1}{2\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x^2)
```

---

**Sympy [B]** time = 0.107827, size = 27, normalized size = 2.25

$$-\frac{1}{2x^{2}\left(a^{3}+3a^{2}b+3ab^{2}+b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)**3,x)
```

```
[Out] -1/(2*x**2*(a**3 + 3*a**2*b + 3*a*b**2 + b**3))
```

---

**Giac [A]** time = 1.15199, size = 22, normalized size = 1.83

$$-\frac{1}{2\,(ax+bx)^2(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^3,x, algorithm="giac")
```

```
[Out] -1/2/((a*x + b*x)^2*(a + b))
```

## 3.324 $\int \frac{1}{ax^2+bx^2}\,dx$

**Optimal**. Leaf size=10

$$-\frac{1}{x(a+b)}$$

```
[Out] -(1/((a + b)*x))
```

---

**Rubi [A]** time = 0.0016056, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6, 12, 30}

$$-\frac{1}{x(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^2)^(-1),x]
```

```
[Out] -(1/((a + b)*x))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax^2+bx^2}\,dx &= \int \frac{1}{(a+b)x^2}\,dx\\ &= \frac{\int \frac{1}{x^2}\,dx}{a+b}\\ &= -\frac{1}{(a+b)x}\end{aligned}$$

**Mathematica [A]** time = 0.0006774, size = 10, normalized size = 1.

$$-\frac{1}{x(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^2)^(-1),x]
```

```
[Out] -(1/((a + b)*x))
```

---

**Maple [A]** time = 0., size = 11, normalized size = 1.1

$$-\frac{1}{(a+b)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+b*x^2),x)
```

```
[Out] -1/(a+b)/x
```

---

**Maxima [A]** time = 1.08684, size = 14, normalized size = 1.4

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^2),x, algorithm="maxima")
```

```
[Out] -1/((a + b)*x)
```

---

**Fricas [A]** time = 0.639769, size = 22, normalized size = 2.2

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^2),x, algorithm="fricas")
```

```
[Out] -1/((a + b)*x)
```

---

**Sympy [A]** time = 0.081292, size = 7, normalized size = 0.7

$$-\frac{1}{x\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+b*x**2),x)
```

```
[Out] -1/(x*(a + b))
```

---

**Giac [A]** time = 1.13651, size = 14, normalized size = 1.4

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^2),x, algorithm="giac")
```

```
[Out] -1/((a + b)*x)
```

## 3.325 $\int \frac{1}{ax^n+bx^n}\,dx$

**Optimal**. Leaf size=20

$$\frac{x^{1-n}}{(1-n)(a+b)}$$

```
[Out] x^(1 - n)/((a + b)*(1 - n))
```

---

**Rubi [A]** time = 0.0058103, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6, 12, 30}

$$\frac{x^{1-n}}{(1-n)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^n + b*x^n)^(-1),x]
```

```
[Out] x^(1 - n)/((a + b)*(1 - n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax^n+bx^n}\,dx &= \int \frac{x^{-n}}{a+b}\,dx\\ &= \frac{\int x^{-n}\,dx}{a+b}\\ &= \frac{x^{1-n}}{(a+b)(1-n)}\end{aligned}$$

**Mathematica [A]** time = 0.0038062, size = 20, normalized size = 1.

$$\frac{x^{1-n}}{(1-n)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^n + b*x^n)^(-1),x]
```

```
[Out] x^(1 - n)/((a + b)*(1 - n))
```

---

**Maple [A]** time = 0.002, size = 19, normalized size = 1.

$$-\frac{x}{(-1+n)\,x^{n}\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n+b*x^n),x)
```

```
[Out] -x/(-1+n)/(x^n)/(a+b)
```

---

**Maxima [A]** time = 1.07105, size = 28, normalized size = 1.4

$$-\frac{x}{(a(n-1)+b(n-1))x^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n),x, algorithm="maxima")
```

```
[Out] -x/((a*(n - 1) + b*(n - 1))*x^n)
```

---

**Fricas [A]** time = 0.872954, size = 41, normalized size = 2.05

$$-\frac{x}{((a+b)n-a-b)x^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n),x, algorithm="fricas")
```

```
[Out] -x/(((a + b)*n - a - b)*x^n)
```

---

**Sympy [A]** time = 0.839191, size = 32, normalized size = 1.6

$$\begin{cases} -\frac{x}{anx^{n}-ax^{n}+bnx^{n}-bx^{n}} & \text{for}\: n\neq 1 \\ \frac{\log{(x)}}{a+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**n+b*x**n),x)
```

```
[Out] Piecewise((-x/(a*n*x**n - a*x**n + b*n*x**n - b*x**n), Ne(n, 1)), (log(x)/(
a + b), True))
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ax^n + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x^n + b*x^n), x)
```

## 3.326 $\int \frac{1}{(ax^n+bx^n)^2} \, dx$

**Optimal**. Leaf size=20

$$\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

```
[Out] x^(1 - 2*n)/((a + b)^2*(1 - 2*n))
```

---

**Rubi [A]** time = 0.0082571, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6, 12, 30}

$$\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^n + b*x^n)^(-2),x]
```

```
[Out] x^(1 - 2*n)/((a + b)^2*(1 - 2*n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax^n+bx^n)^2} \, dx &= \int \frac{x^{-2n}}{(a+b)^2} \, dx\\ &= \frac{\int x^{-2n} \, dx}{(a+b)^2}\\ &= \frac{x^{1-2n}}{(a+b)^2(1-2n)}\end{aligned}$$

**Mathematica [A]** time = 0.0032169, size = 20, normalized size = 1.

$$\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^n + b*x^n)^(-2),x]
```

```
[Out] x^(1 - 2*n)/((a + b)^2*(1 - 2*n))
```

---

**Maple [A]** time = 0.001, size = 21, normalized size = 1.1

$$-\frac{x}{(-1+2n)(x^n)^2(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n+b*x^n)^2,x)
```

```
[Out] -x/(-1+2*n)/(x^n)^2/(a+b)^2
```

---

**Maxima [A]** time = 1.05347, size = 54, normalized size = 2.7

$$-\frac{x}{\left(a^2(2n-1)+2ab(2n-1)+b^2(2n-1)\right)x^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -x/((a^2*(2*n - 1) + 2*a*b*(2*n - 1) + b^2*(2*n - 1))*x^(2*n))
```

---

**Fricas [A]** time = 0.916805, size = 80, normalized size = 4.

$$\frac{x}{\left(a^2+2ab+b^2-2\left(a^2+2ab+b^2\right)n\right)x^{2n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] x/((a^2 + 2*a*b + b^2 - 2*(a^2 + 2*a*b + b^2)*n)*x^(2*n))
```

---

**Sympy [A]** time = 1.38359, size = 82, normalized size = 4.1

$$\begin{cases} -\frac{x}{2a^2nx^{2n}-a^2x^{2n}+4abnx^{2n}-2abx^{2n}+2b^2nx^{2n}-b^2x^{2n}} & \text{for } n \neq \frac{1}{2} \\ \frac{\log(x)}{a^2+2ab+b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**n+b*x**n)**2,x)
```

```
[Out] Piecewise((-x/(2*a**2*n*x**(2*n) - a**2*x**(2*n) + 4*a*b*n*x**(2*n) - 2*a*b
*x**(2*n) + 2*b**2*n*x**(2*n) - b**2*x**(2*n)), Ne(n, 1/2)), (log(x)/(a**2
```

```
+ 2*a*b + b**2), True))
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^n + bx^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((a*x^n + b*x^n)^(-2), x)
```

## 3.327 $\int \frac{1}{(ax^n+bx^n)^3}\,dx$

**Optimal**. Leaf size=20

$$\frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

```
[Out] x^(1 - 3*n)/((a + b)^3*(1 - 3*n))
```

---

**Rubi [A]** time = 0.0089884, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6, 12, 30}

$$\frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^n + b*x^n)^(-3),x]
```

```
[Out] x^(1 - 3*n)/((a + b)^3*(1 - 3*n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax^n+bx^n)^3}\,dx &= \int \frac{x^{-3n}}{(a+b)^3}\,dx\\ &= \frac{\int x^{-3n}\,dx}{(a+b)^3}\\ &= \frac{x^{1-3n}}{(a+b)^3(1-3n)}\end{aligned}$$

**Mathematica [A]** time = 0.0032928, size = 20, normalized size = 1.

$$\frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^n + b*x^n)^(-3),x]
```

```
[Out] x^(1 - 3*n)/((a + b)^3*(1 - 3*n))
```

---

**Maple [A]** time = 0.002, size = 21, normalized size = 1.1

$$-\frac{x}{(-1+3\,n)\,(x^n)^3\,(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n+b*x^n)^3,x)
```

```
[Out] -x/(-1+3*n)/(x^n)^3/(a+b)^3
```

---

**Maxima [B]** time = 1.01413, size = 72, normalized size = 3.6

$$-\frac{x}{\left(a^3(3\,n-1)+3\,a^2b(3\,n-1)+3\,ab^2(3\,n-1)+b^3(3\,n-1)\right)x^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] -x/((a^3*(3*n - 1) + 3*a^2*b*(3*n - 1) + 3*a*b^2*(3*n - 1) + b^3*(3*n - 1))
*x^(3*n))
```

---

**Fricas [B]** time = 0.918571, size = 112, normalized size = 5.6

$$\frac{x}{\left(a^3+3\,a^2b+3\,ab^2+b^3-3\left(a^3+3\,a^2b+3\,ab^2+b^3\right)n\right)x^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] x/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*n)*x^
(3*n))
```

---

**Sympy [A]** time = 1.80554, size = 119, normalized size = 5.95

$$\begin{cases} -\frac{x}{3a^3nx^{3n}-a^3x^{3n}+9a^2bnx^{3n}-3a^2bx^{3n}+9ab^2nx^{3n}-3ab^2x^{3n}+3b^3nx^{3n}-b^3x^{3n}} & \text{for } n \neq \frac{1}{3} \\ \frac{\log(x)}{a^3+3a^2b+3ab^2+b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**n+b*x**n)**3,x)
```

```
[Out] Piecewise((-x/(3*a**3*n*x**(3*n) - a**3*x**(3*n) + 9*a**2*b*n*x**(3*n) - 3*
a**2*b*x**(3*n) + 9*a*b**2*n*x**(3*n) - 3*a*b**2*x**(3*n) + 3*b**3*n*x**(3*
n) - b**3*x**(3*n)), Ne(n, 1/3)), (log(x)/(a**3 + 3*a**2*b + 3*a*b**2 + b**
3), True))
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^n + bx^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x^n + b*x^n)^(-3), x)
```

## 3.328 $\int \left(ax+bx^{14}\right)^{12}\,dx$

**Optimal**. Leaf size=16

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

```
[Out] (a + b*x^13)^13/(169*b)
```

---

**Rubi [A]** time = 0.0044672, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 261}

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^14)^12,x]
```

```
[Out] (a + b*x^13)^13/(169*b)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(ax+bx^{14}\right)^{12}\,dx &= \int x^{12}\left(a+bx^{13}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{13}\right)^{13}}{169b}\end{aligned}$$

**Mathematica [B]** time = 0.0043833, size = 160, normalized size = 10.

$$\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}+\frac{132}{13}a^7b^5x^{78}+\frac{99}{13}a^8b^4x^{65}+\frac{55}{13}a^9b^3x^{52}+\frac{22}{13}a$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^14)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
```

```
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

---

**Maple [B]** time = 0.003, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{169}}{169}+\frac{b^{11}ax^{156}}{13}+\frac{6\,b^{10}a^{2}x^{143}}{13}+\frac{22\,a^{3}b^{9}x^{130}}{13}+\frac{55\,a^{4}b^{8}x^{117}}{13}+\frac{99\,a^{5}b^{7}x^{104}}{13}+\frac{132\,a^{6}b^{6}x^{91}}{13}+\frac{132\,a^{7}b^{5}x^{78}}{13}+\frac{99\,a^{8}b^{4}x^{65}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^14+a*x)^12,x)
```

```
[Out] 1/169*b^12*x^169+1/13*b^11*a*x^156+6/13*b^10*a^2*x^143+22/13*a^3*b^9*x^130+
55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^5*
x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*a^11*b*
x^26+1/13*a^12*x^13
```

---

**Maxima [B]** time = 1.02378, size = 181, normalized size = 11.31

$$\frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^{2}b^{10}x^{143}+\frac{22}{13}\,a^{3}b^{9}x^{130}+\frac{55}{13}\,a^{4}b^{8}x^{117}+\frac{99}{13}\,a^{5}b^{7}x^{104}+\frac{132}{13}\,a^{6}b^{6}x^{91}+\frac{132}{13}\,a^{7}b^{5}x^{78}+\frac{99}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

---

**Fricas [B]** time = 0.597755, size = 365, normalized size = 22.81

$$\frac{1}{169}x^{169}b^{12}+\frac{1}{13}x^{156}b^{11}a+\frac{6}{13}x^{143}b^{10}a^{2}+\frac{22}{13}x^{130}b^{9}a^{3}+\frac{55}{13}x^{117}b^{8}a^{4}+\frac{99}{13}x^{104}b^{7}a^{5}+\frac{132}{13}x^{91}b^{6}a^{6}+\frac{132}{13}x^{78}b^{5}a^{7}+\frac{99}{13}x^{65}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/169*x^169*b^12 + 1/13*x^156*b^11*a + 6/13*x^143*b^10*a^2 + 22/13*x^130*b^
9*a^3 + 55/13*x^117*b^8*a^4 + 99/13*x^104*b^7*a^5 + 132/13*x^91*b^6*a^6 + 1
32/13*x^78*b^5*a^7 + 99/13*x^65*b^4*a^8 + 55/13*x^52*b^3*a^9 + 22/13*x^39*b
^2*a^10 + 6/13*x^26*b*a^11 + 1/13*x^13*a^12
```

---

**Sympy [B]** time = 0.122485, size = 160, normalized size = 10.

$$\frac{a^{12}x^{13}}{13}+\frac{6a^{11}bx^{26}}{13}+\frac{22a^{10}b^{2}x^{39}}{13}+\frac{55a^{9}b^{3}x^{52}}{13}+\frac{99a^{8}b^{4}x^{65}}{13}+\frac{132a^{7}b^{5}x^{78}}{13}+\frac{132a^{6}b^{6}x^{91}}{13}+\frac{99a^{5}b^{7}x^{104}}{13}+\frac{55a^{4}b^{8}x^{117}}{13}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**14+a*x)**12,x)
```

```
[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3
*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*
x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x
**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169
```

---

**Giac [B]** time = 1.15551, size = 181, normalized size = 11.31

$$\frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^{2}b^{10}x^{143}+\frac{22}{13}\,a^{3}b^{9}x^{130}+\frac{55}{13}\,a^{4}b^{8}x^{117}+\frac{99}{13}\,a^{5}b^{7}x^{104}+\frac{132}{13}\,a^{6}b^{6}x^{91}+\frac{132}{13}\,a^{7}b^{5}x^{78}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

## 3.329 $\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx$

**Optimal**. Leaf size=16

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

```
[Out] (a + b*x^25)^13/(325*b)
```

---

**Rubi [A]** time = 0.0114282, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 261}

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx&=\int x^{24}\left(a+bx^{25}\right)^{12}\,dx\\&=\frac{\left(a+bx^{25}\right)^{13}}{325b}\end{aligned}$$

**Mathematica [B]** time = 0.0066342, size = 160, normalized size = 10.

$$\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^6b^6x^{175}+\frac{132}{25}a^7b^5x^{150}+\frac{99}{25}a^8b^4x^{125}+\frac{11}{5}a^9b^3x^{100}+\frac{22}{25}a$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
```

```
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

---

**Maple [B]** time = 0.003, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{325}}{325}+\frac{b^{11}ax^{300}}{25}+\frac{6\,b^{10}a^{2}x^{275}}{25}+\frac{22\,a^{3}b^{9}x^{250}}{25}+\frac{11\,a^{4}b^{8}x^{225}}{5}+\frac{99\,a^{5}b^{7}x^{200}}{25}+\frac{132\,a^{6}b^{6}x^{175}}{25}+\frac{132\,a^{7}b^{5}x^{150}}{25}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^26+a*x)^12,x)
```

```
[Out] 1/325*b^12*x^325+1/25*b^11*a*x^300+6/25*b^10*a^2*x^275+22/25*a^3*b^9*x^250+
11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*
x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*
b*x^50+1/25*a^12*x^25
```

---

**Maxima [B]** time = 1.11586, size = 181, normalized size = 11.31

$$\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

---

**Fricas [B]** time = 0.682468, size = 367, normalized size = 22.94

$$\frac{1}{325}x^{325}b^{12}+\frac{1}{25}x^{300}b^{11}a+\frac{6}{25}x^{275}b^{10}a^{2}+\frac{22}{25}x^{250}b^{9}a^{3}+\frac{11}{5}x^{225}b^{8}a^{4}+\frac{99}{25}x^{200}b^{7}a^{5}+\frac{132}{25}x^{175}b^{6}a^{6}+\frac{132}{25}x^{150}b^{5}a^{7}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^
9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 1
32/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75
*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12
```

---

**Sympy [B]** time = 0.240242, size = 160, normalized size = 10.

$$\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**26+a*x)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

---

**Giac [B]** time = 1.15535, size = 181, normalized size = 11.31

$$\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

# 3.330 $\int x^{24} \left(ax + bx^{38}\right)^{12} \, dx$

**Optimal**. Leaf size=16

$$\frac{\left(a + bx^{37}\right)^{13}}{481b}$$

```
[Out] (a + b*x^37)^13/(481*b)
```

---

**Rubi [A]** time = 0.0117595, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 261}

$$\frac{\left(a + bx^{37}\right)^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int x^{24} \left(ax + bx^{38}\right)^{12} \, dx &= \int x^{36} \left(a + bx^{37}\right)^{12} \, dx \\ &= \frac{\left(a + bx^{37}\right)^{13}}{481b} \end{aligned}$$

**Mathematica [B]** time = 0.008164, size = 160, normalized size = 10.

$$\frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} +$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
```

```
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

---

**Maple [B]** time = 0.001, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{481}}{481}+\frac{b^{11}ax^{444}}{37}+\frac{6\,b^{10}a^{2}x^{407}}{37}+\frac{22\,a^{3}b^{9}x^{370}}{37}+\frac{55\,a^{4}b^{8}x^{333}}{37}+\frac{99\,a^{5}b^{7}x^{296}}{37}+\frac{132\,a^{6}b^{6}x^{259}}{37}+\frac{132\,a^{7}b^{5}x^{222}}{37}+\frac{99\,a^{8}b^{4}x}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(b*x^38+a*x)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*b^11*a*x^444+6/37*b^10*a^2*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

---

**Maxima [B]** time = 1.05626, size = 181, normalized size = 11.31

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

---

**Fricas [B]** time = 0.760761, size = 371, normalized size = 23.19

$$\frac{1}{481}x^{481}b^{12}+\frac{1}{37}x^{444}b^{11}a+\frac{6}{37}x^{407}b^{10}a^{2}+\frac{22}{37}x^{370}b^{9}a^{3}+\frac{55}{37}x^{333}b^{8}a^{4}+\frac{99}{37}x^{296}b^{7}a^{5}+\frac{132}{37}x^{259}b^{6}a^{6}+\frac{132}{37}x^{222}b^{5}a^{7}+\frac{99}{37}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

---

**Sympy [B]** time = 0.23361, size = 160, normalized size = 10.

$$\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^{2}x^{111}}{37}+\frac{55a^{9}b^{3}x^{148}}{37}+\frac{99a^{8}b^{4}x^{185}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24*(b*x**38+a*x)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

---

**Giac [B]** time = 1.11851, size = 181, normalized size = 11.31

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

## 3.331 $\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx$

**Optimal**. Leaf size=27

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

---

**Rubi [A]** time = 0.0147372, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1584, 261}

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}dx&=\int x^{12+12(-1+m)}\left(a+bx^{1+12m}\right)^{12}dx\\&=\frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}\end{aligned}$$

**Mathematica [A]** time = 0.013241, size = 24, normalized size = 0.89

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{156bm+13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b + 156*b*m)
```

---

**Maple [B]** time = 0.106, size = 339, normalized size = 12.6

$$\frac{b^{12}\left(x^{2+12m}\right)^{13}}{(13+156m)\,x^{13}}+\frac{ab^{11}\left(x^{2+12m}\right)^{12}}{(1+12m)\,x^{12}}+6\,\frac{a^{2}b^{10}\left(x^{2+12m}\right)^{11}}{(1+12m)\,x^{11}}+22\,\frac{a^{3}b^{9}\left(x^{2+12m}\right)^{10}}{(1+12m)\,x^{10}}+55\,\frac{a^{4}b^{8}\left(x^{2+12m}\right)^{9}}{(1+12m)\,x^{9}}+99\,\frac{a^{5}b^{7}\left(x^{2+12}\right.}{(1+12m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x)
```

```
[Out] 1/13/(1+12*m)*b^12/x^13*(x^(2+12*m))^13+1/(1+12*m)*a*b^11/x^12*(x^(2+12*m))
^12+6/(1+12*m)*a^2*b^10/x^11*(x^(2+12*m))^11+22/(1+12*m)*a^3*b^9/x^10*(x^(2
+12*m))^10+55/(1+12*m)*a^4*b^8/x^9*(x^(2+12*m))^9+99/(1+12*m)*a^5*b^7/x^8*(
x^(2+12*m))^8+132/(1+12*m)*a^6*b^6/x^7*(x^(2+12*m))^7+132/(1+12*m)*a^7*b^5/
x^6*(x^(2+12*m))^6+99/(1+12*m)*a^8*b^4/x^5*(x^(2+12*m))^5+55/(1+12*m)*a^9*b
^3/x^4*(x^(2+12*m))^4+22/(1+12*m)*a^10*b^2/x^3*(x^(2+12*m))^3+6/(1+12*m)*a^
11*b/x^2*(x^(2+12*m))^2+1/(1+12*m)*a^12/x*x^(2+12*m)
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [B]** time = 0.723535, size = 578, normalized size = 21.41

$$\underline{13\,a^{12}x^{12}x^{12m+2}+78\,a^{11}bx^{11}x^{24m+4}+286\,a^{10}b^{2}x^{10}x^{36m+6}+715\,a^{9}b^{3}x^{9}x^{48m+8}+1287\,a^{8}b^{4}x^{8}x^{60m+10}+1716\,a^{7}b^{5}x^{7}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(13*a^12*x^12*x^(12*m + 2) + 78*a^11*b*x^11*x^(24*m + 4) + 286*a^10*b^
2*x^10*x^(36*m + 6) + 715*a^9*b^3*x^9*x^(48*m + 8) + 1287*a^8*b^4*x^8*x^(60
*m + 10) + 1716*a^7*b^5*x^7*x^(72*m + 12) + 1716*a^6*b^6*x^6*x^(84*m + 14)
+ 1287*a^5*b^7*x^5*x^(96*m + 16) + 715*a^4*b^8*x^4*x^(108*m + 18) + 286*a^3
*b^9*x^3*x^(120*m + 20) + 78*a^2*b^10*x^2*x^(132*m + 22) + 13*a*b^11*x*x^(1
44*m + 24) + b^12*x^(156*m + 26))/((12*m + 1)*x^13)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-12+12*m)*(a*x+b*x**(2+12*m))**12,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax + bx^{12m+2}\right)^{12} x^{12m-12}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(12*m + 2))^12*x^(12*m - 12), x)
```

## 3.332 $\int \left(ax+bx^{14}\right)^{12}\,dx$

**Optimal**. Leaf size=16

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

```
[Out] (a + b*x^13)^13/(169*b)
```

---

**Rubi [A]** time = 0.0044802, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 261}

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^14)^12,x]
```

```
[Out] (a + b*x^13)^13/(169*b)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(ax+bx^{14}\right)^{12}\,dx &= \int x^{12}\left(a+bx^{13}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{13}\right)^{13}}{169b}\end{aligned}$$

**Mathematica [B]** time = 0.0031746, size = 160, normalized size = 10.

$$\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}+\frac{132}{13}a^7b^5x^{78}+\frac{99}{13}a^8b^4x^{65}+\frac{55}{13}a^9b^3x^{52}+\frac{22}{13}a$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^14)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
```

```
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

---

**Maple [B]** time = 0., size = 135, normalized size = 8.4

$$\frac{b^{12}x^{169}}{169}+\frac{b^{11}a x^{156}}{13}+\frac{6\,b^{10}a^2x^{143}}{13}+\frac{22\,a^3b^9x^{130}}{13}+\frac{55\,a^4b^8x^{117}}{13}+\frac{99\,a^5b^7x^{104}}{13}+\frac{132\,a^6b^6x^{91}}{13}+\frac{132\,a^7b^5x^{78}}{13}+\frac{99\,a^8b^4x^{65}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^14+a*x)^12,x)
```

```
[Out] 1/169*b^12*x^169+1/13*b^11*a*x^156+6/13*b^10*a^2*x^143+22/13*a^3*b^9*x^130+
55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^5*
x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*a^11*b*
x^26+1/13*a^12*x^13
```

---

**Maxima [B]** time = 1.10368, size = 181, normalized size = 11.31

$$\frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^2b^{10}x^{143}+\frac{22}{13}\,a^3b^9x^{130}+\frac{55}{13}\,a^4b^8x^{117}+\frac{99}{13}\,a^5b^7x^{104}+\frac{132}{13}\,a^6b^6x^{91}+\frac{132}{13}\,a^7b^5x^{78}+\frac{99}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

---

**Fricas [B]** time = 0.596537, size = 365, normalized size = 22.81

$$\frac{1}{169}x^{169}b^{12}+\frac{1}{13}x^{156}b^{11}a+\frac{6}{13}x^{143}b^{10}a^2+\frac{22}{13}x^{130}b^9a^3+\frac{55}{13}x^{117}b^8a^4+\frac{99}{13}x^{104}b^7a^5+\frac{132}{13}x^{91}b^6a^6+\frac{132}{13}x^{78}b^5a^7+\frac{99}{13}x^{65}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/169*x^169*b^12 + 1/13*x^156*b^11*a + 6/13*x^143*b^10*a^2 + 22/13*x^130*b^
9*a^3 + 55/13*x^117*b^8*a^4 + 99/13*x^104*b^7*a^5 + 132/13*x^91*b^6*a^6 + 1
32/13*x^78*b^5*a^7 + 99/13*x^65*b^4*a^8 + 55/13*x^52*b^3*a^9 + 22/13*x^39*b
^2*a^10 + 6/13*x^26*b*a^11 + 1/13*x^13*a^12
```

---

**Sympy [B]** time = 0.222354, size = 160, normalized size = 10.

$$\frac{a^{12}x^{13}}{13}+\frac{6a^{11}bx^{26}}{13}+\frac{22a^{10}b^2x^{39}}{13}+\frac{55a^9b^3x^{52}}{13}+\frac{99a^8b^4x^{65}}{13}+\frac{132a^7b^5x^{78}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{55a^4b^8x^{117}}{13}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**14+a*x)**12,x)
```

```
[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3
*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*
x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x
**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169
```

---

**Giac [B]** time = 1.15819, size = 181, normalized size = 11.31

$$\frac{1}{169} b^{12} x^{169} + \frac{1}{13} a b^{11} x^{156} + \frac{6}{13} a^{2} b^{10} x^{143} + \frac{22}{13} a^{3} b^{9} x^{130} + \frac{55}{13} a^{4} b^{8} x^{117} + \frac{99}{13} a^{5} b^{7} x^{104} + \frac{132}{13} a^{6} b^{6} x^{91} + \frac{132}{13} a^{7} b^{5} x^{78} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

## 3.333 $\int \left(ax^2+bx^{27}\right)^{12} dx$

**Optimal**. Leaf size=16

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

```
[Out] (a + b*x^25)^13/(325*b)
```

---

**Rubi [A]** time = 0.0045659, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 261}

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^27)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(ax^2+bx^{27}\right)^{12} dx &= \int x^{24}\left(a+bx^{25}\right)^{12} dx\\ &= \frac{\left(a+bx^{25}\right)^{13}}{325b}\end{aligned}$$

**Mathematica [B]** time = 0.0047925, size = 160, normalized size = 10.

$$\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^6b^6x^{175}+\frac{132}{25}a^7b^5x^{150}+\frac{99}{25}a^8b^4x^{125}+\frac{11}{5}a^9b^3x^{100}+\frac{22}{25}a$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^27)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
```

```
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

---

**Maple [B]** time = 0.003, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{325}}{325}+\frac{b^{11}ax^{300}}{25}+\frac{6\,b^{10}a^{2}x^{275}}{25}+\frac{22\,a^{3}b^{9}x^{250}}{25}+\frac{11\,a^{4}b^{8}x^{225}}{5}+\frac{99\,a^{5}b^{7}x^{200}}{25}+\frac{132\,a^{6}b^{6}x^{175}}{25}+\frac{132\,a^{7}b^{5}x^{150}}{25}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^27+a*x^2)^12,x)
```

```
[Out] 1/325*b^12*x^325+1/25*b^11*a*x^300+6/25*b^10*a^2*x^275+22/25*a^3*b^9*x^250+
11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*
x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*
b*x^50+1/25*a^12*x^25
```

---

**Maxima [B]** time = 1.02941, size = 181, normalized size = 11.31

$$\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^27+a*x^2)^12,x, algorithm="maxima")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

---

**Fricas [B]** time = 0.659142, size = 367, normalized size = 22.94

$$\frac{1}{325}x^{325}b^{12}+\frac{1}{25}x^{300}b^{11}a+\frac{6}{25}x^{275}b^{10}a^{2}+\frac{22}{25}x^{250}b^{9}a^{3}+\frac{11}{5}x^{225}b^{8}a^{4}+\frac{99}{25}x^{200}b^{7}a^{5}+\frac{132}{25}x^{175}b^{6}a^{6}+\frac{132}{25}x^{150}b^{5}a^{7}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^27+a*x^2)^12,x, algorithm="fricas")
```

```
[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^
9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 1
32/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75
*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12
```

---

**Sympy [B]** time = 0.171689, size = 160, normalized size = 10.

$$\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**27+a*x**2)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

---

**Giac [B]** time = 1.12765, size = 181, normalized size = 11.31

$$\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^27+a*x^2)^12,x, algorithm="giac")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

## 3.334 $\int \left(ax^3+bx^{40}\right)^{12}\,dx$

**Optimal**. Leaf size=16

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

```
[Out] (a + b*x^37)^13/(481*b)
```

---

**Rubi [A]** time = 0.0044845, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 261}

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^40)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(ax^3+bx^{40}\right)^{12}\,dx &= \int x^{36}\left(a+bx^{37}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{37}\right)^{13}}{481b}\end{aligned}$$

**Mathematica [B]** time = 0.0059641, size = 160, normalized size = 10.

$$\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8b^4x^{185}+\frac{55}{37}a^9b^3x^{148}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^40)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
```

```
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

---

**Maple [B]** time = 0.003, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{481}}{481}+\frac{b^{11}ax^{444}}{37}+\frac{6\,b^{10}a^{2}x^{407}}{37}+\frac{22\,a^{3}b^{9}x^{370}}{37}+\frac{55\,a^{4}b^{8}x^{333}}{37}+\frac{99\,a^{5}b^{7}x^{296}}{37}+\frac{132\,a^{6}b^{6}x^{259}}{37}+\frac{132\,a^{7}b^{5}x^{222}}{37}+\frac{99\,a^{8}b^{4}x}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^40+a*x^3)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*b^11*a*x^444+6/37*b^10*a^2*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

---

**Maxima [B]** time = 1.06418, size = 181, normalized size = 11.31

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

---

**Fricas [B]** time = 0.664963, size = 371, normalized size = 23.19

$$\frac{1}{481}x^{481}b^{12}+\frac{1}{37}x^{444}b^{11}a+\frac{6}{37}x^{407}b^{10}a^{2}+\frac{22}{37}x^{370}b^{9}a^{3}+\frac{55}{37}x^{333}b^{8}a^{4}+\frac{99}{37}x^{296}b^{7}a^{5}+\frac{132}{37}x^{259}b^{6}a^{6}+\frac{132}{37}x^{222}b^{5}a^{7}+\frac{99}{37}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="fricas")
```

```
[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

---

**Sympy [B]** time = 0.163629, size = 160, normalized size = 10.

$$\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^{2}x^{111}}{37}+\frac{55a^{9}b^{3}x^{148}}{37}+\frac{99a^{8}b^{4}x^{185}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**40+a*x**3)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

---

**Giac [B]** time = 1.14712, size = 181, normalized size = 11.31

$$\frac{1}{481}\,b^{12}x^{481} + \frac{1}{37}\,ab^{11}x^{444} + \frac{6}{37}\,a^{2}b^{10}x^{407} + \frac{22}{37}\,a^{3}b^{9}x^{370} + \frac{55}{37}\,a^{4}b^{8}x^{333} + \frac{99}{37}\,a^{5}b^{7}x^{296} + \frac{132}{37}\,a^{6}b^{6}x^{259} + \frac{132}{37}\,a^{7}b^{5}x^{222}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

## 3.335 $\int \left(ax^m + bx^{1+13m}\right)^{12} dx$

**Optimal**. Leaf size=27

$$\frac{\left(a + bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

---

**Rubi [A]** time = 0.0077874, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 261}

$$\frac{\left(a + bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m + b*x^(1 + 13*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(ax^m + bx^{1+13m}\right)^{12} dx &= \int x^{12m} \left(a + bx^{1+12m}\right)^{12} dx \\ &= \frac{\left(a + bx^{1+12m}\right)^{13}}{13b(1+12m)}\end{aligned}$$

**Mathematica [A]** time = 0.0040226, size = 24, normalized size = 0.89

$$\frac{\left(a + bx^{12m+1}\right)^{13}}{156bm + 13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m + b*x^(1 + 13*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b + 156*b*m)
```

---

**Maple [B]** time = 0.041, size = 287, normalized size = 10.6

$$\frac{b^{12}x^{13}\left(x^{m}\right)^{156}}{13+156\,m}+\frac{a b^{11}x^{12}\left(x^{m}\right)^{144}}{1+12\,m}+6\,\frac{a^{2}b^{10}x^{11}\left(x^{m}\right)^{132}}{1+12\,m}+22\,\frac{a^{3}b^{9}x^{10}\left(x^{m}\right)^{120}}{1+12\,m}+55\,\frac{a^{4}b^{8}x^{9}\left(x^{m}\right)^{108}}{1+12\,m}+99\,\frac{a^{5}b^{7}x^{8}\left(x^{m}\right)^{96}}{1+12\,m}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m+b*x^(1+13*m))^12,x)
```

```
[Out] 1/13*b^12*x^13/(1+12*m)*(x^m)^156+a*b^11*x^12/(1+12*m)*(x^m)^144+6*a^2*b^10
*x^11/(1+12*m)*(x^m)^132+22*a^3*b^9*x^10/(1+12*m)*(x^m)^120+55*a^4*b^8*x^9/
(1+12*m)*(x^m)^108+99*a^5*b^7*x^8/(1+12*m)*(x^m)^96+132*a^6*b^6*x^7/(1+12*m
)*(x^m)^84+132*a^7*b^5*x^6/(1+12*m)*(x^m)^72+99*a^8*b^4*x^5/(1+12*m)*(x^m)^
60+55*a^9*b^3*x^4/(1+12*m)*(x^m)^48+22*a^10*b^2*x^3/(1+12*m)*(x^m)^36+6*a^1
1*b*x^2/(1+12*m)*(x^m)^24+a^12/(1+12*m)*x*(x^m)^12
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [B]** time = 0.862526, size = 493, normalized size = 18.26

$$\frac{b^{12}x^{13}x^{156\,m}+13\,a b^{11}x^{12}x^{144\,m}+78\,a^{2}b^{10}x^{11}x^{132\,m}+286\,a^{3}b^{9}x^{10}x^{120\,m}+715\,a^{4}b^{8}x^{9}x^{108\,m}+1287\,a^{5}b^{7}x^{8}x^{96\,m}+1716}{13\,(1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(b^12*x^13*x^(156*m) + 13*a*b^11*x^12*x^(144*m) + 78*a^2*b^10*x^11*x^(
132*m) + 286*a^3*b^9*x^10*x^(120*m) + 715*a^4*b^8*x^9*x^(108*m) + 1287*a^5*
b^7*x^8*x^(96*m) + 1716*a^6*b^6*x^7*x^(84*m) + 1716*a^7*b^5*x^6*x^(72*m) +
1287*a^8*b^4*x^5*x^(60*m) + 715*a^9*b^3*x^4*x^(48*m) + 286*a^10*b^2*x^3*x^(
36*m) + 78*a^11*b*x^2*x^(24*m) + 13*a^12*x*x^(12*m))/(12*m + 1)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m+b*x**(1+13*m))**12,x)
```

```
[Out] Timed out
```

---

**Giac [B]** time = 1.25509, size = 277, normalized size = 10.26

$$\frac{b^{12}x^{13}x^{156\,m} + 13\,ab^{11}x^{12}x^{144\,m} + 78\,a^{2}b^{10}x^{11}x^{132\,m} + 286\,a^{3}b^{9}x^{10}x^{120\,m} + 715\,a^{4}b^{8}x^{9}x^{108\,m} + 1287\,a^{5}b^{7}x^{8}x^{96\,m} + 1716\,a^{6}}{13\,(12\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="giac")
```

```
[Out] 1/13*(b^12*x^13*x^(156*m) + 13*a*b^11*x^12*x^(144*m) + 78*a^2*b^10*x^11*x^(
132*m) + 286*a^3*b^9*x^10*x^(120*m) + 715*a^4*b^8*x^9*x^(108*m) + 1287*a^5*
b^7*x^8*x^(96*m) + 1716*a^6*b^6*x^7*x^(84*m) + 1716*a^7*b^5*x^6*x^(72*m) +
1287*a^8*b^4*x^5*x^(60*m) + 715*a^9*b^3*x^4*x^(48*m) + 286*a^10*b^2*x^3*x^(
36*m) + 78*a^11*b*x^2*x^(24*m) + 13*a^12*x*x^(12*m))/(12*m + 1)
```

## 3.336 $\int \left(ax^m + bx^{1+6m}\right)^5 \, dx$

**Optimal**. Leaf size=27

$$\frac{\left(a + bx^{5m+1}\right)^6}{6b(5m+1)}$$

```
[Out] (a + b*x^(1 + 5*m))^6/(6*b*(1 + 5*m))
```

---

**Rubi [A]** time = 0.0102364, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 261}

$$\frac{\left(a + bx^{5m+1}\right)^6}{6b(5m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m + b*x^(1 + 6*m))^5,x]
```

```
[Out] (a + b*x^(1 + 5*m))^6/(6*b*(1 + 5*m))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(ax^m + bx^{1+6m}\right)^5 \, dx &= \int x^{5m}\left(a + bx^{1+5m}\right)^5 \, dx\\ &= \frac{\left(a + bx^{1+5m}\right)^6}{6b(1+5m)}\end{aligned}$$

**Mathematica [A]** time = 0.0133221, size = 27, normalized size = 1.

$$\frac{\left(a + bx^{5m+1}\right)^6}{6b(5m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m + b*x^(1 + 6*m))^5,x]
```

```
[Out] (a + b*x^(1 + 5*m))^6/(6*b*(1 + 5*m))
```

---

**Maple [B]** time = 0.018, size = 126, normalized size = 4.7

$$\frac{b^{5} x^{6} \left(x^{m}\right)^{30}}{6+30 m}+\frac{a b^{4} x^{5} \left(x^{m}\right)^{25}}{1+5 m}+\frac{5 a^{2} b^{3} x^{4} \left(x^{m}\right)^{20}}{2+10 m}+\frac{10 a^{3} b^{2} x^{3} \left(x^{m}\right)^{15}}{3+15 m}+\frac{5 a^{4} b x^{2} \left(x^{m}\right)^{10}}{2+10 m}+\frac{a^{5} x \left(x^{m}\right)^{5}}{1+5 m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m+b*x^(1+6*m))^5,x)
```

```
[Out] 1/6*b^5*x^6/(1+5*m)*(x^m)^30+a*b^4*x^5/(1+5*m)*(x^m)^25+5/2*a^2*b^3*x^4/(1+
5*m)*(x^m)^20+10/3*a^3*b^2*x^3/(1+5*m)*(x^m)^15+5/2*a^4*b*x^2/(1+5*m)*(x^m)
^10+a^5/(1+5*m)*x*(x^m)^5
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [B]** time = 0.85108, size = 204, normalized size = 7.56

$$\frac{b^{5} x^{6} x^{30 m}+6 a b^{4} x^{5} x^{25 m}+15 a^{2} b^{3} x^{4} x^{20 m}+20 a^{3} b^{2} x^{3} x^{15 m}+15 a^{4} b x^{2} x^{10 m}+6 a^{5} x x^{5 m}}{6 (5 m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(b^5*x^6*x^(30*m) + 6*a*b^4*x^5*x^(25*m) + 15*a^2*b^3*x^4*x^(20*m) + 20
*a^3*b^2*x^3*x^(15*m) + 15*a^4*b*x^2*x^(10*m) + 6*a^5*x*x^(5*m))/(5*m + 1)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m+b*x**(1+6*m))**5,x)
```

```
[Out] Timed out
```

---

**Giac [B]** time = 1.12316, size = 126, normalized size = 4.67

$$\frac{b^{5} x^{6} x^{30 m}+6 a b^{4} x^{5} x^{25 m}+15 a^{2} b^{3} x^{4} x^{20 m}+20 a^{3} b^{2} x^{3} x^{15 m}+15 a^{4} b x^{2} x^{10 m}+6 a^{5} x x^{5 m}}{6 (5 m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="giac")
```

```
[Out] 1/6*(b^5*x^6*x^(30*m) + 6*a*b^4*x^5*x^(25*m) + 15*a^2*b^3*x^4*x^(20*m) + 20
*a^3*b^2*x^3*x^(15*m) + 15*a^4*b*x^2*x^(10*m) + 6*a^5*x*x^(5*m))/(5*m + 1)
```

## 3.337 $\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx$

**Optimal**. Leaf size=27

$$-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

```
[Out] -1/(2*b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)
```

---

**Rubi [A]** time = 0.0137646, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 261}

$$-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1 - 2*m) + a*x^m)^(-3),x]
```

```
[Out] -1/(2*b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx &= \int \frac{x^{-3m}}{\left(a+bx^{1-3m}\right)^3}\,dx\\ &= -\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}\end{aligned}$$

**Mathematica [A]** time = 0.0185168, size = 27, normalized size = 1.

$$-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1 - 2*m) + a*x^m)^(-3),x]
```

```
[Out] -1/(2*b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)
```

---

**Maple [A]** time = 0.02, size = 39, normalized size = 1.4

$$-\frac{\left(2\,a\left(x^{m}\right)^{3}+bx\right)x}{\left(-2+6\,m\right)a^{2}\left(a\left(x^{m}\right)^{3}+bx\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1-2*m)+a*x^m)^3,x)
```

```
[Out] -1/2*x*(2*a*(x^m)^3+b*x)/(-1+3*m)/a^2/(a*(x^m)^3+b*x)^2
```

---

**Maxima [B]** time = 1.10417, size = 89, normalized size = 3.3

$$-\frac{2\,axx^{3\,m}+bx^{2}}{2\left(2\,a^{3}b(3\,m-1)xx^{3\,m}+a^{2}b^{2}(3\,m-1)x^{2}+a^{4}(3\,m-1)x^{6\,m}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*x*x^(3*m) + b*x^2)/(2*a^3*b*(3*m - 1)*x*x^(3*m) + a^2*b^2*(3*m -
1)*x^2 + a^4*(3*m - 1)*x^(6*m))
```

---

**Fricas [B]** time = 0.893229, size = 162, normalized size = 6.

$$-\frac{2\,axx^{3\,m}+bx^{2}}{2\left(2\left(3\,a^{3}bm-a^{3}b\right)xx^{3\,m}+\left(3\,a^{2}b^{2}m-a^{2}b^{2}\right)x^{2}+\left(3\,a^{4}m-a^{4}\right)x^{6\,m}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*x^(3*m) + b*x^2)/(2*(3*a^3*b*m - a^3*b)*x*x^(3*m) + (3*a^2*b^2*
m - a^2*b^2)*x^2 + (3*a^4*m - a^4)*x^(6*m))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1-2*m)+a*x**m)**3,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax^{m} + bx^{-2\,m+1}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*x^(-2*m + 1))^(-3), x)
```

## 3.338 $\int \frac{1}{\frac{b}{x}+ax}\,dx$

**Optimal**. Leaf size=15

$$\frac{\log\left(ax^2+b\right)}{2a}$$

```
[Out] Log[b + a*x^2]/(2*a)
```

---

**Rubi [A]** time = 0.004795, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{\log\left(ax^2+b\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^2]/(2*a)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{b}{x}+ax}\,dx &= \int \frac{x}{b+ax^2}\,dx\\ &= \frac{\log\left(b+ax^2\right)}{2a}\end{aligned}$$

**Mathematica [A]** time = 0.0026931, size = 15, normalized size = 1.

$$\frac{\log\left(ax^2+b\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^2]/(2*a)
```

---

**Maple [A]** time = 0.002, size = 14, normalized size = 0.9

$$\frac{\ln\left(a x^{2}+b\right)}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x+a*x),x)
```

```
[Out] 1/2*ln(a*x^2+b)/a
```

---

**Maxima [A]** time = 1.26634, size = 18, normalized size = 1.2

$$\frac{\log\left(a x^{2}+b\right)}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(a*x^2 + b)/a
```

---

**Fricas [A]** time = 0.651248, size = 30, normalized size = 2.

$$\frac{\log\left(a x^{2}+b\right)}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x, algorithm="fricas")
```

```
[Out] 1/2*log(a*x^2 + b)/a
```

---

**Sympy [A]** time = 0.1141, size = 10, normalized size = 0.67

$$\frac{\log\left(a x^{2}+b\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x)
```

```
[Out] log(a*x**2 + b)/(2*a)
```

---

**Giac [A]** time = 1.12146, size = 19, normalized size = 1.27

$$\frac{\log\left(\left|a x^{2}+b\right|\right)}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(a*x^2 + b))/a
```

## 3.339 $\int \frac{1}{\frac{b}{x^2}+ax}\,dx$

**Optimal**. Leaf size=15

$$\frac{\log \left(ax^3+b\right)}{3a}$$

```
[Out] Log[b + a*x^3]/(3*a)
```

---

**Rubi [A]** time = 0.0060296, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{\log \left(ax^3+b\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^2 + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^3]/(3*a)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\frac{b}{x^2}+ax}\,dx &= \int \frac{x^2}{b+ax^3}\,dx \\ &= \frac{\log \left(b+ax^3\right)}{3a} \end{aligned}$$

**Mathematica [A]** time = 0.0039968, size = 15, normalized size = 1.

$$\frac{\log \left(ax^3+b\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^2 + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^3]/(3*a)
```

---

**Maple [A]** time = 0.001, size = 14, normalized size = 0.9

$$\frac{\ln\left(a x^{3}+b\right)}{3 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^2*b+a*x),x)
```

```
[Out] 1/3*ln(a*x^3+b)/a
```

---

**Maxima [A]** time = 1.08087, size = 18, normalized size = 1.2

$$\frac{\log\left(a x^{3}+b\right)}{3 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2+a*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(a*x^3 + b)/a
```

---

**Fricas [A]** time = 0.728851, size = 30, normalized size = 2.

$$\frac{\log\left(a x^{3}+b\right)}{3 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2+a*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(a*x^3 + b)/a
```

---

**Sympy [A]** time = 0.188406, size = 10, normalized size = 0.67

$$\frac{\log\left(a x^{3}+b\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**2+a*x),x)
```

```
[Out] log(a*x**3 + b)/(3*a)
```

---

**Giac [A]** time = 1.15672, size = 19, normalized size = 1.27

$$\frac{\log\left(\left|a x^{3}+b\right|\right)}{3 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2+a*x),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(a*x^3 + b))/a
```

## 3.340 $\int \frac{1}{\frac{b}{x^3}+ax}\,dx$

**Optimal**. Leaf size=15

$$\frac{\log\left(ax^4+b\right)}{4a}$$

```
[Out] Log[b + a*x^4]/(4*a)
```

---

**Rubi [A]** time = 0.0053283, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{\log\left(ax^4+b\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^3 + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^4]/(4*a)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{b}{x^3}+ax}\,dx &= \int \frac{x^3}{b+ax^4}\,dx\\ &= \frac{\log\left(b+ax^4\right)}{4a}\end{aligned}$$

**Mathematica [A]** time = 0.0047679, size = 15, normalized size = 1.

$$\frac{\log\left(ax^4+b\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^3 + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^4]/(4*a)
```

---

**Maple [A]** time = 0.001, size = 14, normalized size = 0.9

$$\frac{\ln\left(a x^{4}+b\right)}{4 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3+a*x),x)
```

```
[Out] 1/4*ln(a*x^4+b)/a
```

---

**Maxima [A]** time = 1.11956, size = 18, normalized size = 1.2

$$\frac{\log\left(a x^{4}+b\right)}{4 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/4*log(a*x^4 + b)/a
```

---

**Fricas [A]** time = 0.705642, size = 30, normalized size = 2.

$$\frac{\log\left(a x^{4}+b\right)}{4 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(a*x^4 + b)/a
```

---

**Sympy [A]** time = 0.17412, size = 10, normalized size = 0.67

$$\frac{\log\left(a x^{4}+b\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**3+a*x),x)
```

```
[Out] log(a*x**4 + b)/(4*a)
```

---

**Giac [A]** time = 1.09987, size = 19, normalized size = 1.27

$$\frac{\log\left(\left|a x^{4}+b\right|\right)}{4 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(a*x^4 + b))/a
```

## 3.341 $\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx$

**Optimal**. Leaf size=19

$$\frac{x^4}{4b\left(ax^2+b\right)^2}$$

```
[Out] x^4/(4*b*(b + a*x^2)^2)
```

---

**Rubi [A]** time = 0.0054465, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 264}

$$\frac{x^4}{4b\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x + a*x)^(-3),x]
```

```
[Out] x^4/(4*b*(b + a*x^2)^2)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx &= \int \frac{x^3}{\left(b+ax^2\right)^3}\,dx\\ &= \frac{x^4}{4b\left(b+ax^2\right)^2}\end{aligned}$$

**Mathematica [A]** time = 0.0084775, size = 24, normalized size = 1.26

$$-\frac{2ax^2+b}{4a^2\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x + a*x)^(-3),x]
```

```
[Out] -(b + 2*a*x^2)/(4*a^2*(b + a*x^2)^2)
```

---

**Maple [A]** time = 0.007, size = 31, normalized size = 1.6

$$\frac{b}{4\,a^2\left(ax^2+b\right)^2}-\frac{1}{2\,a^2\left(ax^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x+a*x)^3,x)
```

```
[Out] 1/4*b/a^2/(a*x^2+b)^2-1/2/a^2/(a*x^2+b)
```

---

**Maxima [B]** time = 1.15915, size = 49, normalized size = 2.58

$$-\frac{2\,ax^2+b}{4\left(a^4x^4+2\,a^3bx^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

---

**Fricas [B]** time = 0.784184, size = 73, normalized size = 3.84

$$-\frac{2\,ax^2+b}{4\left(a^4x^4+2\,a^3bx^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

---

**Sympy [B]** time = 0.482015, size = 36, normalized size = 1.89

$$-\frac{2ax^2+b}{4a^4x^4+8a^3bx^2+4a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)**3,x)
```

```
[Out] -(2*a*x**2 + b)/(4*a**4*x**4 + 8*a**3*b*x**2 + 4*a**2*b**2)
```

**Giac [A]** time = 1.15051, size = 30, normalized size = 1.58

$$-\frac{2\,ax^2+b}{4\,(ax^2+b)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*a*x^2 + b)/((a*x^2 + b)^2*a^2)
```

## 3.342 $\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx$

**Optimal**. Leaf size=19

$$\frac{x^{10}}{10b\left(ax^5+b\right)^2}$$

```
[Out] x^10/(10*b*(b + a*x^5)^2)
```

---

**Rubi [A]** time = 0.006112, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 264}

$$\frac{x^{10}}{10b\left(ax^5+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^3 + a*x^2)^(-3),x]
```

```
[Out] x^10/(10*b*(b + a*x^5)^2)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx &= \int \frac{x^9}{\left(b+ax^5\right)^3}\,dx\\ &= \frac{x^{10}}{10b\left(b+ax^5\right)^2}\end{aligned}$$

**Mathematica [A]** time = 0.0087693, size = 24, normalized size = 1.26

$$-\frac{2ax^5+b}{10a^2\left(ax^5+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^3 + a*x^2)^(-3),x]
```

```
[Out] -(b + 2*a*x^5)/(10*a^2*(b + a*x^5)^2)
```

---

**Maple [A]** time = 0.004, size = 31, normalized size = 1.6

$$\frac{b}{10\,a^2\left(ax^5+b\right)^2}-\frac{1}{5\,a^2\left(ax^5+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3+a*x^2)^3,x)
```

```
[Out] 1/10*b/a^2/(a*x^5+b)^2-1/5/a^2/(a*x^5+b)
```

---

**Maxima [B]** time = 1.0571, size = 49, normalized size = 2.58

$$-\frac{2\,ax^5+b}{10\left(a^4x^{10}+2\,a^3bx^5+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)
```

---

**Fricas [B]** time = 0.631051, size = 76, normalized size = 4.

$$-\frac{2\,ax^5+b}{10\left(a^4x^{10}+2\,a^3bx^5+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)
```

---

**Sympy [B]** time = 5.42171, size = 36, normalized size = 1.89

$$-\frac{2ax^5+b}{10a^4x^{10}+20a^3bx^5+10a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**3+a*x**2)**3,x)
```

```
[Out] -(2*a*x**5 + b)/(10*a**4*x**10 + 20*a**3*b*x**5 + 10*a**2*b**2)
```

---

**Giac [A]** time = 1.1578, size = 30, normalized size = 1.58

$$-\frac{2\,ax^5+b}{10\,\left(ax^5+b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/10*(2*a*x^5 + b)/((a*x^5 + b)^2*a^2)
```

## 3.343 $\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx$

**Optimal**. Leaf size=19

$$\frac{x^{16}}{16b\left(ax^8+b\right)^2}$$

```
[Out] x^16/(16*b*(b + a*x^8)^2)
```

---

**Rubi [A]** time = 0.0057949, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 264}

$$\frac{x^{16}}{16b\left(ax^8+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^5 + a*x^3)^(-3),x]
```

```
[Out] x^16/(16*b*(b + a*x^8)^2)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx &= \int \frac{x^{15}}{\left(b+ax^8\right)^3}\,dx\\ &= \frac{x^{16}}{16b\left(b+ax^8\right)^2}\end{aligned}$$

**Mathematica [A]** time = 0.0111089, size = 24, normalized size = 1.26

$$-\frac{2ax^8+b}{16a^2\left(ax^8+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^5 + a*x^3)^(-3),x]
```

```
[Out] -(b + 2*a*x^8)/(16*a^2*(b + a*x^8)^2)
```

---

**Maple [A]** time = 0.008, size = 31, normalized size = 1.6

$$\frac{b}{16\,a^{2}\left(a x^{8}+b\right)^{2}}-\frac{1}{8\,a^{2}\left(a x^{8}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^5+a*x^3)^3,x)
```

```
[Out] 1/16*b/a^2/(a*x^8+b)^2-1/8/a^2/(a*x^8+b)
```

---

**Maxima [B]** time = 1.10908, size = 49, normalized size = 2.58

$$-\frac{2\,a x^{8}+b}{16\left(a^{4} x^{16}+2\,a^{3} b x^{8}+a^{2} b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(2*a*x^8 + b)/(a^4*x^16 + 2*a^3*b*x^8 + a^2*b^2)
```

---

**Fricas [B]** time = 0.820367, size = 76, normalized size = 4.

$$-\frac{2\,a x^{8}+b}{16\left(a^{4} x^{16}+2\,a^{3} b x^{8}+a^{2} b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*a*x^8 + b)/(a^4*x^16 + 2*a^3*b*x^8 + a^2*b^2)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**5+a*x**3)**3,x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.17288, size = 30, normalized size = 1.58

$$-\frac{2\,ax^8+b}{16\left(ax^8+b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="giac")
```

```
[Out] -1/16*(2*a*x^8 + b)/((a*x^8 + b)^2*a^2)
```

## 3.344 $\int \left(\frac{a}{x}+bx\right)^2 \, dx$

**Optimal**. Leaf size=24

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

---

**Rubi [A]** time = 0.0109929, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 270}

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a/x + b*x)^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{a}{x}+bx\right)^2 \, dx &= \int \frac{\left(a+bx^2\right)^2}{x^2} \, dx\\ &= \int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right) \, dx\\ &= -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}\end{aligned}$$

**Mathematica [A]** time = 0.001072, size = 24, normalized size = 1.

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x + b*x)^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

---

**Maple [A]** time = 0.003, size = 23, normalized size = 1.

$$-\frac{a^2}{x} + 2\,abx + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x)^2,x)
```

```
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

---

**Maxima [A]** time = 1.10582, size = 30, normalized size = 1.25

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

---

**Fricas [A]** time = 0.726116, size = 50, normalized size = 2.08

$$\frac{b^2x^4 + 6\,abx^2 - 3\,a^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

---

**Sympy [A]** time = 0.317238, size = 19, normalized size = 0.79

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

---

**Giac [A]** time = 1.17389, size = 30, normalized size = 1.25

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

## 3.345 $\int \left(\frac{a}{x}+bx\right)^3 \, dx$

**Optimal**. Leaf size=40

$$3a^2b\log(x)-\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}$$

```
[Out] -a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

---

**Rubi [A]** time = 0.0225053, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1593, 266, 43}

$$3a^2b\log(x)-\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a/x + b*x)^3,x]
```

```
[Out] -a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(\frac{a}{x}+bx\right)^3 \, dx &= \int \frac{\left(a+bx^2\right)^3}{x^3} \, dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)^3}{x^2} \, dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(3ab^2+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b^3x\right) dx, x, x^2\right)\\ &= -\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x)\end{aligned}$$

**Mathematica [A]** time = 0.0068147, size = 40, normalized size = 1.

$$3a^2b\log(x)-\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x + b*x)^3,x]
```

```
[Out] -a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

---

**Maple [A]** time = 0.007, size = 35, normalized size = 0.9

$$-\frac{a^3}{2\,x^2}+\frac{3\,ab^2x^2}{2}+\frac{b^3x^4}{4}+3\,a^2b\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x)^3,x)
```

```
[Out] -1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

---

**Maxima [A]** time = 1.15406, size = 46, normalized size = 1.15

$$\frac{1}{4}\,b^3x^4+\frac{3}{2}\,ab^2x^2+3\,a^2b\log\left(x\right)-\frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3*a^2*b*log(x) - 1/2*a^3/x^2
```

---

**Fricas [A]** time = 0.770119, size = 85, normalized size = 2.12

$$\frac{b^3x^6+6\,ab^2x^4+12\,a^2bx^2\log\left(x\right)-2\,a^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

---

**Sympy [A]** time = 0.419012, size = 37, normalized size = 0.92

$$-\frac{a^3}{2x^2}+3a^2b\log\left(x\right)+\frac{3ab^2x^2}{2}+\frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)**3,x)
```

```
[Out] -a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2 + b**3*x**4/4
```

---

**Giac [A]** time = 1.11189, size = 62, normalized size = 1.55

$$\frac{1}{4}\,b^{3}x^{4} + \frac{3}{2}\,ab^{2}x^{2} + \frac{3}{2}\,a^{2}b\log\left(x^{2}\right) - \frac{3\,a^{2}bx^{2} + a^{3}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*(3*a^2*b*x^2 + a^3)/
x^2
```

## 3.346 $\int \left(\frac{a}{x}+bx\right)^4 dx$

**Optimal**. Leaf size=50

$$6a^2b^2x-\frac{4a^3b}{x}-\frac{a^4}{3x^3}+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

```
[Out] -a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

---

**Rubi [A]** time = 0.0211109, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 270}

$$6a^2b^2x-\frac{4a^3b}{x}-\frac{a^4}{3x^3}+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a/x + b*x)^4,x]
```

```
[Out] -a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{a}{x}+bx\right)^4 dx &= \int \frac{\left(a+bx^2\right)^4}{x^4}\,dx\\ &= \int \left(6a^2b^2+\frac{a^4}{x^4}+\frac{4a^3b}{x^2}+4ab^3x^2+b^4x^4\right)dx\\ &= -\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}\end{aligned}$$

**Mathematica [A]** time = 0.0060251, size = 50, normalized size = 1.

$$6a^2b^2x-\frac{4a^3b}{x}-\frac{a^4}{3x^3}+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x + b*x)^4,x]
```

```
[Out] -a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

---

**Maple [A]** time = 0.006, size = 45, normalized size = 0.9

$$-\frac{a^4}{3x^3}-4\frac{a^3b}{x}+6a^2b^2x+\frac{4ab^3x^3}{3}+\frac{b^4x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x)^4,x)
```

```
[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

---

**Maxima [A]** time = 1.17249, size = 59, normalized size = 1.18

$$\frac{1}{5}b^4x^5+\frac{4}{3}ab^3x^3+6a^2b^2x-\frac{4a^3b}{x}-\frac{a^4}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^4,x, algorithm="maxima")
```

```
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 4*a^3*b/x - 1/3*a^4/x^3
```

---

**Fricas [A]** time = 0.757692, size = 104, normalized size = 2.08

$$\frac{3b^4x^8+20ab^3x^6+90a^2b^2x^4-60a^3bx^2-5a^4}{15x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^4*x^8 + 20*a*b^3*x^6 + 90*a^2*b^2*x^4 - 60*a^3*b*x^2 - 5*a^4)/x^3
```

---

**Sympy [A]** time = 0.350488, size = 48, normalized size = 0.96

$$6a^2b^2x+\frac{4ab^3x^3}{3}+\frac{b^4x^5}{5}-\frac{a^4+12a^3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)**4,x)
```

```
[Out] 6*a**2*b**2*x + 4*a*b**3*x**3/3 + b**4*x**5/5 - (a**4 + 12*a**3*b*x**2)/(3*
x**3)
```

---

**Giac [A]** time = 1.11432, size = 61, normalized size = 1.22

$$\frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{12\,a^3bx^2+a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^4,x, algorithm="giac")
```

```
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

## 3.347 $\int \frac{1}{\frac{1}{x^2}+x^3}\,dx$

**Optimal**. Leaf size=185

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+\frac{1}{5}\log(x+1)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\mathrm{ta}$$

```
[Out] -(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt[2/(5 + Sqrt
[5])]*x])/5 - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] - Sqrt[
(2*(5 + Sqrt[5]))/5]*x])/5 + Log[1 + x]/5 - ((1 + Sqrt[5])*Log[1 - ((1 - Sq
rt[5])*x)/2 + x^2])/20 - ((1 - Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])
/20
```

---

**Rubi [A]** time = 0.345543, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {1593, 293, 634, 618, 204, 628, 31}

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+\frac{1}{5}\log(x+1)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\mathrm{ta}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-2) + x^3)^(-1),x]
```

```
[Out] -(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt[2/(5 + Sqrt
[5])]*x])/5 - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] - Sqrt[
(2*(5 + Sqrt[5]))/5]*x])/5 + Log[1 + x]/5 - ((1 + Sqrt[5])*Log[1 - ((1 - Sq
rt[5])*x)/2 + x^2])/20 - ((1 - Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])
/20
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\frac{1}{x^2}+x^3}\,dx &= \int \frac{x^2}{1+x^5}\,dx\\
&= \frac{2}{5}\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)x}{1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx+\frac{2}{5}\int \frac{\frac{1}{4}\left(-1+\sqrt{5}\right)-\frac{1}{4}\left(1-\sqrt{5}\right)x}{1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx+\frac{1}{5}\int \frac{1}{1+x}\,dx\\
&= \frac{1}{5}\log(1+x)+\frac{\int \frac{1}{1+\frac{1}{2}\left(-1-\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}}-\frac{\int \frac{1}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}}+\frac{1}{20}\left(-1-\sqrt{5}\right)\int \frac{\frac{1}{2}\left(-1+\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2}\\
&= \frac{1}{5}\log(1+x)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(2-x-\sqrt{5}x+2x^2\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(2-x+\sqrt{5}x+2x^2\right)+\frac{\text{Subst}}{}\\
&= \sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}-4x\right)\right)+\frac{1}{5}\text{lo}
\end{aligned}
$$

**Mathematica [A]** time = 0.125276, size = 144, normalized size = 0.78

$$\frac{1}{20}\left(-\left(1+\sqrt{5}\right)\log\left(x^2+\frac{1}{2}\left(\sqrt{5}-1\right)x+1\right)+\left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+4\log(x+1)-2\sqrt{2\left(5+\sqrt{5}\right)}\,\text{ta}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^(-2) + x^3)^(-1),x]
```

```
[Out] (-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*Sqrt[5]]]
- 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]]
 + 4*Log[1 + x] - (1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2] + (-1 +
 Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

**Maple [A]** time = 0.019, size = 156, normalized size = 0.8

$$\frac{\ln(1+x)}{5}+\frac{\ln\left(-x\sqrt{5}+2x^2-x+2\right)\sqrt{5}}{20}-\frac{\ln\left(-x\sqrt{5}+2x^2-x+2\right)}{20}+\frac{2\sqrt{5}}{5\sqrt{10-2\sqrt{5}}}\arctan\left(\frac{-\sqrt{5}+4x-1}{\sqrt{10-2\sqrt{5}}}\right)-\frac{\ln\left(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^2+x^3),x)
```

```
[Out] 1/5*ln(1+x)+1/20*ln(-x*5^(1/2)+2*x^2-x+2)*5^(1/2)-1/20*ln(-x*5^(1/2)+2*x^2-
x+2)+2/5/(10-2*5^(1/2))^(1/2)*arctan((-5^(1/2)+4*x-1)/(10-2*5^(1/2))^(1/2))
*5^(1/2)-1/20*ln(x*5^(1/2)+2*x^2-x+2)*5^(1/2)-1/20*ln(x*5^(1/2)+2*x^2-x+2)-
2/5/(10+2*5^(1/2))^(1/2)*arctan((5^(1/2)+4*x-1)/(10+2*5^(1/2))^(1/2))*5^(1/
2)
```

---

**Maxima [A]** time = 1.54593, size = 167, normalized size = 0.9

$$-\frac{2\sqrt{5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}}+\frac{2\sqrt{5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}}+\frac{\log\left(2x^2-x\left(\sqrt{5}+1\right)+2\right)}{5\left(\sqrt{5}+1\right)}-\frac{\log\left(2x^2+x\left(\sqrt{5}-1\right)+2\right)}{5\left(\sqrt{5}-1\right)}+\frac{1}{5}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^2+x^3),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))/sqrt(2*sqrt(5
) + 10) + 2/5*sqrt(5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10))/sqr
t(-2*sqrt(5) + 10) + 1/5*log(2*x^2 - x*(sqrt(5) + 1) + 2)/(sqrt(5) + 1) - 1
/5*log(2*x^2 + x*(sqrt(5) - 1) + 2)/(sqrt(5) - 1) + 1/5*log(x + 1)
```

---

**Fricas [B]** time = 5.63503, size = 2210, normalized size = 11.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^2+x^3),x, algorithm="fricas")
```

```
[Out] -1/20*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(1/16*(2*sqrt(1/2)*s
qrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + x) + 1/20*(sqrt(5) + 2*sqrt(-3/16*(2*sq
rt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*sqrt(sqrt(5)
- 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 3/16*(2
*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(sqrt(5) - 5)
 + 1/2*sqrt(5) - 5/2) - 1)*log(-1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(
5) + 1)^2 - 1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 1/2*sqrt
(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*s
qrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) -
1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(
sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*(sqrt(5) - 1) + 2*x - 1) + 1/20*(sqrt(5)
- 2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqr
t(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sq
rt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/
```

```
2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2) - 1)*log(-1/16*(2*sqrt(1/2)*sqrt(
sqrt(5) - 5) + sqrt(5) + 1)^2 - 1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(
5) - 1)^2 - 1/2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2
+ 1/8*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)
^2 + sqrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*(sqrt(5) - 1) + 2*x -
 1) + 1/20*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(1/16*(2*sqrt(1
/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + x) + 1/5*log(x + 1)
```

---

**Sympy [A]** time = 1.6551, size = 36, normalized size = 0.19

$$\frac{\log(x+1)}{5} + \operatorname{RootSum}\left(625t^4 + 125t^3 + 25t^2 + 5t + 1, \left(t \mapsto t\log\left(25t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**2+x**3),x)
```

```
[Out] log(x + 1)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda(
_t, _t*log(25*_t**2 + x)))
```

---

**Giac [A]** time = 1.15905, size = 151, normalized size = 0.82

$$\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)-\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)-\frac{1}{10}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{4x+}{\sqrt{2\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^2+x^3),x, algorithm="giac")
```

```
[Out] 1/20*(sqrt(5) - 1)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) - 1/20*(sqrt(5) + 1)*
log(x^2 + 1/2*x*(sqrt(5) - 1) + 1) - 1/10*sqrt(-2*sqrt(5) + 10)*arctan((4*x
 + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) + 1/10*sqrt(2*sqrt(5) + 10)*arctan((4
*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10)) + 1/5*log(abs(x + 1))
```

## 3.348 $\int x^p \left(ax^n + bx^{1+13n+p}\right)^{12} dx$

**Optimal**. Leaf size=29

$$\frac{\left(a + bx^{12n+p+1}\right)^{13}}{13b(12n + p + 1)}$$

```
[Out] (a + b*x^(1 + 12*n + p))^13/(13*b*(1 + 12*n + p))
```

---

**Rubi [A]** time = 0.0172572, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1584, 261}

$$\frac{\left(a + bx^{12n+p+1}\right)^{13}}{13b(12n + p + 1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]
```

```
[Out] (a + b*x^(1 + 12*n + p))^13/(13*b*(1 + 12*n + p))
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int x^p \left(ax^n + bx^{1+13n+p}\right)^{12} dx &= \int x^{12n+p} \left(a + bx^{1+12n+p}\right)^{12} dx \\ &= \frac{\left(a + bx^{1+12n+p}\right)^{13}}{13b(1 + 12n + p)} \end{aligned}$$

**Mathematica [A]** time = 0.0155429, size = 29, normalized size = 1.

$$\frac{\left(a + bx^{12n+p+1}\right)^{13}}{13b(12n + p + 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]
```

```
[Out] (a + b*x^(1 + 12*n + p))^13/(13*b*(1 + 12*n + p))
```

---

**Maple [B]** time = 0.168, size = 363, normalized size = 12.5

$$\frac{(x^n)^{156}\,x^{13}b^{12}\,(x^p)^{13}}{13+156\,n+13\,p}+\frac{(x^n)^{144}\,x^{12}ab^{11}\,(x^p)^{12}}{1+12\,n+p}+6\,\frac{(x^n)^{132}\,x^{11}a^2b^{10}\,(x^p)^{11}}{1+12\,n+p}+22\,\frac{(x^n)^{120}\,x^{10}a^3b^9\,(x^p)^{10}}{1+12\,n+p}+55\,\frac{(x^n)^{108}\,x^9a^4b}{1+12\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*(a*x^n+b*x^(1+13*n+p))^12,x)
```

```
[Out] 1/13*b^12*x^13*(x^n)^156/(1+12*n+p)*(x^p)^13+a*b^11*x^12*(x^n)^144/(1+12*n+
p)*(x^p)^12+6*a^2*b^10*x^11*(x^n)^132/(1+12*n+p)*(x^p)^11+22*a^3*b^9*x^10*(
x^n)^120/(1+12*n+p)*(x^p)^10+55*a^4*b^8*x^9*(x^n)^108/(1+12*n+p)*(x^p)^9+99
*a^5*b^7*x^8*(x^n)^96/(1+12*n+p)*(x^p)^8+132*a^6*b^6*x^7*(x^n)^84/(1+12*n+p
)*(x^p)^7+132*a^7*b^5*x^6*(x^n)^72/(1+12*n+p)*(x^p)^6+99*a^8*b^4*x^5*(x^n)^
60/(1+12*n+p)*(x^p)^5+55*a^9*b^3*x^4*(x^n)^48/(1+12*n+p)*(x^p)^4+22*a^10*b^
2*x^3*(x^n)^36/(1+12*n+p)*(x^p)^3+6*a^11*b*x^2*(x^n)^24/(1+12*n+p)*(x^p)^2+
a^12/(1+12*n+p)*x*(x^n)^12*x^p
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [B]** time = 0.853519, size = 753, normalized size = 25.97

$$78\,a^2b^{10}x^{2\,n}x^{143\,n+11\,p+11}+286\,a^3b^9x^{3\,n}x^{130\,n+10\,p+10}+715\,a^4b^8x^{4\,n}x^{117\,n+9\,p+9}+1287\,a^5b^7x^{5\,n}x^{104\,n+8\,p+8}+1716\,a^6b^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(78*a^2*b^10*x^(2*n)*x^(143*n + 11*p + 11) + 286*a^3*b^9*x^(3*n)*x^(13
0*n + 10*p + 10) + 715*a^4*b^8*x^(4*n)*x^(117*n + 9*p + 9) + 1287*a^5*b^7*x
^(5*n)*x^(104*n + 8*p + 8) + 1716*a^6*b^6*x^(6*n)*x^(91*n + 7*p + 7) + 1716
*a^7*b^5*x^(7*n)*x^(78*n + 6*p + 6) + 1287*a^8*b^4*x^(8*n)*x^(65*n + 5*p +
5) + 715*a^9*b^3*x^(9*n)*x^(52*n + 4*p + 4) + 286*a^10*b^2*x^(10*n)*x^(39*n
 + 3*p + 3) + 78*a^11*b*x^(11*n)*x^(26*n + 2*p + 2) + 13*a^12*x^(12*n)*x^(1
3*n + p + 1) + 13*a*b^11*x^(156*n + 12*p + 12)*x^n + b^12*x^(169*n + 13*p +
 13))/((12*n + p + 1)*x^(13*n))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**p*(a*x**n+b*x**(1+13*n+p))**12,x)
```

```
[Out] Timed out
```

---

**Giac [B]** time = 4.31688, size = 363, normalized size = 12.52

$$b^{12}x^{13}x^{156\,n}x^{13\,p} + 13\,ab^{11}x^{12}x^{144\,n}x^{12\,p} + 78\,a^{2}b^{10}x^{11}x^{132\,n}x^{11\,p} + 286\,a^{3}b^{9}x^{10}x^{120\,n}x^{10\,p} + 715\,a^{4}b^{8}x^{9}x^{108\,n}x^{9\,p} + 1287\,a^{5}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="giac")
```

```
[Out] 1/13*(b^12*x^13*x^(156*n)*x^(13*p) + 13*a*b^11*x^12*x^(144*n)*x^(12*p) + 78
*a^2*b^10*x^11*x^(132*n)*x^(11*p) + 286*a^3*b^9*x^10*x^(120*n)*x^(10*p) + 7
15*a^4*b^8*x^9*x^(108*n)*x^(9*p) + 1287*a^5*b^7*x^8*x^(96*n)*x^(8*p) + 1716
*a^6*b^6*x^7*x^(84*n)*x^(7*p) + 1716*a^7*b^5*x^6*x^(72*n)*x^(6*p) + 1287*a^
8*b^4*x^5*x^(60*n)*x^(5*p) + 715*a^9*b^3*x^4*x^(48*n)*x^(4*p) + 286*a^10*b^
2*x^3*x^(36*n)*x^(3*p) + 78*a^11*b*x^2*x^(24*n)*x^(2*p) + 13*a^12*x*x^(12*n
)*x^p)/(12*n + p + 1)
```

## 3.349 $\int x^{12}\left(a+bx^{13}\right)^{12}\,dx$

**Optimal**. Leaf size=16

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

```
[Out] (a + b*x^13)^13/(169*b)
```

---

**Rubi [A]** time = 0.0029452, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a + b*x^13)^12,x]
```

```
[Out] (a + b*x^13)^13/(169*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{12}\left(a+bx^{13}\right)^{12}\,dx=\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

**Mathematica [B]** time = 0.0047348, size = 160, normalized size = 10.

$$\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}+\frac{132}{13}a^7b^5x^{78}+\frac{99}{13}a^8b^4x^{65}+\frac{55}{13}a^9b^3x^{52}+\frac{22}{13}a$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a + b*x^13)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

---

**Maple [B]** time = 0., size = 135, normalized size = 8.4

$$\frac{b^{12}x^{169}}{169}+\frac{b^{11}ax^{156}}{13}+\frac{6\,b^{10}a^2x^{143}}{13}+\frac{22\,a^3b^9x^{130}}{13}+\frac{55\,a^4b^8x^{117}}{13}+\frac{99\,a^5b^7x^{104}}{13}+\frac{132\,a^6b^6x^{91}}{13}+\frac{132\,a^7b^5x^{78}}{13}+\frac{99\,a^8b^4}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^13+a)^12,x)
```

```
[Out] 1/169*b^12*x^169+1/13*b^11*a*x^156+6/13*b^10*a^2*x^143+22/13*a^3*b^9*x^130+
55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^5*
x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*a^11*b*
x^26+1/13*a^12*x^13
```

---

**Maxima [A]** time = 1.0264, size = 19, normalized size = 1.19

$$\frac{\left(bx^{13}+a\right)^{13}}{169\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="maxima")
```

```
[Out] 1/169*(b*x^13 + a)^13/b
```

---

**Fricas [B]** time = 0.612211, size = 365, normalized size = 22.81

$$\frac{1}{169}x^{169}b^{12}+\frac{1}{13}x^{156}b^{11}a+\frac{6}{13}x^{143}b^{10}a^{2}+\frac{22}{13}x^{130}b^{9}a^{3}+\frac{55}{13}x^{117}b^{8}a^{4}+\frac{99}{13}x^{104}b^{7}a^{5}+\frac{132}{13}x^{91}b^{6}a^{6}+\frac{132}{13}x^{78}b^{5}a^{7}+\frac{99}{13}x^{65}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="fricas")
```

```
[Out] 1/169*x^169*b^12 + 1/13*x^156*b^11*a + 6/13*x^143*b^10*a^2 + 22/13*x^130*b^
9*a^3 + 55/13*x^117*b^8*a^4 + 99/13*x^104*b^7*a^5 + 132/13*x^91*b^6*a^6 + 1
32/13*x^78*b^5*a^7 + 99/13*x^65*b^4*a^8 + 55/13*x^52*b^3*a^9 + 22/13*x^39*b
^2*a^10 + 6/13*x^26*b*a^11 + 1/13*x^13*a^12
```

---

**Sympy [B]** time = 0.122232, size = 160, normalized size = 10.

$$\frac{a^{12}x^{13}}{13}+\frac{6a^{11}bx^{26}}{13}+\frac{22a^{10}b^{2}x^{39}}{13}+\frac{55a^{9}b^{3}x^{52}}{13}+\frac{99a^{8}b^{4}x^{65}}{13}+\frac{132a^{7}b^{5}x^{78}}{13}+\frac{132a^{6}b^{6}x^{91}}{13}+\frac{99a^{5}b^{7}x^{104}}{13}+\frac{55a^{4}b^{8}x^{117}}{13}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**13+a)**12,x)
```

```
[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3
*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*
x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x
**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169
```

---

**Giac [A]** time = 1.21248, size = 19, normalized size = 1.19

$$\frac{\left(bx^{13} + a\right)^{13}}{169\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="giac")
```

```
[Out] 1/169*(b*x^13 + a)^13/b
```

## 3.350 $\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx$

**Optimal**. Leaf size=16

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

```
[Out] (a + b*x^25)^13/(325*b)
```

---

**Rubi [A]** time = 0.0086186, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 261}

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx &= \int x^{24}\left(a+bx^{25}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{25}\right)^{13}}{325b}\end{aligned}$$

**Mathematica [B]** time = 0.0060576, size = 160, normalized size = 10.

$$\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^6b^6x^{175}+\frac{132}{25}a^7b^5x^{150}+\frac{99}{25}a^8b^4x^{125}+\frac{11}{5}a^9b^3x^{100}+\frac{22}{25}a$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
```

```
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

---

**Maple [B]** time = 0., size = 135, normalized size = 8.4

$$\frac{b^{12}x^{325}}{325}+\frac{b^{11}ax^{300}}{25}+\frac{6\,b^{10}a^{2}x^{275}}{25}+\frac{22\,a^{3}b^{9}x^{250}}{25}+\frac{11\,a^{4}b^{8}x^{225}}{5}+\frac{99\,a^{5}b^{7}x^{200}}{25}+\frac{132\,a^{6}b^{6}x^{175}}{25}+\frac{132\,a^{7}b^{5}x^{150}}{25}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^26+a*x)^12,x)
```

```
[Out] 1/325*b^12*x^325+1/25*b^11*a*x^300+6/25*b^10*a^2*x^275+22/25*a^3*b^9*x^250+
11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*
x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*
b*x^50+1/25*a^12*x^25
```

---

**Maxima [B]** time = 1.05386, size = 181, normalized size = 11.31

$$\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

---

**Fricas [B]** time = 0.619714, size = 367, normalized size = 22.94

$$\frac{1}{325}x^{325}b^{12}+\frac{1}{25}x^{300}b^{11}a+\frac{6}{25}x^{275}b^{10}a^{2}+\frac{22}{25}x^{250}b^{9}a^{3}+\frac{11}{5}x^{225}b^{8}a^{4}+\frac{99}{25}x^{200}b^{7}a^{5}+\frac{132}{25}x^{175}b^{6}a^{6}+\frac{132}{25}x^{150}b^{5}a^{7}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^
9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 1
32/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75
*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12
```

---

**Sympy [B]** time = 0.151775, size = 160, normalized size = 10.

$$\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**26+a*x)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

---

**Giac [B]** time = 1.1787, size = 181, normalized size = 11.31

$$\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

## 3.351 $\int x^{12} \left(ax^2+bx^{39}\right)^{12} \, dx$

**Optimal**. Leaf size=16

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

```
[Out] (a + b*x^37)^13/(481*b)
```

---

**Rubi [A]** time = 0.0119976, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 261}

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a*x^2 + b*x^39)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x^{12}\left(ax^2+bx^{39}\right)^{12} dx &= \int x^{36}\left(a+bx^{37}\right)^{12} dx\\ &= \frac{\left(a+bx^{37}\right)^{13}}{481b}\end{aligned}$$

**Mathematica [B]** time = 0.0071078, size = 160, normalized size = 10.

$$\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8b^4x^{185}+\frac{55}{37}a^9b^3x^{148}+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a*x^2 + b*x^39)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
```

```
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

---

**Maple [B]** time = 0.001, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{481}}{481}+\frac{b^{11}ax^{444}}{37}+\frac{6\,b^{10}a^{2}x^{407}}{37}+\frac{22\,a^{3}b^{9}x^{370}}{37}+\frac{55\,a^{4}b^{8}x^{333}}{37}+\frac{99\,a^{5}b^{7}x^{296}}{37}+\frac{132\,a^{6}b^{6}x^{259}}{37}+\frac{132\,a^{7}b^{5}x^{222}}{37}+\frac{99\,a^{8}b^{4}x}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^39+a*x^2)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*b^11*a*x^444+6/37*b^10*a^2*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

---

**Maxima [B]** time = 1.06426, size = 181, normalized size = 11.31

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

---

**Fricas [B]** time = 0.58102, size = 371, normalized size = 23.19

$$\frac{1}{481}x^{481}b^{12}+\frac{1}{37}x^{444}b^{11}a+\frac{6}{37}x^{407}b^{10}a^{2}+\frac{22}{37}x^{370}b^{9}a^{3}+\frac{55}{37}x^{333}b^{8}a^{4}+\frac{99}{37}x^{296}b^{7}a^{5}+\frac{132}{37}x^{259}b^{6}a^{6}+\frac{132}{37}x^{222}b^{5}a^{7}+\frac{99}{37}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="fricas")
```

```
[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

---

**Sympy [B]** time = 0.126827, size = 160, normalized size = 10.

$$\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^{2}x^{111}}{37}+\frac{55a^{9}b^{3}x^{148}}{37}+\frac{99a^{8}b^{4}x^{185}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{55a^{4}b^{8}x^{33}}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**39+a*x**2)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

---

**Giac [B]** time = 1.19325, size = 181, normalized size = 11.31

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

## 3.352 $\int x^{24} \left(a+bx^{25}\right)^{12} \, dx$

**Optimal**. Leaf size=16

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

```
[Out] (a + b*x^25)^13/(325*b)
```

---

**Rubi [A]** time = 0.0027836, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[x^24*(a + b*x^25)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{24} \left(a+bx^{25}\right)^{12} \, dx = \frac{\left(a+bx^{25}\right)^{13}}{325b}$$

**Mathematica [B]** time = 0.0042827, size = 160, normalized size = 10.

$$\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^6b^6x^{175}+\frac{132}{25}a^7b^5x^{150}+\frac{99}{25}a^8b^4x^{125}+\frac{11}{5}a^9b^3x^{100}+\frac{22}{25}a$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24*(a + b*x^25)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

---

**Maple [B]** time = 0.001, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{325}}{325}+\frac{b^{11}ax^{300}}{25}+\frac{6\,b^{10}a^2x^{275}}{25}+\frac{22\,a^3b^9x^{250}}{25}+\frac{11\,a^4b^8x^{225}}{5}+\frac{99\,a^5b^7x^{200}}{25}+\frac{132\,a^6b^6x^{175}}{25}+\frac{132\,a^7b^5x^{150}}{25}+\frac{99\,a^8b^4x}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(b*x^25+a)^12,x)
```

```
[Out] 1/325*b^12*x^325+1/25*b^11*a*x^300+6/25*b^10*a^2*x^275+22/25*a^3*b^9*x^250+
11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*
x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*
b*x^50+1/25*a^12*x^25
```

---

**Maxima [A]** time = 1.10246, size = 19, normalized size = 1.19

$$\frac{\left(bx^{25}+a\right)^{13}}{325\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="maxima")
```

```
[Out] 1/325*(b*x^25 + a)^13/b
```

---

**Fricas [B]** time = 0.680386, size = 367, normalized size = 22.94

$$\frac{1}{325}x^{325}b^{12}+\frac{1}{25}x^{300}b^{11}a+\frac{6}{25}x^{275}b^{10}a^2+\frac{22}{25}x^{250}b^9a^3+\frac{11}{5}x^{225}b^8a^4+\frac{99}{25}x^{200}b^7a^5+\frac{132}{25}x^{175}b^6a^6+\frac{132}{25}x^{150}b^5a^7+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="fricas")
```

```
[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^
9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 1
32/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75
*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12
```

---

**Sympy [B]** time = 0.121134, size = 160, normalized size = 10.

$$\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^2x^{75}}{25}+\frac{11a^9b^3x^{100}}{5}+\frac{99a^8b^4x^{125}}{25}+\frac{132a^7b^5x^{150}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{99a^5b^7x^{200}}{25}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24*(b*x**25+a)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

---

**Giac [A]** time = 1.20377, size = 19, normalized size = 1.19

$$\frac{\left(bx^{25}+a\right)^{13}}{325\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="giac")
```

```
[Out] 1/325*(b*x^25 + a)^13/b
```

## 3.353 $\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx$

**Optimal**. Leaf size=16

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

```
[Out] (a + b*x^37)^13/(481*b)
```

---

**Rubi [A]** time = 0.0083321, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 261}

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx&=\int x^{36}\left(a+bx^{37}\right)^{12}\,dx\\&=\frac{\left(a+bx^{37}\right)^{13}}{481b}\end{aligned}$$

**Mathematica [B]** time = 0.0057769, size = 160, normalized size = 10.

$$\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8b^4x^{185}+\frac{55}{37}a^9b^3x^{148}+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
```

```
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

---

**Maple [B]** time = 0.002, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{481}}{481}+\frac{b^{11}ax^{444}}{37}+\frac{6\,b^{10}a^{2}x^{407}}{37}+\frac{22\,a^{3}b^{9}x^{370}}{37}+\frac{55\,a^{4}b^{8}x^{333}}{37}+\frac{99\,a^{5}b^{7}x^{296}}{37}+\frac{132\,a^{6}b^{6}x^{259}}{37}+\frac{132\,a^{7}b^{5}x^{222}}{37}+\frac{99\,a^{8}b^{4}x}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(b*x^38+a*x)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*b^11*a*x^444+6/37*b^10*a^2*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

---

**Maxima [B]** time = 1.01604, size = 181, normalized size = 11.31

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

---

**Fricas [B]** time = 0.644098, size = 371, normalized size = 23.19

$$\frac{1}{481}x^{481}b^{12}+\frac{1}{37}x^{444}b^{11}a+\frac{6}{37}x^{407}b^{10}a^{2}+\frac{22}{37}x^{370}b^{9}a^{3}+\frac{55}{37}x^{333}b^{8}a^{4}+\frac{99}{37}x^{296}b^{7}a^{5}+\frac{132}{37}x^{259}b^{6}a^{6}+\frac{132}{37}x^{222}b^{5}a^{7}+\frac{99}{37}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

---

**Sympy [B]** time = 0.145064, size = 160, normalized size = 10.

$$\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^{2}x^{111}}{37}+\frac{55a^{9}b^{3}x^{148}}{37}+\frac{99a^{8}b^{4}x^{185}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{55a^{4}b^{8}x^{33}}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24*(b*x**38+a*x)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

---

**Giac [B]** time = 1.17673, size = 181, normalized size = 11.31

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

## 3.354 $\int x^{36} \left(a + bx^{37}\right)^{12} \, dx$

**Optimal**. Leaf size=16

$$\frac{\left(a + bx^{37}\right)^{13}}{481b}$$

```
[Out] (a + b*x^37)^13/(481*b)
```

---

**Rubi [A]** time = 0.0029089, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a + bx^{37}\right)^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[x^36*(a + b*x^37)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{36} \left(a + bx^{37}\right)^{12} \, dx = \frac{\left(a + bx^{37}\right)^{13}}{481b}$$

**Mathematica [B]** time = 0.0058375, size = 160, normalized size = 10.

$$\frac{6}{37}a^2b^{10}x^{407} + \frac{22}{37}a^3b^9x^{370} + \frac{55}{37}a^4b^8x^{333} + \frac{99}{37}a^5b^7x^{296} + \frac{132}{37}a^6b^6x^{259} + \frac{132}{37}a^7b^5x^{222} + \frac{99}{37}a^8b^4x^{185} + \frac{55}{37}a^9b^3x^{148} + \frac{22}{37}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^36*(a + b*x^37)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

---

**Maple [B]** time = 0.001, size = 135, normalized size = 8.4

$$\frac{b^{12}x^{481}}{481} + \frac{b^{11}ax^{444}}{37} + \frac{6\,b^{10}a^2x^{407}}{37} + \frac{22\,a^3b^9x^{370}}{37} + \frac{55\,a^4b^8x^{333}}{37} + \frac{99\,a^5b^7x^{296}}{37} + \frac{132\,a^6b^6x^{259}}{37} + \frac{132\,a^7b^5x^{222}}{37} + \frac{99\,a^8b^4x}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^36*(b*x^37+a)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*b^11*a*x^444+6/37*b^10*a^2*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

---

**Maxima [A]** time = 0.981064, size = 19, normalized size = 1.19

$$\frac{\left(bx^{37} + a\right)^{13}}{481\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="maxima")
```

```
[Out] 1/481*(b*x^37 + a)^13/b
```

---

**Fricas [B]** time = 0.589219, size = 371, normalized size = 23.19

$$\frac{1}{481}x^{481}b^{12} + \frac{1}{37}x^{444}b^{11}a + \frac{6}{37}x^{407}b^{10}a^2 + \frac{22}{37}x^{370}b^9a^3 + \frac{55}{37}x^{333}b^8a^4 + \frac{99}{37}x^{296}b^7a^5 + \frac{132}{37}x^{259}b^6a^6 + \frac{132}{37}x^{222}b^5a^7 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="fricas")
```

```
[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

---

**Sympy [B]** time = 0.133362, size = 160, normalized size = 10.

$$\frac{a^{12}x^{37}}{37} + \frac{6a^{11}bx^{74}}{37} + \frac{22a^{10}b^2x^{111}}{37} + \frac{55a^9b^3x^{148}}{37} + \frac{99a^8b^4x^{185}}{37} + \frac{132a^7b^5x^{222}}{37} + \frac{132a^6b^6x^{259}}{37} + \frac{99a^5b^7x^{296}}{37} + \frac{55a^4b^8}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**36*(b*x**37+a)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

---

**Giac [A]** time = 1.18695, size = 19, normalized size = 1.19

$$\frac{\left(bx^{37}+a\right)^{13}}{481\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="giac")
```

```
[Out] 1/481*(b*x^37 + a)^13/b
```

## 3.355 $\int \frac{1}{ax+bx^n}\,dx$

**Optimal**. Leaf size=23

$$\frac{\log\left(ax^{1-n}+b\right)}{a(1-n)}$$

```
[Out] Log[b + a*x^(1 - n)]/(a*(1 - n))
```

---

**Rubi [A]** time = 0.0108914, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{\log\left(ax^{1-n}+b\right)}{a(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^n)^(-1),x]
```

```
[Out] Log[b + a*x^(1 - n)]/(a*(1 - n))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+bx^n}\,dx &= \int \frac{x^{-n}}{b+ax^{1-n}}\,dx\\ &= \frac{\log\left(b+ax^{1-n}\right)}{a(1-n)}\end{aligned}$$

**Mathematica [A]** time = 0.0081442, size = 23, normalized size = 1.

$$\frac{\log\left(ax^{1-n}+b\right)}{a(1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^n)^(-1),x]
```

```
[Out] Log[b + a*x^(1 - n)]/(a*(1 - n))
```

---

**Maple [A]** time = 0.01, size = 36, normalized size = 1.6

$$\frac{n\ln(x)}{a(-1+n)}-\frac{\ln\left(ax+b\mathrm{e}^{n\ln(x)}\right)}{a(-1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^n),x)
```

```
[Out] n/a/(-1+n)*ln(x)-1/a/(-1+n)*ln(a*x+b*exp(n*ln(x)))
```

**Maxima [A]** time = 1.00649, size = 50, normalized size = 2.17

$$\frac{n\log(x)}{a(n-1)}-\frac{\log\left(\frac{ax+bx^n}{b}\right)}{a(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="maxima")
```

```
[Out] n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n - 1))
```

**Fricas [A]** time = 0.930557, size = 55, normalized size = 2.39

$$\frac{n\log(x)-\log(ax+bx^n)}{an-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="fricas")
```

```
[Out] (n*log(x) - log(a*x + b*x^n))/(a*n - a)
```

**Sympy [A]** time = 0.693469, size = 48, normalized size = 2.09

$$\begin{cases}\frac{\log(x)}{b} & \text{for } a=0\wedge n=1\\ -\frac{x}{b(nx^n-x^n)} & \text{for } a=0\\ \frac{\log(x)}{a+b} & \text{for } n=1\\ \frac{n\log(x)}{an-a}-\frac{\log\left(x+\frac{bx^n}{a}\right)}{an-a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x**n),x)
```

```
[Out] Piecewise((log(x)/b, Eq(a, 0) & Eq(n, 1)), (-x/(b*(n*x**n - x**n)), Eq(a, 0
)), (log(x)/(a + b), Eq(n, 1)), (n*log(x)/(a*n - a) - log(x + b*x**n/a)/(a*
n - a), True))
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ax + bx^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x + b*x^n), x)
```

## 3.356 $\int \frac{1}{ax+bx^{1+n}}\,dx$

**Optimal**. Leaf size=23

$$\frac{\log(x)}{a}-\frac{\log(a+bx^n)}{an}$$

```
[Out] Log[x]/a - Log[a + b*x^n]/(a*n)
```

---

**Rubi [A]** time = 0.0131801, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log(a+bx^n)}{an}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^(1 + n))^(-1),x]
```

```
[Out] Log[x]/a - Log[a + b*x^n]/(a*n)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{ax+bx^{1+n}}\,dx &= \int \frac{1}{x\left(a+bx^n\right)}\,dx \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,x^n\right)}{n} \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{an}-\frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^n\right)}{an} \\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx^n\right)}{an}
\end{aligned}
$$

**Mathematica [A]** time = 0.0062417, size = 22, normalized size = 0.96

$$\frac{n\log(x)-\log\left(a+bx^n\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^(1 + n))^(-1),x]
```

```
[Out] (n*Log[x] - Log[a + b*x^n])/(a*n)
```

---

**Maple [A]** time = 0.008, size = 39, normalized size = 1.7

$$\frac{\ln(x)}{an}+\frac{\ln(x)}{a}-\frac{\ln\left(ax+b\mathrm{e}^{(1+n)\ln(x)}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(1+n)),x)
```

```
[Out] 1/a/n*ln(x)+ln(x)/a-1/a/n*ln(a*x+b*exp((1+n)*ln(x)))
```

---

**Maxima [A]** time = 1.03796, size = 36, normalized size = 1.57

$$\frac{\log(x)}{a}-\frac{\log\left(\frac{bx^n+a}{b}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="maxima")
```

```
[Out] log(x)/a - log((b*x^n + a)/b)/(a*n)
```

---

**Fricas [A]** time = 0.877325, size = 66, normalized size = 2.87

$$\frac{(n+1)\log(x)-\log\left(ax+bx^{n+1}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="fricas")
```

```
[Out] ((n + 1)*log(x) - log(a*x + b*x^(n + 1)))/(a*n)
```

---

**Sympy [A]** time = 1.71174, size = 41, normalized size = 1.78

$$\begin{cases} \tilde{\infty} \log{(x)} & \text{for}\: a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log{(x)}}{a} & \text{for}\: b = 0 \\ \frac{\log{(x)}}{a+b} & \text{for}\: n = 0 \\ -\frac{x^{-n}}{bn} & \text{for}\: a = 0 \\ \frac{\log{(x)}}{a} - \frac{\log{\left(\frac{a}{b} + x^{n} \right)}}{an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x**(1+n)),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a, Eq(b, 0)
), (log(x)/(a + b), Eq(n, 0)), (-x**(-n)/(b*n), Eq(a, 0)), (log(x)/a - log(
a/b + x**n)/(a*n), True))
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ax + bx^{n+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x + b*x^(n + 1)), x)
```

## 3.357 $\int \frac{1}{ax+bx^{1-n}}\,dx$

**Optimal**. Leaf size=15

$$\frac{\log\left(ax^n+b\right)}{an}$$

```
[Out] Log[b + a*x^n]/(a*n)
```

---

**Rubi [A]** time = 0.0076386, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 260}

$$\frac{\log\left(ax^n+b\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^(1 - n))^(-1),x]
```

```
[Out] Log[b + a*x^n]/(a*n)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+bx^{1-n}}\,dx &= \int \frac{x^{-1+n}}{b+ax^n}\,dx\\ &= \frac{\log\left(b+ax^n\right)}{an}\end{aligned}$$

**Mathematica [A]** time = 0.0044175, size = 15, normalized size = 1.

$$\frac{\log\left(ax^n+b\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^(1 - n))^(-1),x]
```

```
[Out] Log[b + a*x^n]/(a*n)
```

---

**Maple [B]** time = 0.01, size = 41, normalized size = 2.7

$$-\frac{\ln(x)}{an}+\frac{\ln(x)}{a}+\frac{\ln\left(ax+b\mathrm{e}^{(1-n)\ln(x)}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(1-n)),x)
```

```
[Out] -1/a/n*ln(x)+ln(x)/a+1/a/n*ln(a*x+b*exp((1-n)*ln(x)))
```

---

**Maxima [A]** time = 1.02379, size = 26, normalized size = 1.73

$$\frac{\log\left(\frac{ax^n+b}{a}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="maxima")
```

```
[Out] log((a*x^n + b)/a)/(a*n)
```

---

**Fricas [A]** time = 0.90674, size = 68, normalized size = 4.53

$$\frac{(n-1)\log(x)+\log\left(ax+bx^{-n+1}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="fricas")
```

```
[Out] ((n - 1)*log(x) + log(a*x + b*x^(-n + 1)))/(a*n)
```

---

**Sympy [A]** time = 1.99376, size = 39, normalized size = 2.6

$$\begin{cases} \tilde{\infty}\log(x) & \text{for}\: a=0\wedge b=0\wedge n=0 \\ \frac{\log(x)}{a} & \text{for}\: b=0 \\ \frac{\log(x)}{a+b} & \text{for}\: n=0 \\ \frac{x^n}{bn} & \text{for}\: a=0 \\ \frac{\log(x)}{a}+\frac{\log\left(\frac{a}{b}+x^{-n}\right)}{an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x**(1-n)),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/a, Eq(b, 0)
), (log(x)/(a + b), Eq(n, 0)), (x**n/(b*n), Eq(a, 0)), (log(x)/a + log(a/b
+ x**(-n))/(a*n), True))
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ax + bx^{-n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x + b*x^(-n + 1)), x)
```

## 3.358 $\int \frac{1}{2x+3x^{1+n}} \, dx$

**Optimal**. Leaf size=22

$$\frac{\log(x)}{2} - \frac{\log(3x^n + 2)}{2n}$$

```
[Out] Log[x]/2 - Log[2 + 3*x^n]/(2*n)
```

---

**Rubi [A]** time = 0.0097024, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{2} - \frac{\log(3x^n + 2)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + 3*x^(1 + n))^(-1),x]

[Out] Log[x]/2 - Log[2 + 3*x^n]/(2*n)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2x+3x^{1+n}}\,dx &= \int \frac{1}{x(2+3x^n)}\,dx\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x(2+3x)}\,dx,x,x^n\right)}{n}\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{2n}-\frac{3\operatorname{Subst}\left(\int \frac{1}{2+3x}\,dx,x,x^n\right)}{2n}\\ &= \frac{\log(x)}{2}-\frac{\log(2+3x^n)}{2n}\end{aligned}$$

**Mathematica [A]** time = 0.0059449, size = 22, normalized size = 1.

$$\frac{n\log(x)-\log(3x^n+2)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + 3*x^(1 + n))^(-1),x]
```

```
[Out] (n*Log[x] - Log[2 + 3*x^n])/(2*n)
```

---

**Maple [A]** time = 0.028, size = 32, normalized size = 1.5

$$\frac{\ln(x)}{2n}+\frac{\ln(x)}{2}-\frac{\ln\left(2x+3\,\mathrm{e}^{(1+n)\ln(x)}\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+3*x^(1+n)),x)
```

```
[Out] 1/2/n*ln(x)+1/2*ln(x)-1/2/n*ln(2*x+3*exp((1+n)*ln(x)))
```

---

**Maxima [A]** time = 1.11082, size = 22, normalized size = 1.

$$-\frac{\log\left(x^n+\frac{2}{3}\right)}{2n}+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1+n)),x, algorithm="maxima")
```

```
[Out] -1/2*log(x^n + 2/3)/n + 1/2*log(x)
```

---

**Fricas [A]** time = 0.831635, size = 66, normalized size = 3.

$$\frac{(n+1)\log(x)-\log\left(3x^{n+1}+2x\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1+n)),x, algorithm="fricas")
```

```
[Out] 1/2*((n + 1)*log(x) - log(3*x^(n + 1) + 2*x))/n
```

---

**Sympy [A]** time = 1.989, size = 20, normalized size = 0.91

$$\begin{cases} \frac{\log(x)}{2} - \frac{\log\left(x^n + \frac{2}{3}\right)}{2n} & \text{for } n \neq 0 \\ \frac{\log(x)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x**(1+n)),x)
```

```
[Out] Piecewise((log(x)/2 - log(x**n + 2/3)/(2*n), Ne(n, 0)), (log(x)/5, True))
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{3\,x^{n+1} + 2\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1+n)),x, algorithm="giac")
```

```
[Out] integrate(1/(3*x^(n + 1) + 2*x), x)
```

## 3.359 $\int \frac{1}{2x+3x^{1-n}}\,dx$

**Optimal**. Leaf size=15

$$\frac{\log(2x^n+3)}{2n}$$

```
[Out] Log[3 + 2*x^n]/(2*n)
```

---

**Rubi [A]** time = 0.0063023, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 260}

$$\frac{\log(2x^n+3)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + 3*x^(1 - n))^(-1),x]
```

```
[Out] Log[3 + 2*x^n]/(2*n)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2x+3x^{1-n}}\,dx &= \int \frac{x^{-1+n}}{3+2x^n}\,dx\\ &= \frac{\log(3+2x^n)}{2n}\end{aligned}$$

**Mathematica [A]** time = 0.0033049, size = 15, normalized size = 1.

$$\frac{\log(2x^n+3)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + 3*x^(1 - n))^(-1),x]
```

```
[Out] Log[3 + 2*x^n]/(2*n)
```

---

**Maple [B]** time = 0.009, size = 34, normalized size = 2.3

$$-\frac{\ln\left(x\right)}{2\,n}+\frac{\ln\left(x\right)}{2}+\frac{\ln\left(2\,x+3\,{\mathrm e}^{\left(1-n\right)\ln\left(x\right)}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+3*x^(1-n)),x)
```

```
[Out] -1/2/n*ln(x)+1/2*ln(x)+1/2/n*ln(2*x+3*exp((1-n)*ln(x)))
```

---

**Maxima [A]** time = 1.04812, size = 15, normalized size = 1.

$$\frac{\log\left(x^{n}+\frac{3}{2}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^n + 3/2)/n
```

---

**Fricas [A]** time = 0.722041, size = 68, normalized size = 4.53

$$\frac{\left(n-1\right)\log\left(x\right)+\log\left(3\,x^{-n+1}+2\,x\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="fricas")
```

```
[Out] 1/2*((n - 1)*log(x) + log(3*x^(-n + 1) + 2*x))/n
```

---

**Sympy [A]** time = 1.64235, size = 22, normalized size = 1.47

$$\begin{cases}\frac{\log{\left(x\right)}}{2}+\frac{\log{\left(\frac{2}{3}+x^{-n}\right)}}{2n} & \text{for}\: n\neq 0\\\frac{\log{\left(x\right)}}{5} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x**(1-n)),x)
```

```
[Out] Piecewise((log(x)/2 + log(2/3 + x**(-n))/(2*n), Ne(n, 0)), (log(x)/5, True)
)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{3\,x^{-n+1}+2\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="giac")
```

```
[Out] integrate(1/(3*x^(-n + 1) + 2*x), x)
```

## 3.360 $\int \frac{1}{-\sqrt{x}+x} \, dx$

**Optimal**. Leaf size=12

$$2 \log \left(1-\sqrt{x}\right)$$

```
[Out] 2*Log[1 - Sqrt[x]]
```

---

**Rubi [A]** time = 0.003826, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$2 \log \left(1-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[x] + x)^(-1),x]
```

```
[Out] 2*Log[1 - Sqrt[x]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-\sqrt{x}+x} \, dx &= \int \frac{1}{\left(-1+\sqrt{x}\right) \sqrt{x}} \, dx \\
&= 2 \log \left(1-\sqrt{x}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0022968, size = 12, normalized size = 1.

$$2 \log \left(1-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[x] + x)^(-1),x]
```

```
[Out] 2*Log[1 - Sqrt[x]]
```

---

**Maple [A]** time = 0.003, size = 12, normalized size = 1.

$$\ln (-1+x) - 2 \operatorname{Artanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-x^(1/2)),x)
```

```
[Out] ln(-1+x)-2*arctanh(x^(1/2))
```

---

**Maxima [A]** time = 1.04106, size = 11, normalized size = 0.92

$$2 \log \left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*log(sqrt(x) - 1)
```

---

**Fricas [A]** time = 0.82284, size = 27, normalized size = 2.25

$$2 \log \left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*log(sqrt(x) - 1)
```

---

**Sympy [A]** time = 0.152274, size = 8, normalized size = 0.67

$$2 \log \left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x**(1/2)),x)
```

```
[Out] 2*log(sqrt(x) - 1)
```

---

**Giac [A]** time = 1.17205, size = 12, normalized size = 1.

$$2 \log \left(\left|\sqrt{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*log(abs(sqrt(x) - 1))
```

## 3.361 $\int \frac{1}{-x^{3/5}+x}\,dx$

**Optimal**. Leaf size=14

$$\frac{5}{2}\log\left(1-x^{2/5}\right)$$

```
[Out] (5*Log[1 - x^(2/5)])/2
```

---

**Rubi [A]** time = 0.0046696, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{5}{2}\log\left(1-x^{2/5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(3/5) + x)^(-1),x]

[Out] (5*Log[1 - x^(2/5)])/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-x^{3/5}+x}\,dx &= \int \frac{1}{\left(-1+x^{2/5}\right)x^{3/5}}\,dx\\ &= \frac{5}{2}\log\left(1-x^{2/5}\right)\end{aligned}$$

**Mathematica [A]** time = 0.0024372, size = 14, normalized size = 1.

$$\frac{5}{2}\log\left(1-x^{2/5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(3/5) + x)^(-1),x]

[Out] (5*Log[1 - x^(2/5)])/2
```

---

**Maple [B]** time = 0.096, size = 116, normalized size = 8.3

$$\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}-\frac{1}{2}\ln\left(-\sqrt{5}\sqrt[5]{x}+2\,x^{2/5}+\sqrt[5]{x}+2\right)-\frac{1}{2}\ln\left(\sqrt{5}\sqrt[5]{x}+2\,x^{2/5}+\sqrt[5]{x}+2\right)+2\ln\left(-1+\sqrt[5]{x}\right)+2\,l$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(3/5)+x),x)
```

```
[Out] 1/2*ln(-1+x)+1/2*ln(1+x)-1/2*ln(-5^(1/2)*x^(1/5)+2*x^(2/5)+x^(1/5)+2)-1/2*l
n(5^(1/2)*x^(1/5)+2*x^(2/5)+x^(1/5)+2)+2*ln(-1+x^(1/5))+2*ln(1+x^(1/5))-1/2
*ln(-5^(1/2)*x^(1/5)+2*x^(2/5)-x^(1/5)+2)-1/2*ln(5^(1/2)*x^(1/5)+2*x^(2/5)-
x^(1/5)+2)
```

---

**Maxima [A]** time = 1.0348, size = 23, normalized size = 1.64

$$\frac{5}{2}\log\left(x^{\frac{1}{5}}+1\right)+\frac{5}{2}\log\left(x^{\frac{1}{5}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(3/5)+x),x, algorithm="maxima")
```

```
[Out] 5/2*log(x^(1/5) + 1) + 5/2*log(x^(1/5) - 1)
```

---

**Fricas [A]** time = 0.787572, size = 30, normalized size = 2.14

$$\frac{5}{2}\log\left(x^{\frac{2}{5}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(3/5)+x),x, algorithm="fricas")
```

```
[Out] 5/2*log(x^(2/5) - 1)
```

---

**Sympy [B]** time = 0.385201, size = 22, normalized size = 1.57

$$\frac{5\log\left(\sqrt[5]{x}-1\right)}{2}+\frac{5\log\left(\sqrt[5]{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(3/5)+x),x)
```

```
[Out] 5*log(x**(1/5) - 1)/2 + 5*log(x**(1/5) + 1)/2
```

---

**Giac [A]** time = 1.17703, size = 24, normalized size = 1.71

$$\frac{5}{2}\log\left(x^{\frac{1}{5}}+1\right)+\frac{5}{2}\log\left(\left|x^{\frac{1}{5}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(3/5)+x),x, algorithm="giac")
```

```
[Out] 5/2*log(x^(1/5) + 1) + 5/2*log(abs(x^(1/5) - 1))
```

## 3.362 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx$

**Optimal**. Leaf size=12

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

```
[Out] (3*Log[1 + x^(4/3)])/4
```

---

**Rubi [A]** time = 0.0035075, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1593, 260}

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 + x^(4/3)])/4
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx &= \int \frac{\sqrt[3]{x}}{1+x^{4/3}}\,dx\\ &= \frac{3}{4}\log\left(1+x^{4/3}\right)\end{aligned}$$

**Mathematica [A]** time = 0.0020223, size = 12, normalized size = 1.

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 + x^(4/3)])/4
```

---

**Maple [A]** time = 0., size = 9, normalized size = 0.8

$$\frac{3}{4}\ln\left(1+x^{\frac{4}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3)+x),x)
```

```
[Out] 3/4*ln(1+x^(4/3))
```

---

**Maxima [A]** time = 1.68293, size = 11, normalized size = 0.92

$$\frac{3}{4}\,\log\left(x^{\frac{4}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="maxima")
```

```
[Out] 3/4*log(x^(4/3) + 1)
```

---

**Fricas [A]** time = 0.846989, size = 30, normalized size = 2.5

$$\frac{3}{4}\,\log\left(x^{\frac{4}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="fricas")
```

```
[Out] 3/4*log(x^(4/3) + 1)
```

---

**Sympy [A]** time = 0.261243, size = 10, normalized size = 0.83

$$\frac{3\log\left(x^{\frac{4}{3}}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/3)+x),x)
```

```
[Out] 3*log(x**(4/3) + 1)/4
```

---

**Giac [B]** time = 1.17784, size = 43, normalized size = 3.58

$$\frac{3}{4}\,\log\left(\sqrt{2}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{3}{4}\,\log\left(-\sqrt{2}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="giac")
```

```
[Out] 3/4*log(sqrt(2)*x^(1/3) + x^(2/3) + 1) + 3/4*log(-sqrt(2)*x^(1/3) + x^(2/3)
 + 1)
```

## 3.363 $\int \frac{1}{x+x^{\sqrt{2}}}\,dx$

**Optimal**. Leaf size=24

$$\log(x) - \left(1+\sqrt{2}\right)\log\left(x^{\sqrt{2}-1}+1\right)$$

```
[Out] Log[x] - (1 + Sqrt[2])*Log[1 + x^(-1 + Sqrt[2])]
```

---

**Rubi [A]** time = 0.0163587, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1593, 266, 36, 29, 31}

$$\log(x) - \left(1+\sqrt{2}\right)\log\left(x^{\sqrt{2}-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^Sqrt[2])^(-1),x]

[Out] Log[x] - (1 + Sqrt[2])*Log[1 + x^(-1 + Sqrt[2])]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x+x^{\sqrt{2}}}\,dx &= \int \frac{1}{x\left(1+x^{-1+\sqrt{2}}\right)}\,dx\\ &= \left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,x^{-1+\sqrt{2}}\right)\\ &= \left(-1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^{-1+\sqrt{2}}\right)+\left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^{-1+\sqrt{2}}\right)\\ &= \log(x)-\left(1+\sqrt{2}\right)\log\left(1+x^{-1+\sqrt{2}}\right)\end{aligned}$$

**Mathematica [A]** time = 0.0208126, size = 24, normalized size = 1.

$$\log(x)-\left(1+\sqrt{2}\right)\log\left(x^{\sqrt{2}-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^Sqrt[2])^(-1),x]
```

```
[Out] Log[x] - (1 + Sqrt[2])*Log[1 + x^(-1 + Sqrt[2])]
```

---

**Maple [A]** time = 0.02, size = 39, normalized size = 1.6

$$\sqrt{2}\ln(x)+2\ln(x)-\ln\left(x+e^{\sqrt{2}\ln(x)}\right)\sqrt{2}-\ln\left(x+e^{\sqrt{2}\ln(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+x^(2^(1/2))),x)
```

```
[Out] 2^(1/2)*ln(x)+2*ln(x)-ln(x+exp(2^(1/2)*ln(x)))*2^(1/2)-ln(x+exp(2^(1/2)*ln(
x)))
```

---

**Maxima [A]** time = 1.61263, size = 42, normalized size = 1.75

$$\frac{\sqrt{2}\log(x)}{\sqrt{2}-1}-\frac{\log\left(x+x^{\left(\sqrt{2}\right)}\right)}{\sqrt{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="maxima")
```

```
[Out] sqrt(2)*log(x)/(sqrt(2) - 1) - log(x + x^sqrt(2))/(sqrt(2) - 1)
```

---

**Fricas [A]** time = 0.774054, size = 78, normalized size = 3.25

$$-\left(\sqrt{2}+1\right)\log\left(x+x^{\left(\sqrt{2}\right)}\right)+\left(\sqrt{2}+2\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="fricas")
```

```
[Out] -(sqrt(2) + 1)*log(x + x^sqrt(2)) + (sqrt(2) + 2)*log(x)
```

---

**Sympy [B]** time = 0.680891, size = 76, normalized size = 3.17

$$\frac{140\sqrt{2}\log(x)}{-338+239\sqrt{2}}-\frac{198\log(x)}{-338+239\sqrt{2}}+\frac{99\sqrt{2}\log\left(x+x^{\sqrt{2}}\right)}{-338+239\sqrt{2}}-\frac{140\log\left(x+x^{\sqrt{2}}\right)}{-338+239\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x**(2**(1/2))),x)
```

```
[Out] 140*sqrt(2)*log(x)/(-338 + 239*sqrt(2)) - 198*log(x)/(-338 + 239*sqrt(2)) +
 99*sqrt(2)*log(x + x**(sqrt(2)))/(-338 + 239*sqrt(2)) - 140*log(x + x**(sq
rt(2)))/(-338 + 239*sqrt(2))
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x+x^{(\sqrt{2})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(1/(x + x^sqrt(2)), x)
```

## 3.364 $\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$

**Optimal**. Leaf size=75

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}$$

```
[Out] (-2*Sqrt[a*x^j + b*x^n])/((j - n)*x^(j/2)) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x^
(j/2))/Sqrt[a*x^j + b*x^n]])/(j - n)
```

---

**Rubi [A]** time = 0.114247, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2028, 2029, 206}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*Sqrt[a*x^j + b*x^n])/((j - n)*x^(j/2)) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x^
(j/2))/Sqrt[a*x^j + b*x^n]])/(j - n)
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx &= -\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+a\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx\\ &= -\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}\\ &= -\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}\end{aligned}$$

**Mathematica [A]** time = 0.200335, size = 104, normalized size = 1.39

$$-\frac{2x^{-j/2}\left(-\sqrt{a}\sqrt{b}x^{\frac{j+n}{2}}\sqrt{\frac{ax^{j-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)+ax^j+bx^n\right)}{(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/b
]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/((j - n)*x^(j/2)*Sqrt[a*x^j +
b*x^n])
```

---

**Maple [F]** time = 0.532, size = 0, normalized size = 0.

$$\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j+bx^n}x^{-\frac{1}{2}j-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j + bx^n}x^{-\frac{1}{2}j-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)
```

## 3.365 $\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$

**Optimal**. Leaf size=99

$$\frac{2\sqrt{a}x^{j/2}(cx)^{-j/2}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}-\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

```
[Out] (-2*Sqrt[a*x^j + b*x^n])/(c*(j - n)*(c*x)^(j/2)) + (2*Sqrt[a]*x^(j/2)*ArcTa
nh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(c*(j - n)*(c*x)^(j/2))
```

---

**Rubi [A]** time = 0.157338, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2031, 2028, 2029, 206}

$$\frac{2\sqrt{a}x^{j/2}(cx)^{-j/2}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}-\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*Sqrt[a*x^j + b*x^n])/(c*(j - n)*(c*x)^(j/2)) + (2*Sqrt[a]*x^(j/2)*ArcTa
nh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(c*(j - n)*(c*x)^(j/2))
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx &= \frac{\left(x^{j/2}(cx)^{-j/2}\right)\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx}{c} \\
&= -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}+\frac{\left(ax^{j/2}(cx)^{-j/2}\right)\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{c} \\
&= -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}+\frac{\left(2ax^{j/2}(cx)^{-j/2}\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)} \\
&= -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}+\frac{2\sqrt{a}x^{j/2}(cx)^{-j/2}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}
\end{aligned}
$$

**Mathematica [A]** time = 0.0644413, size = 109, normalized size = 1.1

$$
-\frac{2(cx)^{-j/2}\left(-\sqrt{a}\sqrt{b}x^{\frac{j+n}{2}}\sqrt{\frac{ax^{j-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)+ax^j+bx^n\right)}{c(j-n)\sqrt{ax^j+bx^n}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/b
]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/(c*(j - n)*(c*x)^(j/2)*Sqrt[a*
x^j + b*x^n])
```

---

**Maple [F]** time = 0.584, size = 0, normalized size = 0.

$$
\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \sqrt{ax^j+bx^n}\,(cx)^{-\frac{1}{2}j-1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx)^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral((c*x)**(-j/2 - 1)*sqrt(a*x**j + b*x**n), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j+bx^n}\,(cx)^{-\frac{1}{2}j-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)
```

## 3.366 $\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx$

**Optimal**. Leaf size=91

$$\frac{2\sqrt{a}\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

```
[Out] (-2*Sqrt[a*x^3 + b*x^n])/(c*(3 - n)*(c*x)^(3/2)) + (2*Sqrt[a]*Sqrt[c*x]*Arc
Tanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(c^3*(3 - n)*Sqrt[x])
```

---

**Rubi [A]** time = 0.155487, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$\frac{2\sqrt{a}\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^n])/(c*(3 - n)*(c*x)^(3/2)) + (2*Sqrt[a]*Sqrt[c*x]*Arc
Tanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(c^3*(3 - n)*Sqrt[x])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx &= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{a\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx}{c^3}\\ &= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{\left(a\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{c^3\sqrt{x}}\\ &= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{\left(2a\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}\\ &= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{2\sqrt{a}\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}\end{aligned}$$

**Mathematica [A]** time = 0.246594, size = 103, normalized size = 1.13

$$\frac{2x\left(-\sqrt{a}\sqrt{b}x^{\frac{n+3}{2}}\sqrt{\frac{ax^{3-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)+ax^3+bx^n\right)}{(n-3)(cx)^{5/2}\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2),x]
```

```
[Out] (2*x*(a*x^3 + b*x^n - Sqrt[a]*Sqrt[b]*x^((3 + n)/2)*Sqrt[1 + (a*x^(3 - n))/
b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]]))/((-3 + n)*(c*x)^(5/2)*Sqrt[a*
x^3 + b*x^n])
```

---

**Maple [F]** time = 0.379, size = 0, normalized size = 0.

$$\int \sqrt{ax^3+bx^n}\,(cx)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)
```

```
[Out] int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b*x**n)**(1/2)/(c*x)**(5/2),x)
```

```
[Out] Integral(sqrt(a*x**3 + b*x**n)/(c*x)**(5/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)
```

## 3.367 $\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx$

**Optimal**. Leaf size=71

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}-\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}$$

```
[Out] (-2*Sqrt[a*x^2 + b*x^n])/(c^2*(2 - n)*x) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/S
qrt[a*x^2 + b*x^n]])/(c^2*(2 - n))
```

---

**Rubi [A]** time = 0.0755826, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2028, 2008, 206}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}-\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^n]/(c^2*x^2),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^n])/(c^2*(2 - n)*x) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/S
qrt[a*x^2 + b*x^n]])/(c^2*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx &= \frac{\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^2} \\
&= -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x} + \frac{a\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx}{c^2} \\
&= -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)} \\
&= -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x} + \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}
\end{aligned}
$$

**Mathematica [A]** time = 0.144159, size = 99, normalized size = 1.39

$$
\frac{2\left(-\sqrt{a}\sqrt{b}x^{\frac{n}{2}+1}\sqrt{\frac{ax^{2-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)+ax^2+bx^n\right)}{c^2(n-2)x\sqrt{ax^2+bx^n}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^n]/(c^2*x^2),x]
```

```
[Out] (2*(a*x^2 + b*x^n - Sqrt[a]*Sqrt[b]*x^(1 + n/2)*Sqrt[1 + (a*x^(2 - n))/b]*A
rcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]]))/(c^2*(-2 + n)*x*Sqrt[a*x^2 + b*x^n]
)
```

**Maple [F]** time = 0.341, size = 0, normalized size = 0.

$$
\int \frac{1}{c^2x^2}\sqrt{ax^2+bx^n}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)
```

```
[Out] int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\frac{\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^2 + b*x^n)/x^2, x)/c^2
```

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x**n)**(1/2)/c**2/x**2,x)
```

```
[Out] Integral(sqrt(a*x**2 + b*x**n)/x**2, x)/c**2
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax^2 + bx^n}}{c^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^2 + b*x^n)/(c^2*x^2), x)
```

## 3.368 $\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx$

**Optimal**. Leaf size=87

$$\frac{2\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

```
[Out] (-2*Sqrt[a*x + b*x^n])/(c*(1 - n)*Sqrt[c*x]) + (2*Sqrt[a]*Sqrt[x]*ArcTanh[(
Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(c*(1 - n)*Sqrt[c*x])
```

---

**Rubi [A]** time = 0.137653, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2028, 2031, 2029, 206}

$$\frac{2\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^n]/(c*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^n])/(c*(1 - n)*Sqrt[c*x]) + (2*Sqrt[a]*Sqrt[x]*ArcTanh[(
Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(c*(1 - n)*Sqrt[c*x])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx &= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}+\frac{a\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx}{c}\\
&= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}+\frac{\left(a\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{ax+bx^n}}\,dx}{c\sqrt{cx}}\\
&= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}+\frac{\left(2a\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}\\
&= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}+\frac{2\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}
\end{aligned}
$$

**Mathematica [A]** time = 0.250924, size = 100, normalized size = 1.15

$$
\frac{x\left(-2\sqrt{a}\sqrt{b}x^{\frac{n+1}{2}}\sqrt{\frac{ax^{1-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)+2ax+2bx^n\right)}{(n-1)(cx)^{3/2}\sqrt{ax+bx^n}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^n]/(c*x)^(3/2),x]
```

```
[Out] (x*(2*a*x + 2*b*x^n - 2*Sqrt[a]*Sqrt[b]*x^((1 + n)/2)*Sqrt[1 + (a*x^(1 - n)
)/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]]))/((-1 + n)*(c*x)^(3/2)*Sqrt[
a*x + b*x^n])
```

---

**Maple [F]** time = 0.383, size = 0, normalized size = 0.

$$
\int \sqrt{ax+bx^n}\,(cx)^{-\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)
```

```
[Out] int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{ax+bx^n}}{(cx)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^n}}{(cx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x**n)**(1/2)/(c*x)**(3/2),x)
```

```
[Out] Integral(sqrt(a*x + b*x**n)/(c*x)**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ax + bx^n}}{(cx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)
```

## 3.369 $\int \frac{\sqrt{a+bx^n}}{cx}\,dx$

**Optimal**. Leaf size=51

$$\frac{2\sqrt{a+bx^n}}{cn}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

```
[Out] (2*Sqrt[a + b*x^n])/(c*n) - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c
*n)
```

---

**Rubi [A]** time = 0.0291036, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {12, 266, 50, 63, 208}

$$\frac{2\sqrt{a+bx^n}}{cn}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n]/(c*x),x]
```

```
[Out] (2*Sqrt[a + b*x^n])/(c*n) - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^n}}{cx}\,dx &= \frac{\int \frac{\sqrt{a+bx^n}}{x}\,dx}{c}\\
&= \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^n\right)}{cn}\\
&= \frac{2\sqrt{a+bx^n}}{cn}+\frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^n\right)}{cn}\\
&= \frac{2\sqrt{a+bx^n}}{cn}+\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^n}\right)}{bcn}\\
&= \frac{2\sqrt{a+bx^n}}{cn}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}
\end{aligned}$$

**Mathematica [A]** time = 0.012858, size = 46, normalized size = 0.9

$$\frac{2\sqrt{a+bx^n}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n]/(c*x),x]
```

```
[Out] (2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)
```

---

**Maple [A]** time = 0.002, size = 39, normalized size = 0.8

$$\frac{1}{cn}\left(2\,\sqrt{a+bx^n}-2\,\sqrt{a}\operatorname{Artanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^(1/2)/c/x,x)
```

```
[Out] 1/c/n*(2*(a+b*x^n)^(1/2)-2*a^(1/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2)))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.9044, size = 223, normalized size = 4.37

$$\left[\frac{\sqrt{a}\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)+2\sqrt{bx^n+a}}{cn}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)+\sqrt{bx^n+a}\right)}{cn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*sqrt(b*x^n
 + a))/(c*n), 2*(sqrt(-a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n +
 a))/(c*n)]
```

---

**Sympy [A]** time = 2.23138, size = 78, normalized size = 1.53

$$\frac{-\frac{2\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{n}+\frac{2ax^{-\frac{n}{2}}}{\sqrt{b}n\sqrt{\frac{ax^{-n}}{b}+1}}+\frac{2\sqrt{b}x^{\frac{n}{2}}}{n\sqrt{\frac{ax^{-n}}{b}+1}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(1/2)/c/x,x)
```

```
[Out] (-2*sqrt(a)*asinh(sqrt(a)*x**(-n/2)/sqrt(b))/n + 2*a*x**(-n/2)/(sqrt(b)*n*s
qrt(a*x**(-n)/b + 1)) + 2*sqrt(b)*x**(n/2)/(n*sqrt(a*x**(-n)/b + 1)))/c
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^n+a}}{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)/(c*x), x)
```

## 3.370 $\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx$

**Optimal**. Leaf size=84

$$\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}$$

```
[Out] (2*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(c*(1 + n)) - (2*Sqrt[a]*Sqrt[x]*ArcTanh[Sq
rt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/((1 + n)*Sqrt[c*x])
```

---

**Rubi [A]** time = 0.171002, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x + b*x^n]/Sqrt[c*x],x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(c*(1 + n)) - (2*Sqrt[a]*Sqrt[x]*ArcTanh[Sq
rt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/((1 + n)*Sqrt[c*x])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx &= \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)}+(ac)\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx\\
&= \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)}+\frac{\left(a\sqrt{x}\right)\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{\sqrt{cx}}\\
&= \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)}-\frac{\left(2a\sqrt{x}\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}\\
&= \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)}-\frac{2\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}
\end{aligned}
$$

**Mathematica [A]** time = 0.062124, size = 85, normalized size = 1.01

$$
\frac{x\sqrt{\frac{a}{x}+bx^n}\left(2\sqrt{a+bx^{n+1}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right)\right)}{(n+1)\sqrt{cx}\sqrt{a+bx^{n+1}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x + b*x^n]/Sqrt[c*x],x]
```

```
[Out] (x*Sqrt[a/x + b*x^n]*(2*Sqrt[a + b*x^(1 + n)] - 2*Sqrt[a]*ArcTanh[Sqrt[a +
b*x^(1 + n)]/Sqrt[a]]))/((1 + n)*Sqrt[c*x]*Sqrt[a + b*x^(1 + n)])
```

---

**Maple [F]** time = 0.368, size = 0, normalized size = 0.

$$
\int \sqrt{\frac{a}{x}+bx^n}\frac{1}{\sqrt{cx}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{\sqrt{bx^n+\frac{a}{x}}}{\sqrt{cx}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{a}{x} + bx^{n}}}{\sqrt{cx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x**n)**(1/2)/(c*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x + b*x**n)/sqrt(c*x), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^{n} + \frac{a}{x}}}{\sqrt{cx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)
```

## 3.371 $\int \sqrt{\frac{a}{x^2}+bx^n}\,dx$

Optimal. Leaf size=61

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}$$

```
[Out] (2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^
2 + b*x^n])])/(2 + n)
```

---

**Rubi [A]** time = 0.0834905, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2007, 2029, 206}

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^2 + b*x^n],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^
2 + b*x^n])])/(2 + n)
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{a}{x^2}+bx^n}\,dx &= \frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx\\ &= \frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}-\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}\\ &= \frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}\end{aligned}$$

**Mathematica [A]** time = 0.0532376, size = 78, normalized size = 1.28

$$\frac{x\sqrt{\frac{a}{x^2}+bx^n}\left(2\sqrt{a+bx^{n+2}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+2}}}{\sqrt{a}}\right)\right)}{(n+2)\sqrt{a+bx^{n+2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^2 + b*x^n],x]
```

```
[Out] (x*Sqrt[a/x^2 + b*x^n]*(2*Sqrt[a + b*x^(2 + n)] - 2*Sqrt[a]*ArcTanh[Sqrt[a
+ b*x^(2 + n)]/Sqrt[a]]))/((2 + n)*Sqrt[a + b*x^(2 + n)])
```

---

**Maple [F]** time = 0.34, size = 0, normalized size = 0.

$$\int \sqrt{\frac{a}{x^2}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^2*a+b*x^n)^(1/2),x)
```

```
[Out] int((1/x^2*a+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^n+\frac{a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a/x^2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{a}{x^2} + bx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**2+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**2 + b*x**n), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^n + \frac{a}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a/x^2), x)
```

# 3.372 $\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$

**Optimal**. Leaf size=85

$$\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2\sqrt{a}c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}$$

```
[Out] (2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])/(c*(3 + n)) - (2*Sqrt[a]*c*Sqrt[x]*ArcT
anh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x])
```

---

**Rubi [A]** time = 0.198622, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2\sqrt{a}c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n],x]
```

```
[Out] (2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])/(c*(3 + n)) - (2*Sqrt[a]*c*Sqrt[x]*ArcT
anh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx &= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} + \left(ac^3\right)\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx \\ &= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} + \frac{\left(ac\sqrt{x}\right)\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{\sqrt{cx}} \\ &= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} - \frac{\left(2ac\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}} \\ &= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} - \frac{2\sqrt{a}c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}\end{aligned}$$

**Mathematica [A]** time = 0.0676533, size = 85, normalized size = 1.

$$\frac{x\sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\left(2\sqrt{a+bx^{n+3}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}}\right)\right)}{(n+3)\sqrt{a+bx^{n+3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n],x]
```

```
[Out] (x*Sqrt[c*x]*Sqrt[a/x^3 + b*x^n]*(2*Sqrt[a + b*x^(3 + n)] - 2*Sqrt[a]*ArcTa
nh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]))/((3 + n)*Sqrt[a + b*x^(3 + n)])
```

---

**Maple [F]** time = 0.346, size = 0, normalized size = 0.

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^n+\frac{a}{x^3}}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(a/x**3+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(c*x)*sqrt(a/x**3 + b*x**n), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^n+\frac{a}{x^3}}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)
```

## 3.373 $\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2}\,dx$

**Optimal**. Leaf size=141

$$\frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}-\frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)}-\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

```
[Out] (-2*a*x^j*Sqrt[a*x^j + b*x^n])/(c*(j - n)*(c*x)^((3*j)/2)) - (2*(a*x^j + b*
x^n)^(3/2))/(3*c*(j - n)*(c*x)^((3*j)/2)) + (2*a^(3/2)*x^((3*j)/2)*ArcTanh[
(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(c*(j - n)*(c*x)^((3*j)/2))
```

---

**Rubi [A]** time = 0.225356, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2031, 2028, 2029, 206}

$$\frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}-\frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)}-\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*a*x^j*Sqrt[a*x^j + b*x^n])/(c*(j - n)*(c*x)^((3*j)/2)) - (2*(a*x^j + b*
x^n)^(3/2))/(3*c*(j - n)*(c*x)^((3*j)/2)) + (2*a^(3/2)*x^((3*j)/2)*ArcTanh[
(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(c*(j - n)*(c*x)^((3*j)/2))
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} dx &= \frac{\left(x^{3j/2}(cx)^{-3j/2}\right) \int x^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} dx}{c} \\
&= -\frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{\left(ax^{3j/2}(cx)^{-3j/2}\right) \int x^{-1-\frac{j}{2}} \sqrt{ax^j+bx^n}\, dx}{c} \\
&= -\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{\left(a^2x^{3j/2}(cx)^{-3j/2}\right) \int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\, dx}{c} \\
&= -\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{\left(2a^2x^{3j/2}(cx)^{-3j/2}\right) \text{Subst}\left(\int \right.}{c(j-n)} \\
&= -\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2} \tanh^{-1}\left( \right.}{c(j-n)}
\end{aligned}
$$

**Mathematica [A]** time = 0.233284, size = 131, normalized size = 0.93

$$
-\frac{2(cx)^{-3j/2}\left(-3a^{3/2}\sqrt{b}x^{\frac{1}{2}(3j+n)}\sqrt{\frac{ax^{j-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)+4a^2x^{2j}+5abx^{j+n}+b^2x^{2n}\right)}{3c(j-n)\sqrt{ax^j+bx^n}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*(4*a^2*x^(2*j) + b^2*x^(2*n) + 5*a*b*x^(j + n) - 3*a^(3/2)*Sqrt[b]*x^((
3*j + n)/2)*Sqrt[1 + (a*x^(j - n))/b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[
b]]))/(3*c*(j - n)*(c*x)^((3*j)/2)*Sqrt[a*x^j + b*x^n])
```

---

**Maple [F]** time = 0.61, size = 0, normalized size = 0.

$$
\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{\frac{3}{2}} dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \left(ax^j+bx^n\right)^{\frac{3}{2}} (cx)^{-\frac{3}{2}j-1} dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-3/2*j)*(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax^j + bx^n\right)^{\frac{3}{2}} (cx)^{-\frac{3}{2}j-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)
```

## 3.374 $\int \frac{\left(ax^3+bx^n\right)^{3/2}}{(cx)^{11/2}}\,dx$

**Optimal**. Leaf size=128

$$\frac{2a^{3/2}\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}-\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

```
[Out] (-2*a*Sqrt[a*x^3 + b*x^n])/(c^4*(3 - n)*(c*x)^(3/2)) - (2*(a*x^3 + b*x^n)^(
3/2))/(3*c*(3 - n)*(c*x)^(9/2)) + (2*a^(3/2)*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(
3/2))/Sqrt[a*x^3 + b*x^n]])/(c^6*(3 - n)*Sqrt[x])
```

---

**Rubi [A]** time = 0.208872, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$\frac{2a^{3/2}\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}-\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] (-2*a*Sqrt[a*x^3 + b*x^n])/(c^4*(3 - n)*(c*x)^(3/2)) - (2*(a*x^3 + b*x^n)^(
3/2))/(3*c*(3 - n)*(c*x)^(9/2)) + (2*a^(3/2)*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(
3/2))/Sqrt[a*x^3 + b*x^n]])/(c^6*(3 - n)*Sqrt[x])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax^3+bx^n\right)^{3/2}}{(cx)^{11/2}}\,dx &= -\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}+\frac{a\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx}{c^3}\\
&= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}+\frac{a^2\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx}{c^6}\\
&= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}+\frac{\left(a^2\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{c^6\sqrt{x}}\\
&= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}+\frac{\left(2a^2\sqrt{cx}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}\\
&= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}+\frac{2a^{3/2}\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}
\end{aligned}$$

**Mathematica [A]** time = 0.293409, size = 126, normalized size = 0.98

$$\frac{2\sqrt{cx}\left(-3a^{3/2}\sqrt{b}x^{\frac{n+9}{2}}\sqrt{\frac{ax^{3-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)+4a^2x^6+5abx^{n+3}+b^2x^{2n}\right)}{3c^6(n-3)x^5\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] (2*Sqrt[c*x]*(4*a^2*x^6 + b^2*x^(2*n) + 5*a*b*x^(3 + n) - 3*a^(3/2)*Sqrt[b]
*x^((9 + n)/2)*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sq
rt[b]]))/(3*c^6*(-3 + n)*x^5*Sqrt[a*x^3 + b*x^n])
```

---

**Maple [F]** time = 0.335, size = 0, normalized size = 0.

$$\int \left(ax^3+bx^n\right)^{\frac{3}{2}}(cx)^{-\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)
```

```
[Out] int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax^3+bx^n\right)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b*x**n)**(3/2)/(c*x)**(11/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax^3 + bx^n\right)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)
```

## 3.375 $\int \frac{\left(ax^2+bx^n\right)^{3/2}}{c^4x^4}\,dx$

**Optimal**. Leaf size=104

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}-\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3c^4(2-n)x^3}$$

```
[Out] (-2*a*Sqrt[a*x^2 + b*x^n])/(c^4*(2 - n)*x) - (2*(a*x^2 + b*x^n)^(3/2))/(3*c
^4*(2 - n)*x^3) + (2*a^(3/2)*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(c^4
*(2 - n))
```

---

**Rubi [A]** time = 0.133196, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2028, 2008, 206}

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}-\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3c^4(2-n)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4),x]
```

```
[Out] (-2*a*Sqrt[a*x^2 + b*x^n])/(c^4*(2 - n)*x) - (2*(a*x^2 + b*x^n)^(3/2))/(3*c
^4*(2 - n)*x^3) + (2*a^(3/2)*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(c^4
*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax^2+bx^n\right)^{3/2}}{c^4x^4}\,dx &= \frac{\int \frac{\left(ax^2+bx^n\right)^{3/2}}{x^4}\,dx}{c^4}\\
&= -\frac{2\left(ax^2+bx^n\right)^{3/2}}{3c^4(2-n)x^3}+\frac{a\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^4}\\
&= -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3c^4(2-n)x^3}+\frac{a^2\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx}{c^4}\\
&= -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3c^4(2-n)x^3}+\frac{\left(2a^2\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}\\
&= -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3c^4(2-n)x^3}+\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}
\end{aligned}
$$

**Mathematica [A]** time = 0.157735, size = 117, normalized size = 1.12

$$\frac{2\left(-3a^{3/2}\sqrt{b}x^{\frac{n}{2}+3}\sqrt{\frac{ax^{2-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)+4a^2x^4+5abx^{n+2}+b^2x^{2n}\right)}{3c^4(n-2)x^3\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4),x]
```

```
[Out] (2*(4*a^2*x^4 + b^2*x^(2*n) + 5*a*b*x^(2 + n) - 3*a^(3/2)*Sqrt[b]*x^(3 + n/
2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]]))/(3*c^
4*(-2 + n)*x^3*Sqrt[a*x^2 + b*x^n])
```

---

**Maple [F]** time = 0.324, size = 0, normalized size = 0.

$$\int \frac{1}{c^4x^4}\left(ax^2+bx^n\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)
```

```
[Out] int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\left(ax^2+bx^n\right)^{\frac{3}{2}}}{x^4}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + b*x^n)^(3/2)/x^4, x)/c^4
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{a\sqrt{ax^2+bx^n}}{x^2}\,dx + \int \frac{bx^n\sqrt{ax^2+bx^n}}{x^4}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x**n)**(3/2)/c**4/x**4,x)
```

```
[Out] (Integral(a*sqrt(a*x**2 + b*x**n)/x**2, x) + Integral(b*x**n*sqrt(a*x**2 +
b*x**n)/x**4, x))/c**4
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ax^2 + bx^n\right)^{\frac{3}{2}}}{c^4x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="giac")
```

```
[Out] integrate((a*x^2 + b*x^n)^(3/2)/(c^4*x^4), x)
```

## 3.376 $\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx$

**Optimal**. Leaf size=122

$$\frac{2a^{3/2}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}-\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2\left(ax+bx^n\right)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

```
[Out] (-2*a*Sqrt[a*x + b*x^n])/(c^2*(1 - n)*Sqrt[c*x]) - (2*(a*x + b*x^n)^(3/2))/
(3*c*(1 - n)*(c*x)^(3/2)) + (2*a^(3/2)*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sq
rt[a*x + b*x^n]])/(c^2*(1 - n)*Sqrt[c*x])
```

---

**Rubi [A]** time = 0.188308, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2028, 2031, 2029, 206}

$$\frac{2a^{3/2}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}-\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2\left(ax+bx^n\right)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^n)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] (-2*a*Sqrt[a*x + b*x^n])/(c^2*(1 - n)*Sqrt[c*x]) - (2*(a*x + b*x^n)^(3/2))/
(3*c*(1 - n)*(c*x)^(3/2)) + (2*a^(3/2)*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sq
rt[a*x + b*x^n]])/(c^2*(1 - n)*Sqrt[c*x])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx &= -\frac{2\,(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{a\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx}{c} \\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2\,(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{a^2\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx}{c^2} \\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2\,(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{\left(a^2\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{ax+bx^n}}\,dx}{c^2\sqrt{cx}} \\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2\,(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{\left(2a^2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}} \\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}} - \frac{2\,(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}} + \frac{2a^{3/2}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}
\end{aligned}$$

**Mathematica [A]** time = 0.250859, size = 120, normalized size = 0.98

$$\frac{x\left(-6a^{3/2}\sqrt{b}x^{\frac{n+3}{2}}\sqrt{\frac{ax^{1-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)+8a^2x^2+10abx^{n+1}+2b^2x^{2n}\right)}{3(n-1)(cx)^{5/2}\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^n)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] (x*(8*a^2*x^2 + 2*b^2*x^(2*n) + 10*a*b*x^(1 + n) - 6*a^(3/2)*Sqrt[b]*x^((3
+ n)/2)*Sqrt[1 + (a*x^(1 - n))/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]])
)/(3*(-1 + n)*(c*x)^(5/2)*Sqrt[a*x + b*x^n])
```

---

**Maple [F]** time = 0.323, size = 0, normalized size = 0.

$$\int (ax+bx^n)^{\frac{3}{2}}(cx)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

```
[Out] int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax+bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x**n)**(3/2)/(c*x)**(5/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)
```

## 3.377 $\int \frac{(a+bx^n)^{3/2}}{cx}\,dx$

**Optimal**. Leaf size=73

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}+\frac{2a\sqrt{a+bx^n}}{cn}+\frac{2(a+bx^n)^{3/2}}{3cn}$$

```
[Out] (2*a*Sqrt[a + b*x^n])/(c*n) + (2*(a + b*x^n)^(3/2))/(3*c*n) - (2*a^(3/2)*Ar
cTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)
```

---

**Rubi [A]** time = 0.0408936, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {12, 266, 50, 63, 208}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}+\frac{2a\sqrt{a+bx^n}}{cn}+\frac{2(a+bx^n)^{3/2}}{3cn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(3/2)/(c*x),x]
```

```
[Out] (2*a*Sqrt[a + b*x^n])/(c*n) + (2*(a + b*x^n)^(3/2))/(3*c*n) - (2*a^(3/2)*Ar
cTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^n)^{3/2}}{cx}\,dx &= \frac{\int \frac{(a+bx^n)^{3/2}}{x}\,dx}{c} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^n\right)}{cn} \\
&= \frac{2(a+bx^n)^{3/2}}{3cn} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^n\right)}{cn} \\
&= \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{cn} \\
&= \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bcn} \\
&= \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}
\end{aligned}$$

**Mathematica [A]** time = 0.0257814, size = 58, normalized size = 0.79

$$\frac{2\sqrt{a+bx^n}\,(4a+bx^n) - 6a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{3cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(3/2)/(c*x),x]
```

```
[Out] (2*Sqrt[a + b*x^n]*(4*a + b*x^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a
]])/(3*c*n)
```

---

**Maple [A]** time = 0.002, size = 51, normalized size = 0.7

$$\frac{1}{cn}\left(\frac{2}{3}(a+bx^n)^{\frac{3}{2}} + 2a\sqrt{a+bx^n} - 2a^{3/2}\text{Artanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^(3/2)/c/x,x)
```

```
[Out] 1/c/n*(2/3*(a+b*x^n)^(3/2)+2*a*(a+b*x^n)^(1/2)-2*a^(3/2)*arctanh((a+b*x^n)^
(1/2)/a^(1/2)))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.967679, size = 277, normalized size = 3.79

$$\left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{bx^{n}-2\,\sqrt{bx^{n}+a}\sqrt{a}+2\,a}{x^{n}}\right)+2\,(bx^{n}+4\,a)\sqrt{bx^{n}+a}}{3\,cn}, \frac{2\left(3\,\sqrt{-a}a\arctan\left(\frac{\sqrt{bx^{n}+a}\sqrt{-a}}{a}\right)+(bx^{n}+4\,a)\sqrt{bx^{n}+a}\right)}{3\,cn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*(b*x
^n + 4*a)*sqrt(b*x^n + a))/(c*n), 2/3*(3*sqrt(-a)*a*arctan(sqrt(b*x^n + a)*
sqrt(-a)/a) + (b*x^n + 4*a)*sqrt(b*x^n + a))/(c*n)]
```

---

**Sympy [A]** time = 4.75678, size = 88, normalized size = 1.21

$$\frac{\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx^{n}}{a}}}{3n}+\frac{a^{\frac{3}{2}}\log\left(\frac{bx^{n}}{a}\right)}{n}-\frac{2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^{n}}{a}}+1\right)}{n}+\frac{2\sqrt{a}bx^{n}\sqrt{1+\frac{bx^{n}}{a}}}{3n}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(3/2)/c/x,x)
```

```
[Out] (8*a**(3/2)*sqrt(1 + b*x**n/a)/(3*n) + a**(3/2)*log(b*x**n/a)/n - 2*a**(3/2
)*log(sqrt(1 + b*x**n/a) + 1)/n + 2*sqrt(a)*b*x**n*sqrt(1 + b*x**n/a)/(3*n)
)/c
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{(bx^{n}+a)^{\frac{3}{2}}}{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)/(c*x), x)
```

## 3.378 $\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}\,dx$

**Optimal**. Leaf size=117

$$-\frac{2a^{3/2}c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}+\frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{n+1}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

```
[Out] (2*a*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(1 + n) + (2*(c*x)^(3/2)*(a/x + b*x^n)^(3
/2))/(3*c*(1 + n)) - (2*a^(3/2)*c*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x
 + b*x^n])])/((1 + n)*Sqrt[c*x])
```

---

**Rubi [A]** time = 0.230101, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$-\frac{2a^{3/2}c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}+\frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{n+1}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]
```

```
[Out] (2*a*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(1 + n) + (2*(c*x)^(3/2)*(a/x + b*x^n)^(3
/2))/(3*c*(1 + n)) - (2*a^(3/2)*c*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x
 + b*x^n])])/((1 + n)*Sqrt[c*x])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2} dx &= \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)} + (ac)\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\, dx \\
&= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n} + \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)} + \left(a^2c^2\right)\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\, dx \\
&= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n} + \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)} + \frac{\left(a^2c\sqrt{x}\right)\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\, dx}{\sqrt{cx}} \\
&= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n} + \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)} - \frac{\left(2a^2c\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\, dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}} \\
&= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n} + \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)} - \frac{2a^{3/2}c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}
\end{aligned}$$

**Mathematica [A]** time = 0.0714926, size = 97, normalized size = 0.83

$$\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}\left(\sqrt{a+bx^{n+1}}\left(4a+bx^{n+1}\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right)\right)}{3(n+1)\sqrt{a+bx^{n+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a/x + b*x^n]*(Sqrt[a + b*x^(1 + n)]*(4*a + b*x^(1 + n)) -
 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]]))/(3*(1 + n)*Sqrt[a + b*x
^(1 + n)])
```

---

**Maple [F]** time = 0.323, size = 0, normalized size = 0.

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}\sqrt{cx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(a/x+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^n + \frac{a}{x}\right)^{\frac{3}{2}} \sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)
```

## 3.379 $\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx$

**Optimal**. Leaf size=98

$$-\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}+\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}$$

```
[Out] (2*a*c^2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) + (2*c^2*x^3*(a/x^2 + b*x^n)^(3/2))
/(3*(2 + n)) - (2*a^(3/2)*c^2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(2
+ n)
```

---

**Rubi [A]** time = 0.171814, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {12, 2028, 2007, 2029, 206}

$$-\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}+\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]
```

```
[Out] (2*a*c^2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) + (2*c^2*x^3*(a/x^2 + b*x^n)^(3/2))
/(3*(2 + n)) - (2*a^(3/2)*c^2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(2
+ n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}dx &= c^2\int x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}dx\\
&= \frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}+\left(ac^2\right)\int\sqrt{\frac{a}{x^2}+bx^n}\,dx\\
&= \frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}+\left(a^2c^2\right)\int\frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx\\
&= \frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}-\frac{\left(2a^2c^2\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}\\
&= \frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}-\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}
\end{aligned}
$$

**Mathematica [A]** time = 0.0678041, size = 94, normalized size = 0.96

$$\frac{2c^2x\sqrt{\frac{a}{x^2}+bx^n}\left(\sqrt{a+bx^{n+2}}\left(4a+bx^{n+2}\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+2}}}{\sqrt{a}}\right)\right)}{3(n+2)\sqrt{a+bx^{n+2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^2*x*Sqrt[a/x^2 + b*x^n]*(Sqrt[a + b*x^(2 + n)]*(4*a + b*x^(2 + n)) - 3
*a^(3/2)*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]]))/(3*(2 + n)*Sqrt[a + b*x^(
2 + n)])
```

---

**Maple [F]** time = 0.322, size = 0, normalized size = 0.

$$\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^2*x^2*(1/x^2*a+b*x^n)^(3/2),x)
```

```
[Out] int(c^2*x^2*(1/x^2*a+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$c^2\int\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] c^2*integrate((b*x^n + a/x^2)^(3/2)*x^2, x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$c^{2}\left(\int a\sqrt{\frac{a}{x^{2}}+bx^{n}}\,dx+\int bx^{2}x^{n}\sqrt{\frac{a}{x^{2}}+bx^{n}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**2*x**2*(a/x**2+b*x**n)**(3/2),x)
```

```
[Out] c**2*(Integral(a*sqrt(a/x**2 + b*x**n), x) + Integral(b*x**2*x**n*sqrt(a/x*
*2 + b*x**n), x))
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(bx^{n}+\frac{a}{x^{2}}\right)^{\frac{3}{2}}c^{2}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a/x^2)^(3/2)*c^2*x^2, x)
```

## 3.380 $\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx$

**Optimal**. Leaf size=122

$$-\frac{2a^{3/2}c^4\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}+\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{n+3}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

```
[Out] (2*a*c^2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])/(3 + n) + (2*(c*x)^(9/2)*(a/x^3 +
 b*x^n)^(3/2))/(3*c*(3 + n)) - (2*a^(3/2)*c^4*Sqrt[x]*ArcTanh[Sqrt[a]/(x^(3
/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x])
```

---

**Rubi [A]** time = 0.276027, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$-\frac{2a^{3/2}c^4\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}+\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{n+3}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]
```

```
[Out] (2*a*c^2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])/(3 + n) + (2*(c*x)^(9/2)*(a/x^3 +
 b*x^n)^(3/2))/(3*c*(3 + n)) - (2*a^(3/2)*c^4*Sqrt[x]*ArcTanh[Sqrt[a]/(x^(3
/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}\,dx &= \frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}+\left(ac^3\right)\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx\\
&= \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}+\left(a^2c^6\right)\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx\\
&= \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}+\frac{\left(a^2c^4\sqrt{x}\right)\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{\sqrt{cx}}\\
&= \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}-\frac{\left(2a^2c^4\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx}}\right.}{(3+n)\sqrt{cx}}\\
&= \frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}-\frac{2a^{3/2}c^4\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}
\end{aligned}
$$

**Mathematica [A]** time = 0.0839508, size = 100, normalized size = 0.82

$$\frac{2c^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}\left(\sqrt{a+bx^{n+3}}\left(4a+bx^{n+3}\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}}\right)\right)}{3(n+3)\sqrt{a+bx^{n+3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]*(Sqrt[a + b*x^(3 + n)]*(4*a + b*x^(3
 + n)) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]))/(3*(3 + n)*Sqrt
[a + b*x^(3 + n)])
```

---

**Maple [F]** time = 0.322, size = 0, normalized size = 0.

$$\int (cx)^{\frac{7}{2}}\left(\frac{a}{x^3}+bx^n\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)*(a/x**3+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^n + \frac{a}{x^3}\right)^{\frac{3}{2}} (cx)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)
```

## 3.381 $\int c^5 x^5 \left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx$

**Optimal**. Leaf size=100

$$-\frac{2a^{3/2}c^5\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{n+4}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}+\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}$$

```
[Out] (2*a*c^5*x^2*Sqrt[a/x^4 + b*x^n])/(4 + n) + (2*c^5*x^6*(a/x^4 + b*x^n)^(3/2
))/(3*(4 + n)) - (2*a^(3/2)*c^5*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/x^4 + b*x^n])])
/(4 + n)
```

---

**Rubi [A]** time = 0.212059, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2028, 2029, 206}

$$-\frac{2a^{3/2}c^5\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{n+4}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}+\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}$$

Antiderivative was successfully verified.

```
[In] Int[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]
```

```
[Out] (2*a*c^5*x^2*Sqrt[a/x^4 + b*x^n])/(4 + n) + (2*c^5*x^6*(a/x^4 + b*x^n)^(3/2
))/(3*(4 + n)) - (2*a^(3/2)*c^5*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/x^4 + b*x^n])])
/(4 + n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}dx &= c^5\int x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}dx\\
&= \frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}+\left(ac^5\right)\int x\sqrt{\frac{a}{x^4}+bx^n}\,dx\\
&= \frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}+\left(a^2c^5\right)\int\frac{1}{x^3\sqrt{\frac{a}{x^4}+bx^n}}\,dx\\
&= \frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}-\frac{\left(2a^2c^5\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{1}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{4+n}\\
&= \frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}-\frac{2a^{3/2}c^5\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{4+n}
\end{aligned}
$$

**Mathematica [A]** time = 0.0721212, size = 96, normalized size = 0.96

$$
\frac{2c^5x^2\sqrt{\frac{a}{x^4}+bx^n}\left(\sqrt{a+bx^{n+4}}\left(4a+bx^{n+4}\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+4}}}{\sqrt{a}}\right)\right)}{3(n+4)\sqrt{a+bx^{n+4}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^5*x^2*Sqrt[a/x^4 + b*x^n]*(Sqrt[a + b*x^(4 + n)]*(4*a + b*x^(4 + n)) -
 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(4 + n)]/Sqrt[a]]))/(3*(4 + n)*Sqrt[a + b*x
^(4 + n)])
```

---

**Maple [F]** time = 0.327, size = 0, normalized size = 0.

$$
\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{\frac{3}{2}}dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)
```

```
[Out] int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
c^5\int\left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}x^5\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] c^5*integrate((b*x^n + a/x^4)^(3/2)*x^5, x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$c^{5}\left(\int ax\sqrt{\frac{a}{x^{4}}+bx^{n}}\,dx+\int bx^{5}x^{n}\sqrt{\frac{a}{x^{4}}+bx^{n}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**5*x**5*(a/x**4+b*x**n)**(3/2),x)
```

```
[Out] c**5*(Integral(a*x*sqrt(a/x**4 + b*x**n), x) + Integral(b*x**5*x**n*sqrt(a/
x**4 + b*x**n), x))
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(bx^{n}+\frac{a}{x^{4}}\right)^{\frac{3}{2}}c^{5}x^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a/x^4)^(3/2)*c^5*x^5, x)
```

## 3.382 $\int \sqrt{\frac{a+bx}{x^2}}\,dx$

**Optimal**. Leaf size=51

$$2x\sqrt{\frac{a}{x^2}+\frac{b}{x}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+\frac{b}{x}}}\right)$$

```
[Out] 2*Sqrt[a/x^2 + b/x]*x - 2*Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[a/x^2 + b/x]*x)]
```

---

**Rubi [A]** time = 0.0765409, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1979, 2007, 2013, 620, 206}

$$2x\sqrt{\frac{a}{x^2}+\frac{b}{x}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+\frac{b}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/x^2],x]
```

```
[Out] 2*Sqrt[a/x^2 + b/x]*x - 2*Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[a/x^2 + b/x]*x)]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{a+bx}{x^2}}\,dx &= \int \sqrt{\frac{a}{x^2}+\frac{b}{x}}\,dx\\
&= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}x + a\int \frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}x^2}\,dx\\
&= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}x - a\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \frac{1}{x}\right)\\
&= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}x - (2a)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}x}\right)\\
&= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}x - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}x}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.0256985, size = 58, normalized size = 1.14

$$
\frac{2x\sqrt{\frac{a+bx}{x^2}}\left(\sqrt{a+bx}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a+bx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/x^2],x]
```

```
[Out] (2*x*Sqrt[(a + b*x)/x^2]*(Sqrt[a + b*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqr
t[a]]))/Sqrt[a + b*x]
```

---

**Maple [A]** time = 0.007, size = 47, normalized size = 0.9

$$
2\,\frac{x}{\sqrt{bx+a}}\sqrt{\frac{bx+a}{x^2}}\left(-\sqrt{a}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{bx+a}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/x^2)^(1/2),x)
```

```
[Out] 2*((b*x+a)/x^2)^(1/2)*x*(-a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^(1
/2))/(b*x+a)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \sqrt{\frac{bx+a}{x^2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x + a)/x^2), x)
```

---

**Fricas [A]** time = 0.916763, size = 231, normalized size = 4.53

$$\left[2\,x\sqrt{\frac{bx+a}{x^2}} + \sqrt{a}\log\left(\frac{bx - 2\,\sqrt{a}x\sqrt{\frac{bx+a}{x^2}} + 2\,a}{x}\right), 2\,x\sqrt{\frac{bx+a}{x^2}} + 2\,\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx+a}{x^2}}}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*x*sqrt((b*x + a)/x^2) + sqrt(a)*log((b*x - 2*sqrt(a)*x*sqrt((b*x + a)/x^
2) + 2*a)/x), 2*x*sqrt((b*x + a)/x^2) + 2*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((
b*x + a)/x^2)/a)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{a+bx}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x)/x**2), x)
```

---

**Giac [A]** time = 1.12506, size = 88, normalized size = 1.73

$$2\left(\frac{a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{bx+a}\right)\operatorname{sgn}(x) - \frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) + \sqrt{-a}\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x + a))*sgn(x) - 2*(a
*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

## 3.383 $\int \sqrt{\frac{a+bx^2}{x^2}}\,dx$

**Optimal**. Leaf size=42

$$x\sqrt{\frac{a}{x^2}+b}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+b}}\right)$$

```
[Out] Sqrt[b + a/x^2]*x - Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[b + a/x^2]*x)]
```

---

**Rubi [A]** time = 0.0229586, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1972, 242, 277, 217, 206}

$$x\sqrt{\frac{a}{x^2}+b}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b + a/x^2]*x - Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[b + a/x^2]*x)]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{a+bx^2}{x^2}}\,dx &= \int \sqrt{b+\frac{a}{x^2}}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{\sqrt{b+ax^2}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{b+\frac{a}{x^2}}x - a\operatorname{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{b+\frac{a}{x^2}}x - a\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{b+\frac{a}{x^2}}x}\right) \\
&= \sqrt{b+\frac{a}{x^2}}x - \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{b+\frac{a}{x^2}}x}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.0280174, size = 62, normalized size = 1.48

$$
x\sqrt{\frac{a}{x^2}+b} - \frac{\sqrt{a}x\sqrt{\frac{a}{x^2}+b}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b + a/x^2]*x - (Sqrt[a]*Sqrt[b + a/x^2]*x*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]])/Sqrt[a + b*x^2]
```

---

**Maple [A]** time = 0.005, size = 61, normalized size = 1.5

$$
x\sqrt{\frac{bx^2+a}{x^2}}\left(\sqrt{bx^2+a} - \sqrt{a}\ln\left(2\,\frac{\sqrt{a}\sqrt{bx^2+a}+a}{x}\right)\right)\frac{1}{\sqrt{bx^2+a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/x^2)^(1/2),x)
```

```
[Out] ((b*x^2+a)/x^2)^(1/2)*x/(b*x^2+a)^(1/2)*((b*x^2+a)^(1/2)-a^(1/2)*ln(2*(a^(1
/2)*(b*x^2+a)^(1/2)+a)/x))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.85997, size = 259, normalized size = 6.17

$$\left[x\sqrt{\frac{bx^2+a}{x^2}}+\frac{1}{2}\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{a}x\sqrt{\frac{bx^2+a}{x^2}}+2a}{x^2}\right),x\sqrt{\frac{bx^2+a}{x^2}}+\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx^2+a}{x^2}}}{bx^2+a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [x*sqrt((b*x^2 + a)/x^2) + 1/2*sqrt(a)*log(-(b*x^2 - 2*sqrt(a)*x*sqrt((b*x^
2 + a)/x^2) + 2*a)/x^2), x*sqrt((b*x^2 + a)/x^2) + sqrt(-a)*arctan(sqrt(-a)
*x*sqrt((b*x^2 + a)/x^2)/(b*x^2 + a))]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{a+bx^2}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x**2)/x**2), x)
```

---

**Giac [A]** time = 1.24815, size = 92, normalized size = 2.19

$$\left(\frac{a\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{bx^2+a}\right)\operatorname{sgn}(x)-\frac{\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] (a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x^2 + a))*sgn(x) - (a
*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

## 3.384 $\int \sqrt{\frac{a+bx^3}{x^2}}\,dx$

**Optimal**. Leaf size=51

$$\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)$$

```
[Out] (2*x*Sqrt[a/x^2 + b*x])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x]
)])/3
```

---

**Rubi [A]** time = 0.0649497, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1979, 2007, 2029, 206}

$$\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x]
)])/3
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{a+bx^3}{x^2}}\,dx &= \int \sqrt{\frac{a}{x^2}+bx}\,dx \\
&= \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}+a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx}}\,dx \\
&= \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x\sqrt{\frac{a}{x^2}+bx}}\right) \\
&= \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.0366667, size = 66, normalized size = 1.29

$$
\frac{2x\sqrt{\frac{a}{x^2}+bx}\left(\sqrt{a+bx^3}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)}{3\sqrt{a+bx^3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x]*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]]))/(3*Sqrt[a + b*x^3])
```

---

**Maple [A]** time = 0.007, size = 55, normalized size = 1.1

$$
\frac{2x}{3}\sqrt{\frac{bx^3+a}{x^2}}\left(-\sqrt{a}\text{Artanh}\left(\sqrt{bx^3+a}\frac{1}{\sqrt{a}}\right)+\sqrt{bx^3+a}\right)\frac{1}{\sqrt{bx^3+a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)/x^2)^(1/2),x)
```

```
[Out] 2/3*((b*x^3+a)/x^2)^(1/2)*x*(-a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))+(b*x
^3+a)^(1/2))/(b*x^3+a)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \sqrt{\frac{bx^3+a}{x^2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^3 + a)/x^2), x)
```

---

**Fricas [A]** time = 0.846674, size = 261, normalized size = 5.12

$$\left[\frac{2}{3}\,x\sqrt{\frac{bx^3+a}{x^2}}+\frac{1}{3}\,\sqrt{a}\log\left(\frac{bx^3-2\,\sqrt{a}x\sqrt{\frac{bx^3+a}{x^2}}+2\,a}{x^3}\right),\frac{2}{3}\,x\sqrt{\frac{bx^3+a}{x^2}}+\frac{2}{3}\,\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx^3+a}{x^2}}}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*x*sqrt((b*x^3 + a)/x^2) + 1/3*sqrt(a)*log((b*x^3 - 2*sqrt(a)*x*sqrt((b
*x^3 + a)/x^2) + 2*a)/x^3), 2/3*x*sqrt((b*x^3 + a)/x^2) + 2/3*sqrt(-a)*arct
an(sqrt(-a)*x*sqrt((b*x^3 + a)/x^2)/a)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.13836, size = 93, normalized size = 1.82

$$\frac{2}{3}\left(\frac{a\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{bx^3+a}\right)\mathrm{sgn}\left(x\right)-\frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\mathrm{sgn}\left(x\right)}{3\,\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(a*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x^3 + a))*sgn(x)
- 2/3*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

## 3.385 $\int \sqrt{\frac{a+bx^n}{x^2}}\,dx$

**Optimal**. Leaf size=61

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^{n-2}}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{n-2}}}\right)}{n}$$

```
[Out] (2*x*Sqrt[a/x^2 + b*x^(-2 + n)])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x
^2 + b*x^(-2 + n)])])/n
```

---

**Rubi [A]** time = 0.0799542, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1979, 2007, 2029, 206}

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^{n-2}}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{n-2}}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x^n)/x^2],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x^(-2 + n)])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x
^2 + b*x^(-2 + n)])])/n
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{a+bx^n}{x^2}}\,dx &= \int \sqrt{\frac{a}{x^2}+bx^{-2+n}}\,dx \\
&= \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n} + a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\,dx \\
&= \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n} - \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\right)}{n} \\
&= \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}
\end{aligned}
$$

**Mathematica [A]** time = 0.0209261, size = 70, normalized size = 1.15

$$
\frac{x\sqrt{\frac{a+bx^n}{x^2}}\left(2\sqrt{a+bx^n}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)}{n\sqrt{a+bx^n}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x^n)/x^2],x]
```

```
[Out] (x*Sqrt[(a + b*x^n)/x^2]*(2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*
x^n]/Sqrt[a]]))/(n*Sqrt[a + b*x^n])
```

---

**Maple [A]** time = 1.267, size = 74, normalized size = 1.2

$$
2\,\frac{x}{n}\sqrt{\frac{a+b\mathrm{e}^{n\ln(x)}}{x^2}} - 2\,\frac{x\sqrt{a}}{n\sqrt{a+b\mathrm{e}^{n\ln(x)}}}\mathrm{Artanh}\left(\frac{\sqrt{a+b\mathrm{e}^{n\ln(x)}}}{\sqrt{a}}\right)\sqrt{\frac{a+b\mathrm{e}^{n\ln(x)}}{x^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a+b*x^n)/x^2)^(1/2),x)
```

```
[Out] 2/n*((a+b*exp(n*ln(x)))/x^2)^(1/2)*x-2*a^(1/2)/n*arctanh((a+b*exp(n*ln(x)))
^(1/2)/a^(1/2))*((a+b*exp(n*ln(x)))/x^2)^(1/2)/(a+b*exp(n*ln(x)))^(1/2)*x
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \sqrt{\frac{bx^n+a}{x^2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^n + a)/x^2), x)
```

---

**Fricas [A]** time = 0.775976, size = 255, normalized size = 4.18

$$\left[\frac{2\,x\sqrt{\frac{bx^{n}+a}{x^{2}}}+\sqrt{a}\log\left(\frac{bx^{n}-2\,\sqrt{a}x\sqrt{\frac{bx^{n}+a}{x^{2}}}+2\,a}{x^{n}}\right)}{n},\ \frac{2\left(x\sqrt{\frac{bx^{n}+a}{x^{2}}}+\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx^{n}+a}{x^{2}}}}{a}\right)\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*x*sqrt((b*x^n + a)/x^2) + sqrt(a)*log((b*x^n - 2*sqrt(a)*x*sqrt((b*x^n
+ a)/x^2) + 2*a)/x^n))/n, 2*(x*sqrt((b*x^n + a)/x^2) + sqrt(-a)*arctan(sqrt
(-a)*x*sqrt((b*x^n + a)/x^2)/a))/n]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x**n)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{bx^{n}+a}{x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^n + a)/x^2), x)
```

## 3.386 $\int \sqrt{\frac{-a+bx}{x^2}}\,dx$

**Optimal**. Leaf size=53

$$2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}+2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{b}{x}-\frac{a}{x^2}}}\right)$$

```
[Out] 2*Sqrt[-(a/x^2) + b/x]*x + 2*Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[-(a/x^2) + b/x]*x
)]
```

---

**Rubi [A]** time = 0.0789457, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1979, 2007, 2013, 620, 203}

$$2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}+2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{b}{x}-\frac{a}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x)/x^2],x]
```

```
[Out] 2*Sqrt[-(a/x^2) + b/x]*x + 2*Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[-(a/x^2) + b/x]*x
)]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{-a+bx}{x^2}}\,dx &= \int \sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,dx\\
&= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x - a\int \frac{1}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x^2}\,dx\\
&= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x + a\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx-ax^2}}\,dx, x, \frac{1}{x}\right)\\
&= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x + (2a)\,\mathrm{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x}\right)\\
&= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x + 2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}x}\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.0295023, size = 66, normalized size = 1.25

$$\frac{2x\sqrt{\frac{bx-a}{x^2}}\left(\sqrt{bx-a}-\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\right)}{\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x)/x^2],x]
```

```
[Out] (2*x*Sqrt[(-a + b*x)/x^2]*(Sqrt[-a + b*x] - Sqrt[a]*ArcTan[Sqrt[-a + b*x]/S
qrt[a]]))/Sqrt[-a + b*x]
```

---

**Maple [A]** time = 0.01, size = 56, normalized size = 1.1

$$-2\,\frac{x}{\sqrt{bx-a}}\sqrt{\frac{bx-a}{x^2}}\left(\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\sqrt{bx-a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x-a)/x^2)^(1/2),x)
```

```
[Out] -2*((b*x-a)/x^2)^(1/2)*x*(a^(1/2)*arctan((b*x-a)^(1/2)/a^(1/2))-(b*x-a)^(1/
2))/(b*x-a)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{bx-a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x - a)/x^2), x)
```

---

**Fricas [A]** time = 0.857931, size = 228, normalized size = 4.3

$$\left[2\,x\sqrt{\frac{bx-a}{x^2}}+\sqrt{-a}\log\left(\frac{bx-2\,\sqrt{-a}x\sqrt{\frac{bx-a}{x^2}}-2\,a}{x}\right),2\,x\sqrt{\frac{bx-a}{x^2}}-2\,\sqrt{a}\arctan\left(\frac{x\sqrt{\frac{bx-a}{x^2}}}{\sqrt{a}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*x*sqrt((b*x - a)/x^2) + sqrt(-a)*log((b*x - 2*sqrt(-a)*x*sqrt((b*x - a)/
x^2) - 2*a)/x), 2*x*sqrt((b*x - a)/x^2) - 2*sqrt(a)*arctan(x*sqrt((b*x - a)
/x^2)/sqrt(a))]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{-a+bx}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x-a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((-a + b*x)/x**2), x)
```

---

**Giac [A]** time = 1.28611, size = 82, normalized size = 1.55

$$-2\left(\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\sqrt{bx-a}\right)\operatorname{sgn}(x)+2\left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)-\sqrt{-a}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*(sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) - sqrt(b*x - a))*sgn(x) + 2*(sqrt
(a)*arctan(sqrt(-a)/sqrt(a)) - sqrt(-a))*sgn(x)
```

## 3.387 $\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx$

**Optimal**. Leaf size=43

$$x\sqrt{b-\frac{a}{x^2}}+\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{b-\frac{a}{x^2}}}\right)$$

```
[Out] Sqrt[b - a/x^2]*x + Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[b - a/x^2]*x)]
```

---

**Rubi [A]** time = 0.023209, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1972, 242, 277, 217, 203}

$$x\sqrt{b-\frac{a}{x^2}}+\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{b-\frac{a}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b - a/x^2]*x + Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[b - a/x^2]*x)]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx &= \int \sqrt{b-\frac{a}{x^2}}\,dx\\ &= -\operatorname{Subst}\left(\int \frac{\sqrt{b-ax^2}}{x^2}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{b-\frac{a}{x^2}}x + a\operatorname{Subst}\left(\int \frac{1}{\sqrt{b-ax^2}}\,dx, x, \frac{1}{x}\right)\\ &= \sqrt{b-\frac{a}{x^2}}x + a\operatorname{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{\sqrt{b-\frac{a}{x^2}}x}\right)\\ &= \sqrt{b-\frac{a}{x^2}}x + \sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{b-\frac{a}{x^2}}x}\right)\end{aligned}$$

**Mathematica [A]** time = 0.0314661, size = 68, normalized size = 1.58

$$x\sqrt{b-\frac{a}{x^2}} - \frac{\sqrt{a}x\sqrt{b-\frac{a}{x^2}}\tan^{-1}\left(\frac{\sqrt{bx^2-a}}{\sqrt{a}}\right)}{\sqrt{bx^2-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b - a/x^2]*x - (Sqrt[a]*Sqrt[b - a/x^2]*x*ArcTan[Sqrt[-a + b*x^2]/Sqrt
[a]])/Sqrt[-a + b*x^2]
```

---

**Maple [B]** time = 0.008, size = 81, normalized size = 1.9

$$x\sqrt{\frac{bx^2-a}{x^2}}\left(a\ln\left(2\frac{\sqrt{-a}\sqrt{bx^2-a}-a}{x}\right)+\sqrt{-a}\sqrt{bx^2-a}\right)\frac{1}{\sqrt{-a}}\frac{1}{\sqrt{bx^2-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2-a)/x^2)^(1/2),x)
```

```
[Out] ((b*x^2-a)/x^2)^(1/2)*x*(a*ln(2*((-a)^(1/2)*(b*x^2-a)^(1/2)-a)/x)+(-a)^(1/2
)*(b*x^2-a)^(1/2))/(-a)^(1/2)/(b*x^2-a)^(1/2)
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.75714, size = 259, normalized size = 6.02

$$\left[x\sqrt{\frac{bx^2-a}{x^2}}+\frac{1}{2}\sqrt{-a}\log\left(-\frac{bx^2-2\sqrt{-a}x\sqrt{\frac{bx^2-a}{x^2}}-2a}{x^2}\right),x\sqrt{\frac{bx^2-a}{x^2}}+\sqrt{a}\arctan\left(\frac{\sqrt{a}x\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [x*sqrt((b*x^2 - a)/x^2) + 1/2*sqrt(-a)*log(-(b*x^2 - 2*sqrt(-a)*x*sqrt((b*
x^2 - a)/x^2) - 2*a)/x^2), x*sqrt((b*x^2 - a)/x^2) + sqrt(a)*arctan(sqrt(a)
*x*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a))]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{-a+bx^2}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2-a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((-a + b*x**2)/x**2), x)
```

---

**Giac [A]** time = 1.22661, size = 86, normalized size = 2.

$$-\left(\sqrt{a}\arctan\left(\frac{\sqrt{bx^2-a}}{\sqrt{a}}\right)-\sqrt{bx^2-a}\right)\operatorname{sgn}(x)+\left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)-\sqrt{-a}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(a)*arctan(sqrt(b*x^2 - a)/sqrt(a)) - sqrt(b*x^2 - a))*sgn(x) + (sqrt
(a)*arctan(sqrt(-a)/sqrt(a)) - sqrt(-a))*sgn(x)
```

## 3.388 $\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx$

**Optimal**. Leaf size=53

$$\frac{2}{3}x\sqrt{bx-\frac{a}{x^2}}+\frac{2}{3}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx-\frac{a}{x^2}}}\right)$$

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x])/3 + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-(a/x^2) +
 b*x])])/3
```

---

**Rubi [A]** time = 0.0679645, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1979, 2007, 2029, 203}

$$\frac{2}{3}x\sqrt{bx-\frac{a}{x^2}}+\frac{2}{3}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx-\frac{a}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x])/3 + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-(a/x^2) +
 b*x])])/3
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx &= \int \sqrt{-\frac{a}{x^2}+bx}\,dx\\ &= \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx} - a\int \frac{1}{x^2\sqrt{-\frac{a}{x^2}+bx}}\,dx\\ &= \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx} + \frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{x\sqrt{-\frac{a}{x^2}+bx}}\right)\\ &= \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx} + \frac{2}{3}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx}}\right)\end{aligned}$$

**Mathematica [A]** time = 0.0403021, size = 73, normalized size = 1.38

$$\frac{2x\sqrt{bx-\frac{a}{x^2}}\left(\sqrt{bx^3-a}-\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bx^3-a}}{\sqrt{a}}\right)\right)}{3\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x]*(Sqrt[-a + b*x^3] - Sqrt[a]*ArcTan[Sqrt[-a + b*x^
3]/Sqrt[a]]))/(3*Sqrt[-a + b*x^3])
```

---

**Maple [A]** time = 0.019, size = 73, normalized size = 1.4

$$\frac{2x}{3}\sqrt{\frac{bx^3-a}{x^2}}\left(\sqrt{bx^3-a}\sqrt{-a}+a\text{Artanh}\left(\sqrt{bx^3-a}\frac{1}{\sqrt{-a}}\right)\right)\frac{1}{\sqrt{bx^3-a}}\frac{1}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3-a)/x^2)^(1/2),x)
```

```
[Out] 2/3*((b*x^3-a)/x^2)^(1/2)*x*((b*x^3-a)^(1/2)*(-a)^(1/2)+a*arctanh((b*x^3-a)
^(1/2)/(-a)^(1/2)))/(b*x^3-a)^(1/2)/(-a)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{bx^3-a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^3 - a)/x^2), x)
```

---

**Fricas [A]** time = 0.821793, size = 258, normalized size = 4.87

$$\left[\frac{2}{3}x\sqrt{\frac{bx^3-a}{x^2}}+\frac{1}{3}\sqrt{-a}\log\left(\frac{bx^3-2\sqrt{-a}x\sqrt{\frac{bx^3-a}{x^2}}-2a}{x^3}\right),\frac{2}{3}x\sqrt{\frac{bx^3-a}{x^2}}-\frac{2}{3}\sqrt{a}\arctan\left(\frac{x\sqrt{\frac{bx^3-a}{x^2}}}{\sqrt{a}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*x*sqrt((b*x^3 - a)/x^2) + 1/3*sqrt(-a)*log((b*x^3 - 2*sqrt(-a)*x*sqrt(
(b*x^3 - a)/x^2) - 2*a)/x^3), 2/3*x*sqrt((b*x^3 - a)/x^2) - 2/3*sqrt(a)*arc
tan(x*sqrt((b*x^3 - a)/x^2)/sqrt(a))]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3-a)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.30164, size = 88, normalized size = 1.66

$$-\frac{2}{3}\left(\sqrt{a}\arctan\left(\frac{\sqrt{bx^3-a}}{\sqrt{a}}\right)-\sqrt{bx^3-a}\right)\operatorname{sgn}(x)+\frac{2}{3}\left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)-\sqrt{-a}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(sqrt(a)*arctan(sqrt(b*x^3 - a)/sqrt(a)) - sqrt(b*x^3 - a))*sgn(x) + 2
/3*(sqrt(a)*arctan(sqrt(-a)/sqrt(a)) - sqrt(-a))*sgn(x)
```

## 3.389 $\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx$

**Optimal**. Leaf size=63

$$\frac{2x\sqrt{bx^{n-2}-\frac{a}{x^2}}}{n}+\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx^{n-2}-\frac{a}{x^2}}}\right)}{n}$$

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x^(-2 + n)])/n + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-
(a/x^2) + b*x^(-2 + n)])])/n
```

---

**Rubi [A]** time = 0.0804348, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1979, 2007, 2029, 203}

$$\frac{2x\sqrt{bx^{n-2}-\frac{a}{x^2}}}{n}+\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx^{n-2}-\frac{a}{x^2}}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x^n)/x^2],x]
```

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x^(-2 + n)])/n + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-
(a/x^2) + b*x^(-2 + n)])])/n
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx &= \int \sqrt{-\frac{a}{x^2}+bx^{-2+n}}\,dx\\ &= \frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n} - a\int \frac{1}{x^2\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\,dx\\ &= \frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}\\ &= \frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n} + \frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}\end{aligned}$$

**Mathematica [A]** time = 0.0293767, size = 78, normalized size = 1.24

$$\frac{x\sqrt{\frac{bx^n-a}{x^2}}\left(2\sqrt{bx^n-a}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bx^n-a}}{\sqrt{a}}\right)\right)}{n\sqrt{bx^n-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x^n)/x^2],x]
```

```
[Out] (x*Sqrt[(-a + b*x^n)/x^2]*(2*Sqrt[-a + b*x^n] - 2*Sqrt[a]*ArcTan[Sqrt[-a +
b*x^n]/Sqrt[a]]))/(n*Sqrt[-a + b*x^n])
```

---

**Maple [A]** time = 0.721, size = 105, normalized size = 1.7

$$-2\,\frac{\left(a-b\mathrm{e}^{n\ln(x)}\right)x}{n\left(b\mathrm{e}^{n\ln(x)}-a\right)}\sqrt{\frac{b\mathrm{e}^{n\ln(x)}-a}{x^2}}-2\,\frac{x\sqrt{a}}{n\sqrt{b\mathrm{e}^{n\ln(x)}-a}}\arctan\left(\frac{\sqrt{b\mathrm{e}^{n\ln(x)}-a}}{\sqrt{a}}\right)\sqrt{\frac{b\mathrm{e}^{n\ln(x)}-a}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^n-a)/x^2)^(1/2),x)
```

```
[Out] -2*(a-b*exp(n*ln(x)))/n/(b*exp(n*ln(x))-a)*((b*exp(n*ln(x))-a)/x^2)^(1/2)*x
-2*a^(1/2)/n*arctan((b*exp(n*ln(x))-a)^(1/2)/a^(1/2))*((b*exp(n*ln(x))-a)/x
^2)^(1/2)/(b*exp(n*ln(x))-a)^(1/2)*x
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{bx^n-a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^n - a)/x^2), x)
```

---

**Fricas [A]** time = 0.928918, size = 252, normalized size = 4.

$$\left[\frac{2\,x\sqrt{\frac{bx^{n}-a}{x^{2}}}+\sqrt{-a}\log\left(\frac{bx^{n}-2\,\sqrt{-a}x\sqrt{\frac{bx^{n}-a}{x^{2}}}-2\,a}{x^{n}}\right)}{n},\ \frac{2\left(x\sqrt{\frac{bx^{n}-a}{x^{2}}}-\sqrt{a}\arctan\left(\frac{x\sqrt{\frac{bx^{n}-a}{x^{2}}}}{\sqrt{a}}\right)\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*x*sqrt((b*x^n - a)/x^2) + sqrt(-a)*log((b*x^n - 2*sqrt(-a)*x*sqrt((b*x^
n - a)/x^2) - 2*a)/x^n))/n, 2*(x*sqrt((b*x^n - a)/x^2) - sqrt(a)*arctan(x*s
qrt((b*x^n - a)/x^2)/sqrt(a)))/n]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+b*x**n)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{bx^{n}-a}{x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^n - a)/x^2), x)
```

## 3.390 $\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx$

**Optimal**. Leaf size=62

$$\frac{2x^{-j/2}(cx)^{j/2}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}c(j-n)}$$

```
[Out] (2*(c*x)^(j/2)*ArcTanh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(Sqrt[a]*c*(
j - n)*x^(j/2))
```

---

**Rubi [A]** time = 0.107995, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2031, 2029, 206}

$$\frac{2x^{-j/2}(cx)^{j/2}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}c(j-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*(c*x)^(j/2)*ArcTanh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(Sqrt[a]*c*(
j - n)*x^(j/2))
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx=\frac{\left(x^{-j/2}(cx)^{j/2}\right)\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{c}$$

$$=\frac{\left(2x^{-j/2}(cx)^{j/2}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}$$

$$=\frac{2x^{-j/2}(cx)^{j/2}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}c(j-n)}$$

**Mathematica [A]** time = 0.140186, size = 98, normalized size = 1.58

$$\frac{2\sqrt{b}(cx)^{j/2}x^{\frac{n-j}{2}}\sqrt{\frac{ax^{j-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}c(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*Sqrt[b]*x^((-j + n)/2)*(c*x)^(j/2)*Sqrt[1 + (a*x^(j - n))/b]*ArcSinh[(Sq
rt[a]*x^((j - n)/2))/Sqrt[b]])/(Sqrt[a]*c*(j - n)*Sqrt[a*x^j + b*x^n])
```

---

**Maple [F]** time = 0.374, size = 0, normalized size = 0.

$$\int (cx)^{-1+\frac{j}{2}}\frac{1}{\sqrt{ax^j+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{1}{2}j-1}}{\sqrt{ax^j+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{j}{2}-1}}{\sqrt{ax^j + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+1/2*j)/(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral((c*x)**(j/2 - 1)/sqrt(a*x**j + b*x**n), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{1}{2}j-1}}{\sqrt{ax^j + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)
```

## 3.391 $\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$

**Optimal**. Leaf size=53

$$\frac{2\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}(3-n)\sqrt{x}}$$

```
[Out] (2*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(Sqrt[a]*(3 -
n)*Sqrt[x])
```

---

**Rubi [A]** time = 0.0987987, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2031, 2029, 206}

$$\frac{2\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}(3-n)\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n],x]
```

```
[Out] (2*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(Sqrt[a]*(3 -
n)*Sqrt[x])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx = \frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{\sqrt{x}}$$
$$= \frac{\left(2\sqrt{cx}\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{(3-n)\sqrt{x}}$$
$$= \frac{2\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}(3-n)\sqrt{x}}$$

**Mathematica [A]** time = 0.153705, size = 89, normalized size = 1.68

$$-\frac{2\sqrt{b}\sqrt{cx}x^{\frac{n-1}{2}}\sqrt{\frac{ax^{3-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}(n-3)\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n],x]
```

```
[Out] (-2*Sqrt[b]*x^((-1 + n)/2)*Sqrt[c*x]*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sqr
t[a]*x^(3/2 - n/2))/Sqrt[b]])/(Sqrt[a]*(-3 + n)*Sqrt[a*x^3 + b*x^n])
```

---

**Maple [F]** time = 0.331, size = 0, normalized size = 0.

$$\int \sqrt{cx}\frac{1}{\sqrt{ax^3+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(a*x**3+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(c*x)/sqrt(a*x**3 + b*x**n), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)
```

## 3.392 $\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx$

**Optimal**. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}(2-n)}$$

```
[Out] (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(Sqrt[a]*(2 - n))
```

---

**Rubi [A]** time = 0.015101, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^2 + b*x^n],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(Sqrt[a]*(2 - n))
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^n}}\right)}{2-n}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}(2-n)}\end{aligned}$$

**Mathematica [B]** time = 0.0681361, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{b}x^{n/2}\sqrt{\frac{ax^{2-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}(n-2)\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^n],x]
```

```
[Out] (-2*Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2))
/Sqrt[b]])/(Sqrt[a]*(-2 + n)*Sqrt[a*x^2 + b*x^n])
```

---

**Maple [F]** time = 0.326, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+b*x^n)^(1/2),x)
```

```
[Out] int(1/(a*x^2+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x^2 + b*x^n), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**n), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*x^2 + b*x^n), x)
```

## 3.393 $\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx$

**Optimal**. Leaf size=51

$$\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}(1-n)\sqrt{cx}}$$

```
[Out] (2*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(Sqrt[a]*(1 - n)*S
qrt[c*x])
```

---

**Rubi [A]** time = 0.0852557, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2031, 2029, 206}

$$\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}(1-n)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]
```

```
[Out] (2*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(Sqrt[a]*(1 - n)*S
qrt[c*x])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx &= \frac{\sqrt{x}\int \frac{1}{\sqrt{x}\sqrt{ax+bx^n}}\,dx}{\sqrt{cx}} \\ &= \frac{\left(2\sqrt{x}\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{(1-n)\sqrt{cx}} \\ &= \frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}(1-n)\sqrt{cx}}\end{aligned}$$

**Mathematica [A]** time = 0.119446, size = 87, normalized size = 1.71

$$-\frac{2\sqrt{b}x^{\frac{n+1}{2}}\sqrt{\frac{ax^{1-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}(n-1)\sqrt{cx}\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]
```

```
[Out] (-2*Sqrt[b]*x^((1 + n)/2)*Sqrt[1 + (a*x^(1 - n))/b]*ArcSinh[(Sqrt[a]*x^(1/2
 - n/2))/Sqrt[b]])/(Sqrt[a]*(-1 + n)*Sqrt[c*x]*Sqrt[a*x + b*x^n])
```

---

**Maple [F]** time = 0.338, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx}}\frac{1}{\sqrt{ax+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
```

```
[Out] int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+bx^n}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(a*x+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*x)*sqrt(a*x + b*x**n)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax+bx^n}\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)
```

## 3.394 $\int \frac{1}{cx\sqrt{a+bx^n}}\,dx$

**Optimal**. Leaf size=31

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}cn}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

---

**Rubi [A]** time = 0.0204733, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}cn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c*x*Sqrt[a + b*x^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{cx\sqrt{a+bx^n}}\,dx &= \frac{\int \frac{1}{x\sqrt{a+bx^n}}\,dx}{c}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^n\right)}{cn}\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^n}\right)}{bcn}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}cn}\end{aligned}$$

**Mathematica [A]** time = 0.0063857, size = 31, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c*x*Sqrt[a + b*x^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

---

**Maple [A]** time = 0.004, size = 26, normalized size = 0.8

$$-2\,\frac{1}{cn\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c/x/(a+b*x^n)^(1/2),x)
```

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/c/n/a^(1/2)
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.790381, size = 169, normalized size = 5.45

$$\left[\frac{\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)}{\sqrt{a}cn}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)}{acn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n)/(sqrt(a)*c*n), 2*sqrt(-
a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a)/(a*c*n)]
```

---

**Sympy [A]** time = 1.89404, size = 27, normalized size = 0.87

$$-\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x**n)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)*x**(-n/2)/sqrt(b))/(sqrt(a)*c*n)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^n + a}cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a)*c*x), x)
```

## 3.395 $\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx$

**Optimal**. Leaf size=54

$$-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}c(n+1)\sqrt{cx}}$$

```
[Out] (-2*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/(Sqrt[a]*c*(1 + n
)*Sqrt[c*x])
```

---

**Rubi [A]** time = 0.110832, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2031, 2029, 206}

$$-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}c(n+1)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]
```

```
[Out] (-2*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/(Sqrt[a]*c*(1 + n
)*Sqrt[c*x])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx=\frac{\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{c\sqrt{cx}}$$

$$=-\frac{\left(2\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{c(1+n)\sqrt{cx}}$$

$$=-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}c(1+n)\sqrt{cx}}$$

**Mathematica [A]** time = 0.0480262, size = 68, normalized size = 1.26

$$-\frac{2x\sqrt{a+bx^{n+1}}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right)}{\sqrt{a}(n+1)(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^(1 + n)]*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]])/(Sqrt[a
]*(1 + n)*(c*x)^(3/2)*Sqrt[a/x + b*x^n])
```

---

**Maple [F]** time = 0.339, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{3}{2}}\frac{1}{\sqrt{\frac{a}{x}+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x)
```

```
[Out] int(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^n+\frac{a}{x}}\,(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx)^{\frac{3}{2}} \sqrt{\frac{a}{x} + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(a/x+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/((c*x)**(3/2)*sqrt(a/x + b*x**n)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^n + \frac{a}{x}}\,(cx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)
```

## 3.396 $\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx$

**Optimal**. Leaf size=40

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}c^2(n+2)}$$

```
[Out] (-2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(Sqrt[a]*c^2*(2 + n))
```

---

**Rubi [A]** time = 0.0737836, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {12, 2029, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}c^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(Sqrt[a]*c^2*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx = \frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^2}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{c^2(2+n)}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}c^2(2+n)}$$

**Mathematica [A]** time = 0.0466403, size = 66, normalized size = 1.65

$$-\frac{2\sqrt{a+bx^{n+2}}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+2}}}{\sqrt{a}}\right)}{\sqrt{a}c^2(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]
```

```
[Out] (-2*Sqrt[a + b*x^(2 + n)]*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]])/(Sqrt[a]*
c^2*(2 + n)*x*Sqrt[a/x^2 + b*x^n])
```

---

**Maple [F]** time = 0.319, size = 0, normalized size = 0.

$$\int \frac{1}{c^2x^2}\frac{1}{\sqrt{\frac{a}{x^2}+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c^2/x^2/(1/x^2*a+b*x^n)^(1/2),x)
```

```
[Out] int(1/c^2/x^2/(1/x^2*a+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{bx^n+\frac{a}{x^2}}x^2}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x^2)*x^2), x)/c^2
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c**2/x**2/(a/x**2+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a/x**2 + b*x**n)), x)/c**2
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^n + \frac{a}{x^2}}c^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x^2)*c^2*x^2), x)
```

## 3.397 $\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx$

**Optimal**. Leaf size=54

$$-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}c^2(n+3)\sqrt{cx}}$$

```
[Out] (-2*Sqrt[x]*ArcTanh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(Sqrt[a]*c^2*(3
 + n)*Sqrt[c*x])
```

---

**Rubi [A]** time = 0.135793, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2031, 2029, 206}

$$-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}c^2(n+3)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]),x]
```

```
[Out] (-2*Sqrt[x]*ArcTanh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(Sqrt[a]*c^2*(3
 + n)*Sqrt[c*x])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx &= \frac{\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{c^2\sqrt{cx}}\\ &= -\frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{c^2(3+n)\sqrt{cx}}\\ &= -\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}c^2(3+n)\sqrt{cx}}\end{aligned}$$

**Mathematica [A]** time = 0.0500456, size = 68, normalized size = 1.26

$$-\frac{2x\sqrt{a+bx^{n+3}}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}}\right)}{\sqrt{a}(n+3)(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^(3 + n)]*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]])/(Sqrt[a
]*(3 + n)*(c*x)^(5/2)*Sqrt[a/x^3 + b*x^n])
```

---

**Maple [F]** time = 0.329, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{5}{2}}\frac{1}{\sqrt{\frac{a}{x^3}+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x)
```

```
[Out] int(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^n+\frac{a}{x^3}}\,(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(a/x**3+b*x**n)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^n + \frac{a}{x^3}}\,(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)
```

## 3.398 $\int \frac{(cx)^{-1+\frac{3j}{2}}}{\left(ax^j+bx^n\right)^{3/2}}\,dx$

**Optimal**. Leaf size=107

$$\frac{2x^{-3j/2}(cx)^{3j/2}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}-\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}$$

```
[Out] (-2*(c*x)^((3*j)/2))/(a*c*(j - n)*x^j*Sqrt[a*x^j + b*x^n]) + (2*(c*x)^((3*j
)/2)*ArcTanh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(a^(3/2)*c*(j - n)*x^(
(3*j)/2))
```

---

**Rubi [A]** time = 0.187196, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2031, 2030, 2029, 206}

$$\frac{2x^{-3j/2}(cx)^{3j/2}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}-\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*(c*x)^((3*j)/2))/(a*c*(j - n)*x^j*Sqrt[a*x^j + b*x^n]) + (2*(c*x)^((3*j
)/2)*ArcTanh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(a^(3/2)*c*(j - n)*x^(
(3*j)/2))
```

### Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

### Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

### Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(cx)^{-1+\frac{3j}{2}}}{\left(ax^j+bx^n\right)^{3/2}}\,dx &= \frac{\left(x^{-3j/2}(cx)^{3j/2}\right)\int \frac{x^{-1+\frac{3j}{2}}}{\left(ax^j+bx^n\right)^{3/2}}\,dx}{c}\\
&= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}+\frac{\left(x^{-3j/2}(cx)^{3j/2}\right)\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{ac}\\
&= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}+\frac{\left(2x^{-3j/2}(cx)^{3j/2}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{ac(j-n)}\\
&= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}+\frac{2x^{-3j/2}(cx)^{3j/2}\tanh^{-1}\left(\frac{\sqrt{a}x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}
\end{aligned}
$$

**Mathematica [A]** time = 0.17613, size = 117, normalized size = 1.09

$$
-\frac{2x^{-3j/2}(cx)^{3j/2}\left(\sqrt{a}x^{j/2}-\sqrt{b}x^{n/2}\sqrt{\frac{ax^{j-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}c(j-n)\sqrt{ax^j+bx^n}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*(c*x)^((3*j)/2)*(Sqrt[a]*x^(j/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(j - n
))/b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/(a^(3/2)*c*(j - n)*x^((3*j
)/2)*Sqrt[a*x^j + b*x^n])
```

---

**Maple [F]** time = 0.37, size = 0, normalized size = 0.

$$
\int (cx)^{-1+\frac{3j}{2}}\left(ax^j+bx^n\right)^{-\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \frac{(cx)^{\frac{3}{2}j-1}}{\left(ax^j+bx^n\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+3/2*j)/(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{3}{2}j-1}}{\left(ax^j + bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)
```

## 3.399 $\int \frac{(cx)^{7/2}}{\left(ax^3+bx^n\right)^{3/2}}\,dx$

**Optimal**. Leaf size=94

$$\frac{2c^3\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

```
[Out] (-2*c^2*(c*x)^(3/2))/(a*(3 - n)*Sqrt[a*x^3 + b*x^n]) + (2*c^3*Sqrt[c*x]*Arc
Tanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(a^(3/2)*(3 - n)*Sqrt[x])
```

---

**Rubi [A]** time = 0.159572, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2030, 2031, 2029, 206}

$$\frac{2c^3\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2),x]
```

```
[Out] (-2*c^2*(c*x)^(3/2))/(a*(3 - n)*Sqrt[a*x^3 + b*x^n]) + (2*c^3*Sqrt[c*x]*Arc
Tanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(a^(3/2)*(3 - n)*Sqrt[x])
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{7/2}}{\left(ax^3+bx^n\right)^{3/2}}\,dx &= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{c^3\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx}{a}\\
&= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{\left(c^3\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{a\sqrt{x}}\\
&= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{\left(2c^3\sqrt{cx}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a(3-n)\sqrt{x}}\\
&= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{2c^3\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}
\end{aligned}$$

**Mathematica [A]** time = 0.187117, size = 109, normalized size = 1.16

$$\frac{2c^3\sqrt{cx}\left(\sqrt{a}x^{3/2}-\sqrt{b}x^{n/2}\sqrt{\frac{ax^{3-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(n-3)\sqrt{x}\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^3*Sqrt[c*x]*(Sqrt[a]*x^(3/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(3 - n))/
b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]]))/(a^(3/2)*(-3 + n)*Sqrt[x]*Sqr
t[a*x^3 + b*x^n])
```

---

**Maple [F]** time = 0.322, size = 0, normalized size = 0.

$$\int (cx)^{\frac{7}{2}}\left(ax^3+bx^n\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(ax^3+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(a*x**3+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(ax^3 + bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)
```

## 3.400 $\int \frac{c^2x^2}{\left(ax^2+bx^n\right)^{3/2}} \, dx$

**Optimal**. Leaf size=72

$$\frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}-\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}$$

```
[Out] (-2*c^2*x)/(a*(2 - n)*Sqrt[a*x^2 + b*x^n]) + (2*c^2*ArcTanh[(Sqrt[a]*x)/Sqr
t[a*x^2 + b*x^n]])/(a^(3/2)*(2 - n))
```

---

**Rubi [A]** time = 0.0880041, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2030, 2008, 206}

$$\frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}-\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2),x]
```

```
[Out] (-2*c^2*x)/(a*(2 - n)*Sqrt[a*x^2 + b*x^n]) + (2*c^2*ArcTanh[(Sqrt[a]*x)/Sqr
t[a*x^2 + b*x^n]])/(a^(3/2)*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c^2x^2}{\left(ax^2+bx^n\right)^{3/2}}\,dx &= c^2\int \frac{x^2}{\left(ax^2+bx^n\right)^{3/2}}\,dx\\ &= -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}+\frac{c^2\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx}{a}\\ &= -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}+\frac{\left(2c^2\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^n}}\right)}{a(2-n)}\\ &= -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}+\frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}\end{aligned}$$

**Mathematica [A]** time = 0.138729, size = 91, normalized size = 1.26

$$\frac{2c^2\left(\sqrt{a}x-\sqrt{b}x^{n/2}\sqrt{\frac{ax^{2-n}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}x^{1-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(n-2)\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^2*(Sqrt[a]*x - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt
[a]*x^(1 - n/2))/Sqrt[b]]))/(a^(3/2)*(-2 + n)*Sqrt[a*x^2 + b*x^n])
```

---

**Maple [F]** time = 0.319, size = 0, normalized size = 0.

$$\int c^2x^2\left(ax^2+bx^n\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)
```

```
[Out] int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$c^2\int \frac{x^2}{\left(ax^2+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^2/(a*x^2 + b*x^n)^(3/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$c^2 \int \frac{x^2}{ax^2\sqrt{ax^2+bx^n}+bx^n\sqrt{ax^2+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**2*x**2/(a*x**2+b*x**n)**(3/2),x)
```

```
[Out] c**2*Integral(x**2/(a*x**2*sqrt(a*x**2 + b*x**n) + b*x**n*sqrt(a*x**2 + b*x
**n)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{c^2x^2}{\left(ax^2+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(c^2*x^2/(a*x^2 + b*x^n)^(3/2), x)
```

## 3.401 $\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx$

**Optimal**. Leaf size=85

$$\frac{2c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$

```
[Out] (-2*Sqrt[c*x])/(a*(1 - n)*Sqrt[a*x + b*x^n]) + (2*c*Sqrt[x]*ArcTanh[(Sqrt[a
]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(a^(3/2)*(1 - n)*Sqrt[c*x])
```

---

**Rubi [A]** time = 0.137894, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2030, 2031, 2029, 206}

$$\frac{2c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a*x + b*x^n)^(3/2),x]
```

```
[Out] (-2*Sqrt[c*x])/(a*(1 - n)*Sqrt[a*x + b*x^n]) + (2*c*Sqrt[x]*ArcTanh[(Sqrt[a
]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(a^(3/2)*(1 - n)*Sqrt[c*x])
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx &= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}+\frac{c\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx}{a}\\
&= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}+\frac{\left(c\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{ax+bx^n}}\,dx}{a\sqrt{cx}}\\
&= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}+\frac{\left(2c\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a(1-n)\sqrt{cx}}\\
&= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}+\frac{2c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}\end{aligned}$$

**Mathematica [A]** time = 0.186556, size = 104, normalized size = 1.22

$$\frac{2\sqrt{cx}\left(\sqrt{a}\sqrt{x}-\sqrt{b}x^{n/2}\sqrt{\frac{ax^{1-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(n-1)\sqrt{x}\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a*x + b*x^n)^(3/2),x]
```

```
[Out] (2*Sqrt[c*x]*(Sqrt[a]*Sqrt[x] - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(1 - n))/b]*A
rcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]]))/(a^(3/2)*(-1 + n)*Sqrt[x]*Sqrt[a*
x + b*x^n])
```

---

**Maple [F]** time = 0.335, size = 0, normalized size = 0.

$$\int \sqrt{cx}\,(ax+bx^n)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(ax + bx^n)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(a*x+b*x**n)**(3/2),x)
```

```
[Out] Integral(sqrt(c*x)/(a*x + b*x**n)**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx}}{(ax + bx^n)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)
```

## 3.402 $\int \frac{1}{cx(a+bx^n)^{3/2}}\,dx$

**Optimal**. Leaf size=54

$$\frac{2}{acn\sqrt{a+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

```
[Out] 2/(a*c*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2)*c
*n)
```

---

**Rubi [A]** time = 0.0332688, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {12, 266, 51, 63, 208}

$$\frac{2}{acn\sqrt{a+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c*x*(a + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2)*c
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{cx(a+bx^n)^{3/2}}\,dx &= \frac{\int \frac{1}{x(a+bx^n)^{3/2}}\,dx}{c}\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^n\right)}{cn}\\ &= \frac{2}{acn\sqrt{a+bx^n}}+\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{acn}\\ &= \frac{2}{acn\sqrt{a+bx^n}}+\frac{2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{abcn}\\ &= \frac{2}{acn\sqrt{a+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}\end{aligned}$$

**Mathematica [C]** time = 0.0140132, size = 40, normalized size = 0.74

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^n}{a}+1\right)}{acn\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c*x*(a + b*x^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^n)/a])/(a*c*n*Sqrt[a + b*x^n])
```

---

**Maple [A]** time = 0.003, size = 42, normalized size = 0.8

$$\frac{1}{cn}\left(-2\frac{1}{a^{3/2}}\operatorname{Artanh}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)+2\frac{1}{a\sqrt{a+bx^n}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c/x/(a+b*x^n)^(3/2),x)
```

```
[Out] 1/c/n*(-2/a^(3/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2))+2/a/(a+b*x^n)^(1/2))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.846647, size = 335, normalized size = 6.2

$$\left[\frac{\left(\sqrt{a}bx^{n}+a^{\frac{3}{2}}\right)\log\left(\frac{bx^{n}-2\sqrt{bx^{n}+a}\sqrt{a}+2a}{x^{n}}\right)+2\sqrt{bx^{n}+a}a}{a^{2}bcnx^{n}+a^{3}cn}, \frac{2\left(\left(\sqrt{-a}bx^{n}+\sqrt{-a}a\right)\arctan\left(\frac{\sqrt{bx^{n}+a}\sqrt{-a}}{a}\right)+\sqrt{bx^{n}+a}a\right)}{a^{2}bcnx^{n}+a^{3}cn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [((sqrt(a)*b*x^n + a^(3/2))*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x
^n) + 2*sqrt(b*x^n + a)*a)/(a^2*b*c*n*x^n + a^3*c*n), 2*((sqrt(-a)*b*x^n +
sqrt(-a)*a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n + a)*a)/(a^2*b*
c*n*x^n + a^3*c*n)]
```

---

**Sympy [B]** time = 3.49617, size = 185, normalized size = 3.43

$$\frac{\frac{2a^{3}\sqrt{1+\frac{bx^{n}}{a}}}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^{n}}+\frac{a^{3}\log\left(\frac{bx^{n}}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^{n}}-\frac{2a^{3}\log\left(\sqrt{1+\frac{bx^{n}}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^{n}}+\frac{a^{2}bx^{n}\log\left(\frac{bx^{n}}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^{n}}-\frac{2a^{2}bx^{n}\log\left(\sqrt{1+\frac{bx^{n}}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^{n}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x**n)**(3/2),x)
```

```
[Out] (2*a**3*sqrt(1 + b*x**n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**3*log(b*x*
*n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) - 2*a**3*log(sqrt(1 + b*x**n/a) + 1)
/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**2*b*x**n*log(b*x**n/a)/(a**(9/2)*n +
 a**(7/2)*b*n*x**n) - 2*a**2*b*x**n*log(sqrt(1 + b*x**n/a) + 1)/(a**(9/2)*n
 + a**(7/2)*b*n*x**n))/c
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^{n}+a)^{\frac{3}{2}}cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^(3/2)*c*x), x)
```

## 3.403 $\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx$

**Optimal**. Leaf size=90

$$\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(n+1)\sqrt{cx}}$$

```
[Out] 2/(a*c^2*(1 + n)*Sqrt[c*x]*Sqrt[a/x + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt[a]/
(Sqrt[x]*Sqrt[a/x + b*x^n])])/(a^(3/2)*c^2*(1 + n)*Sqrt[c*x])
```

---

**Rubi [A]** time = 0.193509, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2030, 2031, 2029, 206}

$$\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(n+1)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^2*(1 + n)*Sqrt[c*x]*Sqrt[a/x + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt[a]/
(Sqrt[x]*Sqrt[a/x + b*x^n])])/(a^(3/2)*c^2*(1 + n)*Sqrt[c*x])
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx &= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} + \frac{\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{ac} \\
&= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} + \frac{\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{ac^2\sqrt{cx}} \\
&= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} - \frac{\left(2\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{ac^2(1+n)\sqrt{cx}} \\
&= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} - \frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(1+n)\sqrt{cx}}
\end{aligned}
$$

**Mathematica [C]** time = 0.046005, size = 55, normalized size = 0.61

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^{n+1}}{a}+1\right)}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(1 + n))/a])/(a*c^2*(1 + n)*Sqr
t[c*x]*Sqrt[a/x + b*x^n])
```

---

**Maple [F]** time = 0.328, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{5}{2}}\left(\frac{a}{x}+bx^n\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x)
```

```
[Out] int(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(a/x+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^n + \frac{a}{x}\right)^{\frac{3}{2}} (cx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)
```

## 3.404 $\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx$

**Optimal**. Leaf size=72

$$\frac{2}{ac^4(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(n+2)}$$

```
[Out] 2/(a*c^4*(2 + n)*x*Sqrt[a/x^2 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2
+ b*x^n])])/(a^(3/2)*c^4*(2 + n))
```

---

**Rubi [A]** time = 0.155999, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2030, 2029, 206}

$$\frac{2}{ac^4(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^4*(2 + n)*x*Sqrt[a/x^2 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2
+ b*x^n])])/(a^(3/2)*c^4*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx = \frac{\int \frac{1}{x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx}{c^4}$$

$$= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}} + \frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{ac^4}$$

$$= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{ac^4(2+n)}$$

$$= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(2+n)}$$

**Mathematica [C]** time = 0.0408202, size = 51, normalized size = 0.71

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^{n+2}}{a}+1\right)}{ac^4(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(2 + n))/a])/(a*c^4*(2 + n)*x*S
qrt[a/x^2 + b*x^n])
```

---

**Maple [F]** time = 0.326, size = 0, normalized size = 0.

$$\int \frac{1}{c^4x^4}\left(\frac{a}{x^2}+bx^n\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c^4/x^4/(1/x^2*a+b*x^n)^(3/2),x)
```

```
[Out] int(1/c^4/x^4/(1/x^2*a+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}x^4}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a/x^2)^(3/2)*x^4), x)/c^4
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{ax^2\sqrt{\frac{a}{x^2}+bx^n}+bx^4x^n\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c**4/x**4/(a/x**2+b*x**n)**(3/2),x)
```

```
[Out] Integral(1/(a*x**2*sqrt(a/x**2 + b*x**n) + b*x**4*x**n*sqrt(a/x**2 + b*x**n
)), x)/c**4
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}c^4x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a/x^2)^(3/2)*c^4*x^4), x)
```

## 3.405 $\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx$

**Optimal**. Leaf size=90

$$\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(n+3)\sqrt{cx}}$$

```
[Out] 2/(a*c^4*(3 + n)*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt
[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(a^(3/2)*c^5*(3 + n)*Sqrt[c*x])
```

---

**Rubi [A]** time = 0.216698, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2030, 2031, 2029, 206}

$$\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(n+3)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^4*(3 + n)*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt
[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(a^(3/2)*c^5*(3 + n)*Sqrt[c*x])
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx &= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}+\frac{\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{ac^3}\\
&= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}+\frac{\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{ac^5\sqrt{cx}}\\
&= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{\left(2\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{ac^5(3+n)\sqrt{cx}}\\
&= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(3+n)\sqrt{cx}}
\end{aligned}$$

**Mathematica [C]** time = 0.0519262, size = 55, normalized size = 0.61

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^{n+3}}{a}+1\right)}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(3 + n))/a])/(a*c^4*(3 + n)*(c*
x)^(3/2)*Sqrt[a/x^3 + b*x^n])
```

---

**Maple [F]** time = 0.327, size = 0, normalized size = 0.

$$\int (cx)^{-\frac{11}{2}}\left(\frac{a}{x^3}+bx^n\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x)
```

```
[Out] int(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(a/x**3+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^n + \frac{a}{x^3}\right)^{\frac{3}{2}} (cx)^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)
```

## 3.406 $\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx$

**Optimal**. Leaf size=72

$$\frac{2}{ac^7(n+4)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{a^{3/2}c^7(n+4)}$$

```
[Out] 2/(a*c^7*(4 + n)*x^2*Sqrt[a/x^4 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/
x^4 + b*x^n])])/(a^(3/2)*c^7*(4 + n))
```

---

**Rubi [A]** time = 0.152161, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2030, 2029, 206}

$$\frac{2}{ac^7(n+4)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{a^{3/2}c^7(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^7*(4 + n)*x^2*Sqrt[a/x^4 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/
x^4 + b*x^n])])/(a^(3/2)*c^7*(4 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx &= \frac{\int \frac{1}{x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx}{c^7}\\ &= \frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}}+\frac{\int \frac{1}{x^3\sqrt{\frac{a}{x^4}+bx^n}}\,dx}{ac^7}\\ &= \frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{ac^7(4+n)}\\ &= \frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{a^{3/2}c^7(4+n)}\end{aligned}$$

**Mathematica [C]** time = 0.0420332, size = 51, normalized size = 0.71

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^{n+4}}{a}+1\right)}{ac^7(n+4)x^2\sqrt{\frac{a}{x^4}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(4 + n))/a])/(a*c^7*(4 + n)*x^2
*Sqrt[a/x^4 + b*x^n])
```

---

**Maple [F]** time = 0.336, size = 0, normalized size = 0.

$$\int \frac{1}{c^7x^7}\left(\frac{a}{x^4}+bx^n\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x)
```

```
[Out] int(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x)
```

---

**Maxima [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{ax^3\sqrt{\frac{a}{x^4}+bx^n}+bx^7x^n\sqrt{\frac{a}{x^4}+bx^n}}\,dx}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c**7/x**7/(a/x**4+b*x**n)**(3/2),x)
```

```
[Out] Integral(1/(a*x**3*sqrt(a/x**4 + b*x**n) + b*x**7*x**n*sqrt(a/x**4 + b*x**n
)), x)/c**7
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^7x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a/x^4)^(3/2)*c^7*x^7), x)
```

## 3.407 $\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}} \, dx$

**Optimal**. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x + b*x^2]])/(3*Sqrt[b])
```

---

**Rubi [A]** time = 0.0144815, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x^3)/x],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x + b*x^2]])/(3*Sqrt[b])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}} \, dx &= \int \frac{1}{\sqrt{\frac{a}{x}+bx^2}} \, dx \\
&= \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x}{\sqrt{\frac{a}{x}+bx^2}}\right) \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}
\end{aligned}$$

**Mathematica [A]** time = 0.0238737, size = 63, normalized size = 1.97

$$\frac{2\sqrt{a+bx^3}\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)}{3\sqrt{b}\sqrt{x}\sqrt{\frac{a+bx^3}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x^3)/x],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/(3*Sqrt[b]*S
qrt[x]*Sqrt[(a + b*x^3)/x])
```

---

**Maple [C]** time = 0.014, size = 477, normalized size = 14.9

$$-4\frac{\left(bx^3+a\right)\left(-1+i\sqrt{3}\right)\left(-bx+\sqrt[3]{-b^2a}\right)^2}{b^2\sqrt{\left(bx^3+a\right)x}\left(i\sqrt{3}-3\right)}\sqrt{-\frac{\left(i\sqrt{3}-3\right)xb}{\left(-1+i\sqrt{3}\right)\left(-bx+\sqrt[3]{-b^2a}\right)}}\sqrt{\frac{i\sqrt{3}\sqrt[3]{-b^2a}+2\,bx+\sqrt[3]{-b^2a}}{\left(1+i\sqrt{3}\right)\left(-bx+\sqrt[3]{-b^2a}\right)}}\sqrt{\frac{i\sqrt{3}\sqrt[3]{-b^2a}}{\left(-1+i\sqrt{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^3+a)/x)^(1/2),x)
```

```
[Out] -4*(b*x^3+a)*(-1+I*3^(1/2))*(-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*
a)^(1/3)))^(1/2)*(-b*x+(-b^2*a)^(1/3))^2*((I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(
-b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(-b^2
*a)^(1/3)-2*b*x-(-b^2*a)^(1/3))/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2)
/b^2*(EllipticF((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(
1/2),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))-Elli
pticPi((-(I*3^(1/2)-3)*x*b/(-1+I*3^(1/2))/(-b*x+(-b^2*a)^(1/3)))^(1/2),(-1+
I*3^(1/2))/(I*3^(1/2)-3),((I*3^(1/2)+3)*(-1+I*3^(1/2))/(1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2)))/((b*x^3+a)/x)^(1/2)/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/(1/b
^2*x*(-b*x+(-b^2*a)^(1/3))*(I*3^(1/2)*(-b^2*a)^(1/3)+2*b*x+(-b^2*a)^(1/3))*
(I*3^(1/2)*(-b^2*a)^(1/3)-2*b*x-(-b^2*a)^(1/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{bx^3+a}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^3 + a)/x), x)
```

---

**Fricas [A]** time = 1.1653, size = 236, normalized size = 7.38

$$\left[\frac{\log\left(-8\,b^{2}x^{6}-8\,abx^{3}-a^{2}-4\left(2\,bx^{5}+ax^{2}\right)\sqrt{b}\sqrt{\frac{bx^{3}+a}{x}}\right)}{6\,\sqrt{b}},\ -\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{-b}x^{2}\sqrt{\frac{bx^{3}+a}{x}}}{2\,bx^{3}+a}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^5 + a*x^2)*sqrt(b)*sqrt((b
*x^3 + a)/x))/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(-b)*x^2*sqrt((b*x^3 + a)
/x)/(2*b*x^3 + a))/b]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**3+a)/x)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.408 $\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx$

**Optimal**. Leaf size=32

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^2 + b*x^2]]/(2*Sqrt[b])
```

---

**Rubi [A]** time = 0.0159634, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1979, 2008, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x^4)/x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^2 + b*x^2]]/(2*Sqrt[b])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x^2}+bx^2}}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}\end{aligned}$$

**Mathematica [A]** time = 0.0207978, size = 59, normalized size = 1.84

$$\frac{\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}x\sqrt{\frac{a+bx^4}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x^4)/x^2],x]
```

```
[Out] (Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]*x*Sqrt[
(a + b*x^4)/x^2])
```

---

**Maple [A]** time = 0.051, size = 49, normalized size = 1.5

$$\frac{1}{2x}\sqrt{bx^4+a}\ln\left(x^2\sqrt{b}+\sqrt{bx^4+a}\right)\frac{1}{\sqrt{\frac{bx^4+a}{x^2}}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^4+a)/x^2)^(1/2),x)
```

```
[Out] 1/2/((b*x^4+a)/x^2)^(1/2)/x*(b*x^4+a)^(1/2)*ln(x^2*b^(1/2)+(b*x^4+a)^(1/2))
/b^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$b\int\frac{x^5}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx+\frac{x^2}{2\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x^5/(b*x^4 + a)^(3/2), x) + 1/2*x^2/sqrt(b*x^4 + a)
```

---

**Fricas [A]** time = 0.835654, size = 196, normalized size = 6.12

$$\left[\frac{\log\left(-2\,bx^4-2\,\sqrt{b}x^3\sqrt{\frac{bx^4+a}{x^2}}-a\right)}{4\,\sqrt{b}},-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x^3\sqrt{\frac{bx^4+a}{x^2}}}{bx^4+a}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-2*b*x^4 - 2*sqrt(b)*x^3*sqrt((b*x^4 + a)/x^2) - a)/sqrt(b), -1/2*
sqrt(-b)*arctan(sqrt(-b)*x^3*sqrt((b*x^4 + a)/x^2)/(b*x^4 + a))/b]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**4+a)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.409 $\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx$

**Optimal**. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^3 + b*x^2]])/(5*Sqrt[b])
```

---

**Rubi [A]** time = 0.0158393, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x^5)/x^3],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^3 + b*x^2]])/(5*Sqrt[b])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x^3}+bx^2}}\,dx\\
&= \frac{2}{5}\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}
\end{aligned}$$

**Mathematica [A]** time = 0.0236792, size = 63, normalized size = 1.97

$$\frac{2\sqrt{a+bx^5}\tanh^{-1}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{a+bx^5}}\right)}{5\sqrt{b}x^{3/2}\sqrt{\frac{a+bx^5}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x^5)/x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^5]*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a + b*x^5]])/(5*Sqrt[b]*x
^(3/2)*Sqrt[(a + b*x^5)/x^3])
```

---

**Maple [F]** time = 0.026, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{bx^5+a}{x^3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^5+a)/x^3)^(1/2),x)
```

```
[Out] int(1/((b*x^5+a)/x^3)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{bx^5+a}{x^3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^5 + a)/x^3), x)
```

---

**Fricas [A]** time = 2.59082, size = 244, normalized size = 7.62

$$\left[\frac{\log\left(-8\,b^2x^{10} - 8\,abx^5 - a^2 - 4\left(2\,bx^9 + ax^4\right)\sqrt{b}\sqrt{\frac{bx^5+a}{x^3}}\right)}{10\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{-b}x^4\sqrt{\frac{bx^5+a}{x^3}}}{2\,bx^5+a}\right)}{5\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/10*log(-8*b^2*x^10 - 8*a*b*x^5 - a^2 - 4*(2*b*x^9 + a*x^4)*sqrt(b)*sqrt(
(b*x^5 + a)/x^3))/sqrt(b), -1/5*sqrt(-b)*arctan(2*sqrt(-b)*x^4*sqrt((b*x^5
```

```
+ a)/x^3)/(2*b*x^5 + a))/b]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**5+a)/x**3)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

# 3.410 $\int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}}\,dx$

**Optimal**. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^{2-n}+bx^2}}\right)}{\sqrt{b}n}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^(2 - n)]])/(Sqrt[b]*n)
```

---

**Rubi [A]** time = 0.0239837, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^{2-n}+bx^2}}\right)}{\sqrt{b}n}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^(2 - n)*(a + b*x^n)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^(2 - n)]])/(Sqrt[b]*n)
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx &= \int \frac{1}{\sqrt{bx^2+ax^{2-n}}}\,dx\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+ax^{2-n}}}\right)}{n}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^{2-n}}}\right)}{\sqrt{b}n}
\end{aligned}$$

**Mathematica [B]** time = 0.0387811, size = 76, normalized size = 2.05

$$\frac{2\sqrt{a}x^{1-\frac{n}{2}}\sqrt{\frac{bx^n}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}n\sqrt{x^{2-n}\left(a+bx^n\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^(2 - n)*(a + b*x^n)],x]
```

```
[Out] (2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 + (b*x^n)/a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a
]])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a + b*x^n)])
```

---

**Maple [F]** time = 0.358, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2-n)*(a+b*x^n))^(1/2),x)
```

```
[Out] int(1/(x^(2-n)*(a+b*x^n))^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(bx^n+a)x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)
```

---

**Fricas [A]** time = 0.911825, size = 216, normalized size = 5.84

$$\left[\frac{\log\left(\frac{2\,bxx^n+ax+2\,\sqrt{b}x^n\sqrt{\frac{bx^2x^n+ax^2}{x^n}}}{x}\right)}{\sqrt{b}n}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{bx^2x^n+ax^2}{x^n}}}{bx}\right)}{bn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] [log((2*b*x*x^n + a*x + 2*sqrt(b)*x^n*sqrt((b*x^2*x^n + a*x^2)/x^n))/x)/(sq
rt(b)*n), -2*sqrt(-b)*arctan(sqrt(-b)*sqrt((b*x^2*x^n + a*x^2)/x^n)/(b*x))/
(b*n)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(2-n)*(a+b*x**n))**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(bx^n + a)x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)
```

## 3.411 $\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx$

**Optimal**. Leaf size=33

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x - b*x^2]])/(3*Sqrt[b])
```

---

**Rubi [A]** time = 0.0140644, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a - b*x^3)/x],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x - b*x^2]])/(3*Sqrt[b])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x}-bx^2}}\,dx\\ &= \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x}-bx^2}}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}\end{aligned}$$

**Mathematica [A]** time = 0.0259932, size = 66, normalized size = 2.

$$\frac{2\sqrt{a-bx^3}\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a-bx^3}}\right)}{3\sqrt{b}\sqrt{x}\sqrt{\frac{a-bx^3}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a - b*x^3)/x],x]
```

```
[Out] (2*Sqrt[a - b*x^3]*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a - b*x^3]])/(3*Sqrt[b]*Sq
rt[x]*Sqrt[(a - b*x^3)/x])
```

---

**Maple [C]** time = 0.566, size = 471, normalized size = 14.3

$$4\,\frac{\left(bx^3-a\right)\left(1+i\sqrt{3}\right)\left(-bx+\sqrt[3]{b^2a}\right)^2}{b^2\sqrt{-\left(bx^3-a\right)x}\left(i\sqrt{3}+3\right)}\sqrt{-\frac{\left(i\sqrt{3}+3\right)xb}{\left(1+i\sqrt{3}\right)\left(-bx+\sqrt[3]{b^2a}\right)}}\sqrt{\frac{i\sqrt{3}\sqrt[3]{b^2a}-2\,bx-\sqrt[3]{b^2a}}{\left(-1+i\sqrt{3}\right)\left(-bx+\sqrt[3]{b^2a}\right)}}\sqrt{\frac{i\sqrt{3}\sqrt[3]{b^2a}+2\,bx+}{\left(1+i\sqrt{3}\right)\left(-bx+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x^3+a)/x)^(1/2),x)
```

```
[Out] 4*(b*x^3-a)*(1+I*3^(1/2))*(-(I*3^(1/2)+3)*x*b/(1+I*3^(1/2))/(-b*x+(b^2*a)^(
1/3)))^(1/2)*(-b*x+(b^2*a)^(1/3))^2*((I*3^(1/2)*(b^2*a)^(1/3)-2*b*x-(b^2*a)
^(1/3))/(-1+I*3^(1/2))/(-b*x+(b^2*a)^(1/3)))^(1/2)*((I*3^(1/2)*(b^2*a)^(1/3
)+2*b*x+(b^2*a)^(1/3))/(1+I*3^(1/2))/(-b*x+(b^2*a)^(1/3)))^(1/2)/b^2*(Ellip
ticF((-(I*3^(1/2)+3)*x*b/(1+I*3^(1/2))/(-b*x+(b^2*a)^(1/3)))^(1/2),((I*3^(1
/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)+3)/(-1+I*3^(1/2)))^(1/2))-EllipticPi((-(I*3
^(1/2)+3)*x*b/(1+I*3^(1/2))/(-b*x+(b^2*a)^(1/3)))^(1/2),(1+I*3^(1/2))/(I*3^
(1/2)+3),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)+3)/(-1+I*3^(1/2)))^(1/2)))
/(-(b*x^3-a)/x)^(1/2)/(-(b*x^3-a)*x)^(1/2)/(I*3^(1/2)+3)/(-1/b^2*x*(-b*x+(b
^2*a)^(1/3))*(I*3^(1/2)*(b^2*a)^(1/3)-2*b*x-(b^2*a)^(1/3))*(I*3^(1/2)*(b^2*
a)^(1/3)+2*b*x+(b^2*a)^(1/3)))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\frac{bx^3-a}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^3 - a)/x), x)
```

---

**Fricas [A]** time = 1.54122, size = 240, normalized size = 7.27

$$\left[-\frac{\sqrt{-b}\log\left(-8\,b^2x^6+8\,abx^3-a^2+4\left(2\,bx^5-ax^2\right)\sqrt{-b}\sqrt{-\frac{bx^3-a}{x}}\right)}{6\,b}, -\frac{\arctan\left(\frac{2\,\sqrt{b}x^2\sqrt{-\frac{bx^3-a}{x}}}{2\,bx^3-a}\right)}{3\,\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*sqrt(-b)*log(-8*b^2*x^6 + 8*a*b*x^3 - a^2 + 4*(2*b*x^5 - a*x^2)*sqrt(
-b)*sqrt(-(b*x^3 - a)/x))/b, -1/3*arctan(2*sqrt(b)*x^2*sqrt(-(b*x^3 - a)/x)
/(2*b*x^3 - a))/sqrt(b)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x**3+a)/x)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.412 $\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx$

**Optimal**. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a/x^2 - b*x^2]]/(2*Sqrt[b])
```

---

**Rubi [A]** time = 0.0149095, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1979, 2008, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a - b*x^4)/x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a/x^2 - b*x^2]]/(2*Sqrt[b])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x^2}-bx^2}}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}
\end{aligned}$$

**Mathematica [A]** time = 0.0219543, size = 62, normalized size = 1.88

$$\frac{\sqrt{a-bx^4}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}x\sqrt{\frac{a-bx^4}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a - b*x^4)/x^2],x]
```

```
[Out] (Sqrt[a - b*x^4]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]*x*Sqrt[(
a - b*x^4)/x^2])
```

---

**Maple [B]** time = 0.013, size = 53, normalized size = 1.6

$$\frac{1}{2x}\sqrt{-bx^4+a}\arctan\left(x^2\sqrt{b}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{-\frac{bx^4-a}{x^2}}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x^4+a)/x^2)^(1/2),x)
```

```
[Out] 1/2/(-(b*x^4-a)/x^2)^(1/2)/x*(-b*x^4+a)^(1/2)/b^(1/2)*arctan(x^2*b^(1/2)/(-
b*x^4+a)^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$b\int\frac{x^5}{\left(bx^4-a\right)\sqrt{-bx^4+a}}\,dx+\frac{x^2}{2\sqrt{-bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x^5/((b*x^4 - a)*sqrt(-b*x^4 + a)), x) + 1/2*x^2/sqrt(-b*x^4 +
a)
```

---

**Fricas [A]** time = 1.02574, size = 198, normalized size = 6.

$$\left[-\frac{\sqrt{-b}\log\left(2\,bx^4-2\,\sqrt{-b}x^3\sqrt{-\frac{bx^4-a}{x^2}}-a\right)}{4\,b},-\frac{\arctan\left(\frac{\sqrt{b}x^3\sqrt{-\frac{bx^4-a}{x^2}}}{bx^4-a}\right)}{2\,\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b)*x^3*sqrt(-(b*x^4 - a)/x^2) - a)/b,
-1/2*arctan(sqrt(b)*x^3*sqrt(-(b*x^4 - a)/x^2)/(b*x^4 - a))/sqrt(b)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x**4+a)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.413 $\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx$

**Optimal**. Leaf size=33

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}$$

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x^3 - b*x^2]])/(5*Sqrt[b])
```

---

**Rubi [A]** time = 0.0161781, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a - b*x^5)/x^3],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x^3 - b*x^2]])/(5*Sqrt[b])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x^3}-bx^2}}\,dx\\
&= \frac{2}{5}\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}
\end{aligned}$$

**Mathematica [A]** time = 0.0248042, size = 66, normalized size = 2.

$$\frac{2\sqrt{a-bx^5}\tan^{-1}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{a-bx^5}}\right)}{5\sqrt{b}x^{3/2}\sqrt{\frac{a-bx^5}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a - b*x^5)/x^3],x]
```

```
[Out] (2*Sqrt[a - b*x^5]*ArcTan[(Sqrt[b]*x^(5/2))/Sqrt[a - b*x^5]])/(5*Sqrt[b]*x^
(3/2)*Sqrt[(a - b*x^5)/x^3])
```

---

**Maple [F]** time = 0.031, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\frac{-bx^5+a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x^5+a)/x^3)^(1/2),x)
```

```
[Out] int(1/((-b*x^5+a)/x^3)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\frac{bx^5-a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^5 - a)/x^3), x)
```

---

**Fricas [A]** time = 3.25213, size = 248, normalized size = 7.52

$$\left[-\frac{\sqrt{-b}\log\left(-8\,b^2x^{10}+8\,abx^5-a^2+4\left(2\,bx^9-ax^4\right)\sqrt{-b}\sqrt{-\frac{bx^5-a}{x^3}}\right)}{10\,b},\ -\frac{\arctan\left(\frac{2\,\sqrt{b}x^4\sqrt{-\frac{bx^5-a}{x^3}}}{2\,bx^5-a}\right)}{5\,\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/10*sqrt(-b)*log(-8*b^2*x^10 + 8*a*b*x^5 - a^2 + 4*(2*b*x^9 - a*x^4)*sqr
t(-b)*sqrt(-(b*x^5 - a)/x^3))/b, -1/5*arctan(2*sqrt(b)*x^4*sqrt(-(b*x^5 - a
```

```
)/x^3)/(2*b*x^5 - a))/sqrt(b)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x**5+a)/x**3)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.414 $\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx$

**Optimal**. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^{2-n}-bx^2}}\right)}{\sqrt{b}n}$$

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^(2 - n)]])/(Sqrt[b]*n)
```

---

**Rubi [A]** time = 0.0226047, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^{2-n}-bx^2}}\right)}{\sqrt{b}n}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^(2 - n)*(a - b*x^n)],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^(2 - n)]])/(Sqrt[b]*n)
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^n\right)}}\,dx &= \int \frac{1}{\sqrt{-bx^2+ax^{2-n}}}\,dx\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^{2-n}}}\right)}{n}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^{2-n}}}\right)}{\sqrt{b}n}\end{aligned}$$

**Mathematica [B]** time = 0.0428816, size = 78, normalized size = 2.05

$$\frac{2\sqrt{a}x^{1-\frac{n}{2}}\sqrt{1-\frac{bx^n}{a}}\sin^{-1}\left(\frac{\sqrt{b}x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}n\sqrt{x^{2-n}\left(a-bx^n\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^(2 - n)*(a - b*x^n)],x]
```

```
[Out] (2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 - (b*x^n)/a]*ArcSin[(Sqrt[b]*x^(n/2))/Sqrt[a]
])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a - b*x^n)])
```

---

**Maple [F]** time = 0.586, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^n\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2-n)*(a-b*x^n))^(1/2),x)
```

```
[Out] int(1/(x^(2-n)*(a-b*x^n))^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(bx^n-a\right)x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)
```

---

**Fricas [A]** time = 1.00405, size = 219, normalized size = 5.76

$$\left[-\frac{\sqrt{-b}\log\left(-\frac{2\,bxx^n-ax-2\,\sqrt{-b}x^n\sqrt{-\frac{bx^2x^n-ax^2}{x^n}}}{x}\right)}{bn}, -\frac{2\,\arctan\left(\frac{\sqrt{-\frac{bx^2x^n-ax^2}{x^n}}}{\sqrt{b}x}\right)}{\sqrt{b}n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-(2*b*x*x^n - a*x - 2*sqrt(-b)*x^n*sqrt(-(b*x^2*x^n - a*x^2)
/x^n))/x)/(b*n), -2*arctan(sqrt(-(b*x^2*x^n - a*x^2)/x^n)/(sqrt(b)*x))/(sqr
t(b)*n)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(2-n)*(a-b*x**n))**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(bx^n - a)x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)
```

## 3.415 $\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx$

**Optimal**. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

---

**Rubi [A]** time = 0.0233351, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^n*(a + b*x^(2 - n))],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx &= \int \frac{1}{\sqrt{bx^2+ax^n}}\,dx\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}
\end{aligned}$$

**Mathematica [B]** time = 0.0754614, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{a}x^{n/2}\sqrt{\frac{bx^{2-n}}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(n-2)\sqrt{ax^n+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^n*(a + b*x^(2 - n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

---

**Maple [F]** time = 0.349, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^n*(a+b*x^(2-n)))^(1/2),x)
```

```
[Out] int(1/(x^n*(a+b*x^(2-n)))^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(bx^{-n+2}+a\right)x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**n*(a+b*x**(2-n)))**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(bx^{-n+2}+a\right)x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)
```

## 3.416 $\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}} \, dx$

**Optimal**. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

---

**Rubi [A]** time = 0.0193817, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}} \, dx &= \int \frac{1}{\sqrt{bx^2+ax^n}} \, dx \\
&= \frac{2\operatorname{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n} \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}
\end{aligned}$$

**Mathematica [B]** time = 0.021973, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{a}x^{n/2}\sqrt{\frac{bx^{2-n}}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(n-2)\sqrt{ax^n+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

---

**Maple [F]** time = 2.16, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b+a*x^(-2+n)))^(1/2),x)
```

```
[Out] int(1/(x^2*(b+a*x^(-2+n)))^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(ax^{n-2}+b\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)
```

---

**Fricas [A]** time = 1.02494, size = 252, normalized size = 6.81

$$\left[\frac{\sqrt{b}\log\left(\frac{axx^{n-2}+2\,bx-2\,\sqrt{ax^2x^{n-2}+bx^2}\sqrt{b}}{xx^{n-2}}\right)}{bn-2\,b}, \frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{ax^2x^{n-2}+bx^2}\sqrt{-b}}{bx}\right)}{bn-2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b)*log((a*x*x^(n - 2) + 2*b*x - 2*sqrt(a*x^2*x^(n - 2) + b*x^2)*sqrt(
b))/(x*x^(n - 2)))/(b*n - 2*b), 2*sqrt(-b)*arctan(sqrt(a*x^2*x^(n - 2) + b*
x^2)*sqrt(-b)/(b*x))/(b*n - 2*b)]
```

---

**Sympy [F(-1)]**   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(b+a*x**(-2+n)))**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]**   time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(ax^{n-2}+b\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)
```

## 3.417 $\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx$

**Optimal**. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

---

**Rubi [A]** time = 0.0183542, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx &= \int \frac{1}{\sqrt{bx^2+ax^n}}\,dx\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}\end{aligned}$$

**Mathematica [B]** time = 0.0220208, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{a}x^{n/2}\sqrt{\frac{bx^{2-n}}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(n-2)\sqrt{ax^n+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

---

**Maple [F]** time = 0.334, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x+a*x^(-1+n)))^(1/2),x)
```

```
[Out] int(1/(x*(b*x+a*x^(-1+n)))^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(ax^{n-1}+bx\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x**(-1+n)))**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(ax^{n-1} + bx)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)
```

## 3.418 $\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx$

**Optimal**. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}(2-n)}$$

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

---

**Rubi [A]** time = 0.0209641, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^n*(a - b*x^(2 - n))],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx &= \int \frac{1}{\sqrt{-bx^2+ax^n}}\,dx\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}\end{aligned}$$

**Mathematica [B]** time = 0.0795574, size = 80, normalized size = 2.11

$$-\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\sin^{-1}\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(n-2)\sqrt{ax^n-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^n*(a - b*x^(2 - n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/
Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

---

**Maple [F]** time = 0.346, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^n*(a-b*x^(2-n)))^(1/2),x)
```

```
[Out] int(1/(x^n*(a-b*x^(2-n)))^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(bx^{-n+2}-a\right)x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**n*(a-b*x**(2-n)))**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(bx^{-n+2}-a\right)x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)
```

## 3.419 $\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx$

**Optimal**. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}(2-n)}$$

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

---

**Rubi [A]** time = 0.0198648, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx &= \int \frac{1}{\sqrt{-bx^2+ax^n}}\,dx\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}\end{aligned}$$

**Mathematica [B]** time = 0.0224835, size = 80, normalized size = 2.11

$$-\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\sin^{-1}\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(n-2)\sqrt{ax^n-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/
Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

---

**Maple [F]** time = 0.813, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x)
```

```
[Out] int(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(ax^{n-2}-b\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)
```

---

**Fricas [A]** time = 1.01108, size = 251, normalized size = 6.61

$$\left[-\frac{\sqrt{-b}\log\left(\frac{axx^{n-2}-2\,bx-2\,\sqrt{ax^2x^{n-2}-bx^2}\sqrt{-b}}{xx^{n-2}}\right)}{bn-2\,b},\ \frac{2\,\sqrt{b}\arctan\left(\frac{\sqrt{ax^2x^{n-2}-bx^2}}{\sqrt{b}x}\right)}{bn-2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log((a*x*x^(n - 2) - 2*b*x - 2*sqrt(a*x^2*x^(n - 2) - b*x^2)*sqr
t(-b))/(x*x^(n - 2)))/(b*n - 2*b), 2*sqrt(b)*arctan(sqrt(a*x^2*x^(n - 2) -
b*x^2)/(sqrt(b)*x))/(b*n - 2*b)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(-b+a*x**(-2+n)))**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(ax^{n-2}-b\right)x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)
```

## 3.420 $\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx$

**Optimal**. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}(2-n)}$$

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

---

**Rubi [A]** time = 0.0188031, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]

[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx &= \int \frac{1}{\sqrt{-bx^2+ax^n}}\,dx\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}(2-n)}\end{aligned}$$

**Mathematica [B]** time = 0.0233586, size = 80, normalized size = 2.11

$$-\frac{2\sqrt{a}x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\sin^{-1}\left(\frac{\sqrt{b}x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}(n-2)\sqrt{ax^n-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/
Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

---

**Maple [F]** time = 0.345, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x)
```

```
[Out] int(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\left(ax^{n-1}-bx\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x**(-1+n)))**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(ax^{n-1} - bx)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)
```

## 3.421 $\int (cx)^m \left(ax^j+bx^n\right)^{3/2}\,dx$

**Optimal**. Leaf size=107

$$\frac{2bx^{n+1}(cx)^m\sqrt{ax^j+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{m+\frac{3n}{2}+1}{j-n};\frac{m+\frac{3n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m+3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

```
[Out] (2*b*x^(1 + n)*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + m +
 (3*n)/2)/(j - n), 1 + (1 + m + (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/((2
+ 2*m + 3*n)*Sqrt[1 + (a*x^(j - n))/b])
```

---

**Rubi [A]** time = 0.102115, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 365, 364}

$$\frac{2bx^{n+1}(cx)^m\sqrt{ax^j+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{m+\frac{3n}{2}+1}{j-n};\frac{m+\frac{3n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m+3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*b*x^(1 + n)*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + m +
 (3*n)/2)/(j - n), 1 + (1 + m + (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/((2
+ 2*m + 3*n)*Sqrt[1 + (a*x^(j - n))/b])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int (cx)^m \left(ax^j+bx^n\right)^{3/2}\,dx &= \frac{\left(x^{-m-\frac{n}{2}}(cx)^m\sqrt{ax^j+bx^n}\right)\int x^{m+\frac{3n}{2}}\left(b+ax^{j-n}\right)^{3/2}\,dx}{\sqrt{b+ax^{j-n}}}\\
&= \frac{\left(bx^{-m-\frac{n}{2}}(cx)^m\sqrt{ax^j+bx^n}\right)\int x^{m+\frac{3n}{2}}\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}\,dx}{\sqrt{1+\frac{ax^{j-n}}{b}}}\\
&= \frac{2bx^{1+n}(cx)^m\sqrt{ax^j+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{1+m+\frac{3n}{2}}{j-n};1+\frac{1+m+\frac{3n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{(2+2m+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}
\end{aligned}
$$

**Mathematica [B]** time = 0.355433, size = 218, normalized size = 2.04

$$
\frac{2(cx)^m \Bigg(3a^2(j-n)^2x^{2j+1}\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{4j+2m-n+2}{2j-2n};\frac{6j+2m-3n+2}{2j-2n};-\frac{ax^{j-n}}{b}\right)+x^{-m}(4j+2m-n+2)\left(ax^j+bx^n\right)\big(a(-j+2}{(2m+3n+2)(4j+2m-n+2)(2j+2m+n+2)\sqrt{ax^j+bx^n}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*(c*x)^m*(((2 + 4*j + 2*m - n)*(a*x^j + b*x^n)*(a*(2 - j + 2*m + 4*n)*x^(
1 + j + m) + b*(2 + 2*j + 2*m + n)*x^(1 + m + n)))/x^m + 3*a^2*(j - n)^2*x^
(1 + 2*j)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 4*j + 2*m -
 n)/(2*j - 2*n), (2 + 6*j + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(
(2 + 4*j + 2*m - n)*(2 + 2*j + 2*m + n)*(2 + 2*m + 3*n)*Sqrt[a*x^j + b*x^n]
)
```

---

**Maple [F]** time = 0.692, size = 0, normalized size = 0.

$$
\int (cx)^m \left(ax^j+bx^n\right)^{\frac{3}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^m*(a*x^j+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \left(ax^j+bx^n\right)^{\frac{3}{2}}(cx)^m\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^m, x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax^j + bx^n\right)^{\frac{3}{2}} (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^m, x)
```

## 3.422 $\int (cx)^m\sqrt{ax^j+bx^n}\,dx$

**Optimal**. Leaf size=100

$$\frac{2x(cx)^m\sqrt{ax^j+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{m+\frac{n}{2}+1}{j-n};\frac{2m+n+2}{2j-2n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m+n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

```
[Out] (2*x*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (1 + m + n/2)/(j -
 n), 1 + (2 + 2*m + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2*m + n)*Sqr
t[1 + (a*x^(j - n))/b])
```

---

**Rubi [A]** time = 0.094578, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 365, 364}

$$\frac{2x(cx)^m\sqrt{ax^j+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{m+\frac{n}{2}+1}{j-n};\frac{2m+n+2}{2j-2n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m+n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (1 + m + n/2)/(j -
 n), 1 + (2 + 2*m + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2*m + n)*Sqr
t[1 + (a*x^(j - n))/b])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int (cx)^m \sqrt{ax^j + bx^n}\, dx &= \frac{\left(x^{-m-\frac{n}{2}} (cx)^m \sqrt{ax^j + bx^n}\right) \int x^{m+\frac{n}{2}} \sqrt{b + ax^{j-n}}\, dx}{\sqrt{b + ax^{j-n}}} \\
&= \frac{\left(x^{-m-\frac{n}{2}} (cx)^m \sqrt{ax^j + bx^n}\right) \int x^{m+\frac{n}{2}} \sqrt{1 + \frac{ax^{j-n}}{b}}\, dx}{\sqrt{1 + \frac{ax^{j-n}}{b}}} \\
&= \frac{2x(cx)^m \sqrt{ax^j + bx^n}\, {}_2F_1\left(-\frac{1}{2}, \frac{1+m+\frac{n}{2}}{j-n}; 1 + \frac{2+2m+n}{2j-2n}; -\frac{ax^{j-n}}{b}\right)}{(2 + 2m + n)\sqrt{1 + \frac{ax^{j-n}}{b}}}
\end{aligned}
$$

**Mathematica [A]** time = 0.191334, size = 156, normalized size = 1.56

$$
\frac{2x(cx)^m \left((2j + 2m - n + 2)\left(ax^j + bx^n\right) - a(j-n)x^j \sqrt{\frac{ax^{j-n}}{b} + 1}\, {}_2F_1\left(\frac{1}{2}, \frac{2j+2m-n+2}{2j-2n}; \frac{4j+2m-3n+2}{2j-2n}; -\frac{ax^{j-n}}{b}\right)\right)}{(2m + n + 2)(2j + 2m - n + 2)\sqrt{ax^j + bx^n}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(c*x)^m*((2 + 2*j + 2*m - n)*(a*x^j + b*x^n) - a*(j - n)*x^j*Sqrt[1 +
(a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 2*j + 2*m - n)/(2*j - 2*n), (2
 + 4*j + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/((2 + 2*j + 2*m - n)
*(2 + 2*m + n)*Sqrt[a*x^j + b*x^n])
```

---

**Maple [F]** time = 0.512, size = 0, normalized size = 0.

$$
\int (cx)^m \sqrt{ax^j + bx^n}\, dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^m*(a*x^j+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int \sqrt{ax^j + bx^n}\,(cx)^m\, dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^m, x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx)^m \sqrt{ax^j + bx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral((c*x)**m*sqrt(a*x**j + b*x**n), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j + bx^n}\,(cx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^m, x)
```

## 3.423 $\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx$

**Optimal**. Leaf size=102

$$\frac{2x(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m-\frac{n}{2}+1}{j-n};\frac{m-\frac{n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m-n+2)\sqrt{ax^j+bx^n}}$$

```
[Out] (2*x*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (1 + m - n/2)
/(j - n), 1 + (1 + m - n/2)/(j - n), -((a*x^(j - n))/b)])/((2 + 2*m - n)*Sq
rt[a*x^j + b*x^n])
```

---

**Rubi [A]** time = 0.097152, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 365, 364}

$$\frac{2x(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{m-\frac{n}{2}+1}{j-n};\frac{m-\frac{n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m-n+2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (1 + m - n/2)
/(j - n), 1 + (1 + m - n/2)/(j - n), -((a*x^(j - n))/b)])/((2 + 2*m - n)*Sq
rt[a*x^j + b*x^n])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{b+ax^{j-n}}\right)\int \frac{x^{m-\frac{n}{2}}}{\sqrt{b+ax^{j-n}}}\,dx}{\sqrt{ax^j+bx^n}}$$

$$= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{m-\frac{n}{2}}}{\sqrt{1+\frac{ax^{j-n}}{b}}}\,dx}{\sqrt{ax^j+bx^n}}$$

$$= \frac{2x(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m-\frac{n}{2}}{j-n};1+\frac{1+m-\frac{n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{(2+2m-n)\sqrt{ax^j+bx^n}}$$

**Mathematica [A]** time = 0.0811581, size = 106, normalized size = 1.04

$$\frac{2x(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{2m-n+2}{2j-2n};\frac{2m-n+2}{2j-2n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m-n+2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 2*m - n)
/(2*j - 2*n), 1 + (2 + 2*m - n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2*m
 - n)*Sqrt[a*x^j + b*x^n])
```

---

**Maple [F]** time = 0.37, size = 0, normalized size = 0.

$$\int (cx)^m \frac{1}{\sqrt{ax^j+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^m/(a*x^j+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/sqrt(a*x^j + b*x^n), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral((c*x)**m/sqrt(a*x**j + b*x**n), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/sqrt(a*x^j + b*x^n), x)
```

## 3.424 $\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{3/2}}\,dx$

**Optimal**. Leaf size=111

$$\frac{2x^{1-n}(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{3}{2},\frac{m-\frac{3n}{2}+1}{j-n};\frac{m-\frac{3n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b(2m-3n+2)\sqrt{ax^j+bx^n}}$$

```
[Out] (2*x^(1 - n)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[3/2, (1 +
m - (3*n)/2)/(j - n), 1 + (1 + m - (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/(
b*(2 + 2*m - 3*n)*Sqrt[a*x^j + b*x^n])
```

---

**Rubi [A]** time = 0.107765, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 365, 364}

$$\frac{2x^{1-n}(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{3}{2},\frac{m-\frac{3n}{2}+1}{j-n};\frac{m-\frac{3n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b(2m-3n+2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*x^(1 - n)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[3/2, (1 +
m - (3*n)/2)/(j - n), 1 + (1 + m - (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/(
b*(2 + 2*m - 3*n)*Sqrt[a*x^j + b*x^n])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{3/2}}\,dx &= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{b+ax^{j-n}}\right)\int \frac{x^{m-\frac{3n}{2}}}{\left(b+ax^{j-n}\right)^{3/2}}\,dx}{\sqrt{ax^j+bx^n}}\\ &= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{m-\frac{3n}{2}}}{\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}}\,dx}{b\sqrt{ax^j+bx^n}}\\ &= \frac{2x^{1-n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{3}{2},\frac{1+m-\frac{3n}{2}}{j-n};1+\frac{1+m-\frac{3n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b(2+2m-3n)\sqrt{ax^j+bx^n}}\end{aligned}$$

**Mathematica [A]** time = 0.128102, size = 116, normalized size = 1.05

$$\frac{2x^{1-j}(cx)^m\left(\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{-2j+2m-n+2}{2j-2n};\frac{2m-3n+2}{2j-2n};-\frac{ax^{j-n}}{b}\right)-1\right)}{a(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*x^(1 - j)*(c*x)^m*(-1 + Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2,
 (2 - 2*j + 2*m - n)/(2*j - 2*n), (2 + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j -
n))/b)]))/(a*(j - n)*Sqrt[a*x^j + b*x^n])
```

---

**Maple [F]** time = 0.372, size = 0, normalized size = 0.

$$\int (cx)^m\left(ax^j+bx^n\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^m/(a*x^j+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((c*x)**m/(a*x**j + b*x**n)**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

## 3.425 $\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{5/2}}\,dx$

**Optimal**. Leaf size=111

$$\frac{2x^{1-2n}(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{5}{2},\frac{m-\frac{5n}{2}+1}{j-n};\frac{m-\frac{5n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b^2(2m-5n+2)\sqrt{ax^j+bx^n}}$$

```
[Out] (2*x^(1 - 2*n)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1
+ m - (5*n)/2)/(j - n), 1 + (1 + m - (5*n)/2)/(j - n), -((a*x^(j - n))/b)])
/(b^2*(2 + 2*m - 5*n)*Sqrt[a*x^j + b*x^n])
```

---

**Rubi [A]** time = 0.106485, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 365, 364}

$$\frac{2x^{1-2n}(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{5}{2},\frac{m-\frac{5n}{2}+1}{j-n};\frac{m-\frac{5n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b^2(2m-5n+2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(a*x^j + b*x^n)^(5/2),x]
```

```
[Out] (2*x^(1 - 2*n)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1
+ m - (5*n)/2)/(j - n), 1 + (1 + m - (5*n)/2)/(j - n), -((a*x^(j - n))/b)])
/(b^2*(2 + 2*m - 5*n)*Sqrt[a*x^j + b*x^n])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{5/2}}\,dx = \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{b+ax^{j-n}}\right)\int \frac{x^{m-\frac{5n}{2}}}{\left(b+ax^{j-n}\right)^{5/2}}\,dx}{\sqrt{ax^j+bx^n}}$$

$$= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{m-\frac{5n}{2}}}{\left(1+\frac{ax^{j-n}}{b}\right)^{5/2}}\,dx}{b^2\sqrt{ax^j+bx^n}}$$

$$= \frac{2x^{1-2n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\;{}_2F_1\left(\frac{5}{2},\frac{1+m-\frac{5n}{2}}{j-n};1+\frac{1+m-\frac{5n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b^2(2+2m-5n)\sqrt{ax^j+bx^n}}$$

**Mathematica [A]** time = 0.376667, size = 166, normalized size = 1.5

$$\frac{2x^{1-2j}(cx)^m\left(-(2j-2m+3n-2)\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{1}{2},\frac{-4j+2m-n+2}{2j-2n};\frac{-2j+2m-3n+2}{2j-2n};-\frac{ax^{j-n}}{b}\right)-\frac{a(j-n)x^j}{ax^j+bx^n}+2j-2m+3n-2\right)}{3a^2(j-n)^2\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(a*x^j + b*x^n)^(5/2),x]
```

```
[Out] (2*x^(1 - 2*j)*(c*x)^m*(-2 + 2*j - 2*m + 3*n - (a*(j - n)*x^j)/(a*x^j + b*x
^n) - (-2 + 2*j - 2*m + 3*n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/
2, (2 - 4*j + 2*m - n)/(2*j - 2*n), (2 - 2*j + 2*m - 3*n)/(2*j - 2*n), -((a
*x^(j - n))/b)]))/(3*a^2*(j - n)^2*Sqrt[a*x^j + b*x^n])
```

---

**Maple [F]** time = 0.357, size = 0, normalized size = 0.

$$\int (cx)^m\left(ax^j+bx^n\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^j+b*x^n)^(5/2),x)
```

```
[Out] int((c*x)^m/(a*x^j+b*x^n)^(5/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(a*x**j+b*x**n)**(5/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{\left(ax^j + bx^n\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

## 3.426 $\int \left(ax^j+bx^n\right)^{3/2}\,dx$

**Optimal**. Leaf size=97

$$\frac{2bx^{n+1}\sqrt{ax^j+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{\frac{3n}{2}+1}{j-n};\frac{2j+n+2}{2(j-n)};-\frac{ax^{j-n}}{b}\right)}{(3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

```
[Out] (2*b*x^(1 + n)*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + (3*n)/2)/(j
 - n), (2 + 2*j + n)/(2*(j - n)), -((a*x^(j - n))/b)])/((2 + 3*n)*Sqrt[1 +
(a*x^(j - n))/b])
```

---

**Rubi [A]** time = 0.05521, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2011, 365, 364}

$$\frac{2bx^{n+1}\sqrt{ax^j+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{\frac{3n}{2}+1}{j-n};\frac{2j+n+2}{2(j-n)};-\frac{ax^{j-n}}{b}\right)}{(3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*b*x^(1 + n)*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + (3*n)/2)/(j
 - n), (2 + 2*j + n)/(2*(j - n)), -((a*x^(j - n))/b)])/((2 + 3*n)*Sqrt[1 +
(a*x^(j - n))/b])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(ax^j+bx^n\right)^{3/2}\,dx &= \frac{\left(x^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{3n/2}\left(b+ax^{j-n}\right)^{3/2}\,dx}{\sqrt{b+ax^{j-n}}}\\ &= \frac{\left(bx^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{3n/2}\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}\,dx}{\sqrt{1+\frac{ax^{j-n}}{b}}}\\ &= \frac{2bx^{1+n}\sqrt{ax^j+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{1+\frac{3n}{2}}{j-n};\frac{2+2j+n}{2(j-n)};-\frac{ax^{j-n}}{b}\right)}{(2+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}\end{aligned}$$

**Mathematica [A]** time = 0.194312, size = 177, normalized size = 1.82

$$\frac{2x\Bigg(3a^2(j-n)^2x^{2j}\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{4j-n+2}{2j-2n};\frac{6j-3n+2}{2j-2n};-\frac{ax^{j-n}}{b}\right)+(4j-n+2)\left(ax^j+bx^n\right)\Big(a(-j+4n+2)x^j+b(2j+n+2)x^n}{(3n+2)(4j-n+2)(2j+n+2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*x*((2 + 4*j - n)*(a*x^j + b*x^n)*(a*(2 - j + 4*n)*x^j + b*(2 + 2*j + n)*
x^n) + 3*a^2*(j - n)^2*x^(2*j)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[
1/2, (2 + 4*j - n)/(2*j - 2*n), (2 + 6*j - 3*n)/(2*j - 2*n), -((a*x^(j - n)
)/b)]))/((2 + 4*j - n)*(2 + 2*j + n)*(2 + 3*n)*Sqrt[a*x^j + b*x^n])
```

---

**Maple [F]** time = 0.398, size = 0, normalized size = 0.

$$\int \left(ax^j+bx^n\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((a*x^j+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax^j+bx^n\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax^{j} + bx^{n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((a*x**j + b*x**n)**(3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax^{j} + bx^{n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2), x)
```

## 3.427 $\int \sqrt{ax^j + bx^n}\,dx$

**Optimal**. Leaf size=87

$$\frac{2x\sqrt{ax^j + bx^n}\,{}_2F_1\left(-\frac{1}{2}, \frac{n+2}{2(j-n)}; \frac{n+2}{2j-2n} + 1; -\frac{ax^{j-n}}{b}\right)}{(n+2)\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

```
[Out] (2*x*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (2 + n)/(2*(j - n)), 1 + (
2 + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + n)*Sqrt[1 + (a*x^(j - n))/b]
)
```

---

**Rubi [A]** time = 0.0525132, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2011, 365, 364}

$$\frac{2x\sqrt{ax^j + bx^n}\,{}_2F_1\left(-\frac{1}{2}, \frac{n+2}{2(j-n)}; \frac{n+2}{2j-2n} + 1; -\frac{ax^{j-n}}{b}\right)}{(n+2)\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (2 + n)/(2*(j - n)), 1 + (
2 + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + n)*Sqrt[1 + (a*x^(j - n))/b]
)
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{ax^j+bx^n}\,dx &= \frac{\left(x^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{n/2}\sqrt{b+ax^{j-n}}\,dx}{\sqrt{b+ax^{j-n}}}\\ &= \frac{\left(x^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\,dx}{\sqrt{1+\frac{ax^{j-n}}{b}}}\\ &= \frac{2x\sqrt{ax^j+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{2+n}{2(j-n)};1+\frac{2+n}{2j-2n};-\frac{ax^{j-n}}{b}\right)}{(2+n)\sqrt{1+\frac{ax^{j-n}}{b}}}\end{aligned}$$

**Mathematica [A]** time = 0.142623, size = 134, normalized size = 1.54

$$\frac{2x\left(a(j-n)x^j\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{2j-n+2}{2j-2n};\frac{4j-3n+2}{2j-2n};-\frac{ax^{j-n}}{b}\right)-(2j-n+2)\left(ax^j+bx^n\right)\right)}{(n+2)(-2j+n-2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(-((2 + 2*j - n)*(a*x^j + b*x^n)) + a*(j - n)*x^j*Sqrt[1 + (a*x^(j - n
))/b]*Hypergeometric2F1[1/2, (2 + 2*j - n)/(2*j - 2*n), (2 + 4*j - 3*n)/(2*
j - 2*n), -((a*x^(j - n))/b)]))/((2 + n)*(-2 - 2*j + n)*Sqrt[a*x^j + b*x^n]
)
```

---

**Maple [F]** time = 0.387, size = 0, normalized size = 0.

$$\int \sqrt{ax^j+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((a*x^j+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**j + b*x**n), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ax^j + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n), x)
```

## 3.428 $\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx$

**Optimal**. Leaf size=93

$$\frac{2x\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2(j-n)};\frac{1-\frac{n}{2}}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

```
[Out] (2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 - n)/(2*(j - n)),
1 + (1 - n/2)/(j - n), -((a*x^(j - n))/b)])/((2 - n)*Sqrt[a*x^j + b*x^n])
```

---

**Rubi [A]** time = 0.0511551, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2011, 365, 364}

$$\frac{2x\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2(j-n)};\frac{1-\frac{n}{2}}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 - n)/(2*(j - n)),
1 + (1 - n/2)/(j - n), -((a*x^(j - n))/b)])/((2 - n)*Sqrt[a*x^j + b*x^n])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx=\frac{\left(x^{n/2}\sqrt{b+ax^{j-n}}\right)\int \frac{x^{-n/2}}{\sqrt{b+ax^{j-n}}}\,dx}{\sqrt{ax^j+bx^n}}$$

$$=\frac{\left(x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{-n/2}}{\sqrt{1+\frac{ax^{j-n}}{b}}}\,dx}{\sqrt{ax^j+bx^n}}$$

$$=\frac{2x\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2(j-n)};1+\frac{1-\frac{n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

**Mathematica [A]** time = 0.0510112, size = 88, normalized size = 0.95

$$-\frac{2x\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2(n-j)};\frac{n-2}{2(n-j)}+1;-\frac{ax^{j-n}}{b}\right)}{(n-2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (-2 + n)/(2*(-j + n)
), 1 + (-2 + n)/(2*(-j + n)), -((a*x^(j - n))/b)])/((-2 + n)*Sqrt[a*x^j + b
*x^n])
```

---

**Maple [F]** time = 0.357, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int(1/(a*x^j+b*x^n)^(1/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x^j + b*x^n), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^j + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**j + b*x**n), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^j + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*x^j + b*x^n), x)
```

## 3.429 $\int \frac{1}{\left(ax^j+bx^n\right)^{3/2}}\,dx$

**Optimal**. Leaf size=101

$$\frac{2x^{1-n}\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n};\frac{1-\frac{3n}{2}}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}$$

```
[Out] (2*x^(1 - n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[3/2, (1 - (3*n)/2)
/(j - n), 1 + (1 - (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b*(2 - 3*n)*Sqrt
[a*x^j + b*x^n])
```

---

**Rubi [A]** time = 0.0568401, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2011, 365, 364}

$$\frac{2x^{1-n}\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n};\frac{1-\frac{3n}{2}}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^j + b*x^n)^(-3/2),x]
```

```
[Out] (2*x^(1 - n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[3/2, (1 - (3*n)/2)
/(j - n), 1 + (1 - (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b*(2 - 3*n)*Sqrt
[a*x^j + b*x^n])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(ax^j+bx^n\right)^{3/2}}\,dx=\frac{\left(x^{n/2}\sqrt{b+ax^{j-n}}\right)\int \frac{x^{-3n/2}}{\left(b+ax^{j-n}\right)^{3/2}}\,dx}{\sqrt{ax^j+bx^n}}$$

$$=\frac{\left(x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{-3n/2}}{\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}}\,dx}{b\sqrt{ax^j+bx^n}}$$

$$=\frac{2x^{1-n}\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n};1+\frac{1-\frac{3n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}$$

**Mathematica [A]** time = 0.0827265, size = 104, normalized size = 1.03

$$\frac{2x^{1-j}\left(\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{1}{2},-\frac{2j+n-2}{2(j-n)};\frac{2-3n}{2j-2n};-\frac{ax^{j-n}}{b}\right)-1\right)}{a(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^j + b*x^n)^(-3/2),x]
```

```
[Out] (2*x^(1 - j)*(-1 + Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, -(-2 +
2*j + n)/(2*(j - n)), (2 - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(a*(j -
n)*Sqrt[a*x^j + b*x^n])
```

---

**Maple [F]** time = 0.36, size = 0, normalized size = 0.

$$\int \left(ax^j+bx^n\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int(1/(a*x^j+b*x^n)^(3/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax^j+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^(-3/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax^{j} + bx^{n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((a*x**j + b*x**n)**(-3/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax^{j} + bx^{n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(-3/2), x)
```

## 3.430 $\int \frac{1}{\left(ax^j+bx^n\right)^{5/2}}\,dx$

**Optimal**. Leaf size=101

$$\frac{2x^{1-2n}\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n};\frac{1-\frac{5n}{2}}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}$$

```
[Out] (2*x^(1 - 2*n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1 - (5*n)/
2)/(j - n), 1 + (1 - (5*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b^2*(2 - 5*n)*
Sqrt[a*x^j + b*x^n])
```

---

**Rubi [A]** time = 0.0565193, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2011, 365, 364}

$$\frac{2x^{1-2n}\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n};\frac{1-\frac{5n}{2}}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^j + b*x^n)^(-5/2),x]
```

```
[Out] (2*x^(1 - 2*n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1 - (5*n)/
2)/(j - n), 1 + (1 - (5*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b^2*(2 - 5*n)*
Sqrt[a*x^j + b*x^n])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(ax^j+bx^n\right)^{5/2}}\,dx=\frac{\left(x^{n/2}\sqrt{b+ax^{j-n}}\right)\int \frac{x^{-5n/2}}{\left(b+ax^{j-n}\right)^{5/2}}\,dx}{\sqrt{ax^j+bx^n}}$$

$$=\frac{\left(x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{-5n/2}}{\left(1+\frac{ax^{j-n}}{b}\right)^{5/2}}\,dx}{b^2\sqrt{ax^j+bx^n}}$$

$$=\frac{2x^{1-2n}\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n};1+\frac{1-\frac{5n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}$$

**Mathematica [A]** time = 0.243232, size = 185, normalized size = 1.83

$$\frac{2x^{1-2j}\Bigg(\left(8j^2+2j(7n-6)+3n^2-8n+4\right)\sqrt{\frac{ax^{j-n}}{b}+1}\left(ax^j+bx^n\right){}_2F_1\left(\frac{1}{2},-\frac{4j+n-2}{2(j-n)};\frac{-2j-3n+2}{2j-2n};-\frac{ax^{j-n}}{b}\right)-(4j+n-2)\big(a(j+4}{3a^2(-4j-n+2)(j-n)^2\left(ax^j+bx^n\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^j + b*x^n)^(-5/2),x]
```

```
[Out] (2*x^(1 - 2*j)*(-((-2 + 4*j + n)*(a*(-2 + j + 4*n)*x^j + b*(-2 + 2*j + 3*n)
*x^n)) + (4 + 8*j^2 - 8*n + 3*n^2 + 2*j*(-6 + 7*n))*Sqrt[1 + (a*x^(j - n))/
b]*(a*x^j + b*x^n)*Hypergeometric2F1[1/2, -(-2 + 4*j + n)/(2*(j - n)), (2 -
 2*j - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(3*a^2*(2 - 4*j - n)*(j - n)
^2*(a*x^j + b*x^n)^(3/2))
```

---

**Maple [F]** time = 0.348, size = 0, normalized size = 0.

$$\int \left(ax^j+bx^n\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^j+b*x^n)^(5/2),x)
```

```
[Out] int(1/(a*x^j+b*x^n)^(5/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax^j+bx^n\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^(-5/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax^j + bx^n\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**j+b*x**n)**(5/2),x)
```

```
[Out] Integral((a*x**j + b*x**n)**(-5/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(ax^j + bx^n\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(-5/2), x)
```

## 3.431 $\int \sqrt{\frac{1+x}{x^5}}\,dx$

**Optimal**. Leaf size=18

$$-\frac{2}{3}\left(\frac{1}{x^4}+\frac{1}{x^5}\right)^{3/2}x^6$$

```
[Out] (-2*(x^(-5) + x^(-4))^(3/2)*x^6)/3
```

---

**Rubi [A]** time = 0.006895, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1979, 2000}

$$-\frac{2}{3}\left(\frac{1}{x^4}+\frac{1}{x^5}\right)^{3/2}x^6$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/x^5],x]
```

```
[Out] (-2*(x^(-5) + x^(-4))^(3/2)*x^6)/3
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1+x}{x^5}}\,dx &= \int \sqrt{\frac{1}{x^5}+\frac{1}{x^4}}\,dx\\ &= -\frac{2}{3}\left(\frac{1}{x^5}+\frac{1}{x^4}\right)^{3/2}x^6\end{aligned}$$

**Mathematica [A]** time = 0.0052456, size = 19, normalized size = 1.06

$$-\frac{2}{3}x(x+1)\sqrt{\frac{x+1}{x^5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/x^5],x]
```

```
[Out] (-2*x*(1 + x)*Sqrt[(1 + x)/x^5])/3
```

---

**Maple [A]** time = 0.024, size = 16, normalized size = 0.9

$$-\frac{2x(1+x)}{3}\sqrt{\frac{1+x}{x^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/x^5)^(1/2),x)
```

```
[Out] -2/3*x*(1+x)*((1+x)/x^5)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x+1}{x^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x^5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x + 1)/x^5), x)
```

---

**Fricas [A]** time = 0.92097, size = 46, normalized size = 2.56

$$-\frac{2}{3}\left(x^2+x\right)\sqrt{\frac{x+1}{x^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x^5)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(x^2 + x)*sqrt((x + 1)/x^5)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x+1}{x^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x**5)**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)/x**5), x)
```

---

**Giac [B]** time = 1.2883, size = 68, normalized size = 3.78

$$\frac{2\left(3\left(x-\sqrt{x^2+x}\right)^2\operatorname{sgn}(x)+3\left(x-\sqrt{x^2+x}\right)\operatorname{sgn}(x)+\operatorname{sgn}(x)\right)}{3\left(x-\sqrt{x^2+x}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x^5)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*(x - sqrt(x^2 + x))^2*sgn(x) + 3*(x - sqrt(x^2 + x))*sgn(x) + sgn(x)
)/(x - sqrt(x^2 + x))^3
```

## 3.432 $\int \sqrt{x+x^{5/2}}\,dx$

**Optimal**. Leaf size=20

$$\frac{4\left(x^{5/2}+x\right)^{3/2}}{9x^{3/2}}$$

```
[Out] (4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

---

**Rubi [A]** time = 0.003314, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2000}

$$\frac{4\left(x^{5/2}+x\right)^{3/2}}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + x^(5/2)],x]
```

```
[Out] (4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \sqrt{x+x^{5/2}}\,dx=\frac{4\left(x+x^{5/2}\right)^{3/2}}{9x^{3/2}}$$

**Mathematica [A]** time = 0.0099119, size = 20, normalized size = 1.

$$\frac{4\left(x^{5/2}+x\right)^{3/2}}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + x^(5/2)],x]
```

```
[Out] (4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

---

**Maple [A]** time = 0.009, size = 18, normalized size = 0.9

$$\frac{4}{9}\sqrt{x+x^{\frac{5}{2}}}\left(1+x^{\frac{3}{2}}\right)\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+x^(5/2))^(1/2),x)
```

```
[Out] 4/9*(x+x^(5/2))^(1/2)/x^(1/2)*(1+x^(3/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^{\frac{5}{2}} + x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(5/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^(5/2) + x), x)
```

---

**Fricas [A]** time = 1.1921, size = 55, normalized size = 2.75

$$\frac{4\,\sqrt{x^{\frac{5}{2}} + x}\left(x^{2} + \sqrt{x}\right)}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(5/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/9*sqrt(x^(5/2) + x)*(x^2 + sqrt(x))/x
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^{\frac{5}{2}} + x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x**(5/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x**(5/2) + x), x)
```

---

**Giac [A]** time = 1.27574, size = 15, normalized size = 0.75

$$\frac{4}{9}\left(x^{\frac{3}{2}} + 1\right)^{\frac{3}{2}} - \frac{4}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(5/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/9*(x^(3/2) + 1)^(3/2) - 4/9
```

## 3.433 $\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx$

**Optimal**. Leaf size=8

$$2\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*ArcTan[Sqrt[x]]
```

---

**Rubi [A]** time = 0.003541, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 63, 203}

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + x^(3/2))^(-1),x]

[Out] 2*ArcTan[Sqrt[x]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx &= \int \frac{1}{\sqrt{x}(1+x)}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\ &= 2\tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

**Mathematica [A]** time = 0.0018928, size = 8, normalized size = 1.

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + x^(3/2))^(-1),x]
```

```
[Out] 2*ArcTan[Sqrt[x]]
```

---

**Maple [A]** time = 0.004, size = 7, normalized size = 0.9

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)+x^(1/2)),x)
```

```
[Out] 2*arctan(x^(1/2))
```

---

**Maxima [A]** time = 1.71752, size = 8, normalized size = 1.

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(x))
```

---

**Fricas [A]** time = 0.958751, size = 26, normalized size = 3.25

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x))
```

---

**Sympy [A]** time = 0.231157, size = 7, normalized size = 0.88

$$2\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(3/2)+x**(1/2)),x)
```

```
[Out] 2*atan(sqrt(x))
```

---

**Giac [A]** time = 1.32652, size = 8, normalized size = 1.

$$2 \arctan \left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

## 3.434 $\int x\sqrt{x^2\left(a+bx^3\right)}\,dx$

**Optimal**. Leaf size=25

$$\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

```
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

---

**Rubi [A]** time = 0.010443, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[x^2*(a + b*x^3)],x]
```

```
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx=\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

**Mathematica [A]** time = 0.0119565, size = 25, normalized size = 1.

$$\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[x^2*(a + b*x^3)],x]
```

```
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

---

**Maple [A]** time = 0.005, size = 29, normalized size = 1.2

$$\frac{2\,bx^3+2\,a}{9\,bx}\sqrt{x^2\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2*(b*x^3+a))^(1/2),x)
```

```
[Out] 2/9*(b*x^3+a)*(x^2*(b*x^3+a))^(1/2)/b/x
```

---

**Maxima [A]** time = 1.24923, size = 19, normalized size = 0.76

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

---

**Fricas [A]** time = 0.908743, size = 58, normalized size = 2.32

$$\frac{2\sqrt{bx^5+ax^2}\left(bx^3+a\right)}{9bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 + a)/(b*x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2*(b*x**3+a))**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.45822, size = 36, normalized size = 1.44

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}\operatorname{sgn}(x)}{9b}-\frac{2a^{\frac{3}{2}}\operatorname{sgn}(x)}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)*sgn(x)/b - 2/9*a^(3/2)*sgn(x)/b
```

## 3.435 $\int x\sqrt{ax^2+bx^5}\,dx$

**Optimal**. Leaf size=25

$$\frac{2\left(ax^2+bx^5\right)^{3/2}}{9bx^3}$$

```
[Out] (2*(a*x^2 + b*x^5)^(3/2))/(9*b*x^3)
```

---

**Rubi [A]** time = 0.0105747, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{2\left(ax^2+bx^5\right)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*(a*x^2 + b*x^5)^(3/2))/(9*b*x^3)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2\left(ax^2+bx^5\right)^{3/2}}{9bx^3}$$

**Mathematica [A]** time = 0.0023885, size = 25, normalized size = 1.

$$\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

---

**Maple [A]** time = 0.002, size = 29, normalized size = 1.2

$$\frac{2\,bx^3+2\,a}{9\,bx}\sqrt{bx^5+ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 2/9*(b*x^3+a)*(b*x^5+a*x^2)^(1/2)/b/x
```

---

**Maxima [A]** time = 1.13338, size = 19, normalized size = 0.76

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

---

**Fricas [A]** time = 0.937805, size = 58, normalized size = 2.32

$$\frac{2\,\sqrt{bx^5+ax^2}\left(bx^3+a\right)}{9\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 + a)/(b*x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(a + b*x**3)), x)
```

---

**Giac [A]** time = 1.17149, size = 36, normalized size = 1.44

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}\operatorname{sgn}\left(x\right)}{9\,b}-\frac{2\,a^{\frac{3}{2}}\operatorname{sgn}\left(x\right)}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)*sgn(x)/b - 2/9*a^(3/2)*sgn(x)/b
```

## 3.436 $\int \sqrt{x^4 (a + bx^3)}\, dx$

**Optimal**. Leaf size=25

$$\frac{2\left(ax^4+bx^7\right)^{3/2}}{9bx^6}$$

```
[Out] (2*(a*x^4 + b*x^7)^(3/2))/(9*b*x^6)
```

---

**Rubi [A]** time = 0.0078477, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1979, 2000}

$$\frac{2\left(ax^4+bx^7\right)^{3/2}}{9bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^4*(a + b*x^3)],x]
```

```
[Out] (2*(a*x^4 + b*x^7)^(3/2))/(9*b*x^6)
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned} \int \sqrt{x^4\left(a+bx^3\right)}\, dx &= \int \sqrt{ax^4+bx^7}\, dx\\ &= \frac{2\left(ax^4+bx^7\right)^{3/2}}{9bx^6} \end{aligned}$$

**Mathematica [A]** time = 0.010894, size = 25, normalized size = 1.

$$\frac{2\left(x^4\left(a+bx^3\right)\right)^{3/2}}{9bx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^4*(a + b*x^3)],x]
```

```
[Out] (2*(x^4*(a + b*x^3))^(3/2))/(9*b*x^6)
```

---

**Maple [A]** time = 0.004, size = 29, normalized size = 1.2

$$\frac{2bx^3+2a}{9bx^2}\sqrt{x^4\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(b*x^3+a))^(1/2),x)
```

```
[Out] 2/9*(b*x^3+a)*(x^4*(b*x^3+a))^(1/2)/b/x^2
```

---

**Maxima [A]** time = 1.1652, size = 19, normalized size = 0.76

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

---

**Fricas [A]** time = 0.918432, size = 61, normalized size = 2.44

$$\frac{2\sqrt{bx^7+ax^4}\left(bx^3+a\right)}{9bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*sqrt(b*x^7 + a*x^4)*(b*x^3 + a)/(b*x^2)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4*(b*x**3+a))**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.24237, size = 19, normalized size = 0.76

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

## 3.437 $\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx$

**Optimal**. Leaf size=988

result too large to display

```
[Out] (-45*a^2*(a + 2*b*x^(1/3))*(-((b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3))/(14*
2^(1/3)*b^3*(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/
3))*(a*x^(1/3) + b*x^(2/3))^(1/3)) - (45*a*(a + b*x^(1/3))*x^(1/3))/(28*b^2
*(a*x^(1/3) + b*x^(2/3))^(1/3)) + (9*(a + b*x^(1/3))*x^(2/3))/(7*b*(a*x^(1/
3) + b*x^(2/3))^(1/3)) - (45*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^4*(1 - 2^(2/3)*(-(
(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((b*(a + b*x^(
1/3))*x^(1/3))/a^2))^(1/3) + 2*2^(1/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))
^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^
2]*(-((b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3)*EllipticE[ArcSin[(1 + Sqrt[3]
 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/
3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))], -7 + 4*Sqrt[3]])/(28*2^(1/
3)*b^3*Sqrt[-((1 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 -
 Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2)]*(a + 2*b
*x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(1/3)) + (15*3^(3/4)*a^4*(1 - 2^(2/3)*(-(
(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((b*(a + b*x^(
1/3))*x^(1/3))/a^2))^(1/3) + 2*2^(1/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))
^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^
2]*(-((b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3)*EllipticF[ArcSin[(1 + Sqrt[3]
 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/
3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))], -7 + 4*Sqrt[3]])/(7*2^(5/6
)*b^3*Sqrt[-((1 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 -
Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2)]*(a + 2*b*
x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(1/3))
```

---

**Rubi [A]** time = 2.24545, antiderivative size = 988, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {2011, 341, 50, 61, 622, 619, 235, 304, 219, 1879}

$$-\frac{45\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}\sqrt[3]{-\frac{b\left(\sqrt[3]{x}a+bx^{2/3}\right)}{a^2}}E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{b(a+b}{}}}{-2^{2/3}\sqrt[3]{-\frac{b(a+b}{}}}\right.\right.}{28\sqrt[3]{2}b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{\sqrt[3]{x}a+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^(1/3) + b*x^(2/3))^(-1/3),x]
```

```
[Out] (-45*a^2*(a + 2*b*x^(1/3))*(-((b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3))/(14*
2^(1/3)*b^3*(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/
3))*(a*x^(1/3) + b*x^(2/3))^(1/3)) - (45*a*(a + b*x^(1/3))*x^(1/3))/(28*b^2
*(a*x^(1/3) + b*x^(2/3))^(1/3)) + (9*(a + b*x^(1/3))*x^(2/3))/(7*b*(a*x^(1/
3) + b*x^(2/3))^(1/3)) - (45*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^4*(1 - 2^(2/3)*(-(
(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((b*(a + b*x^(
```

```
1/3))*x^(1/3))/a^2))^(1/3) + 2*2^(1/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))
^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^
2]*(-((b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3)*EllipticE[ArcSin[(1 + Sqrt[3]
 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/
3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))], -7 + 4*Sqrt[3]])/(28*2^(1/
3)*b^3*Sqrt[-((1 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 -
 Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2)]*(a + 2*b
*x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(1/3)) + (15*3^(3/4)*a^4*(1 - 2^(2/3)*(-(
(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((b*(a + b*x^(
1/3))*x^(1/3))/a^2))^(1/3) + 2*2^(1/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))
^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^
2]*(-((b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3)*EllipticF[ArcSin[(1 + Sqrt[3]
 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/
3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))], -7 + 4*Sqrt[3]])/(7*2^(5/6
)*b^3*Sqrt[-((1 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 -
Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2)]*(a + 2*b*
x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(1/3))
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/(a*c + (b*c + a*d)*x + b*d*x^2)^m, Int[(a*c + (b*c
 + a*d)*x + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d,
0] && LtQ[-1, m, 0] && LeQ[3, Denominator[m], 4] && AtomQ[b*c + a*d]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

### Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

### Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

### Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

### Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

### Rubi steps

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx = \frac{\left(\sqrt[3]{a+b\sqrt[3]{x}}\sqrt[9]{x}\right)\int \frac{1}{\sqrt[3]{a+b\sqrt[3]{x}}\sqrt[9]{x}}\,dx}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= \frac{\left(3\sqrt[3]{a+b\sqrt[3]{x}}\sqrt[9]{x}\right)\text{Subst}\left(\int \frac{x^{5/3}}{\sqrt[3]{a+bx}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - \frac{\left(15a\sqrt[3]{a+b\sqrt[3]{x}}\sqrt[9]{x}\right)\text{Subst}\left(\int \frac{x^{2/3}}{\sqrt[3]{a+bx}}\,dx, x, \sqrt[3]{x}\right)}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{\left(15a^2\sqrt[3]{a+b\sqrt[3]{x}}\sqrt[9]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\sqrt[3]{a+bx}}\,dx, x, \sqrt[3]{x}\right)}{14b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{ax+bx^2}}\,dx, x, \sqrt[3]{x}\right)}{14b^2}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{\left(15a^2\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}}\,dx, x, \sqrt[3]{x}\right)}{14b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - \frac{\left(15a^4\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}\right)\text{Subst}\Bigg(\int \frac{1}{\sqrt[3]{1-\frac{a^2x^2}{b^2}}}\,dx, x, -\frac{b(a+2}{a^2}}{14\sqrt[3]{2}b^4\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - \frac{\left(45a^4\sqrt{-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}\right)\text{Subst}\Bigg(\int \frac{x}{\sqrt{-1+x^3}}}{28\sqrt[3]{2}b^3\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{a\sqrt[3]{x}+bx^{2/}}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{\left(45a^4\sqrt{-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}\right)\text{Subst}\Bigg(\int \frac{1+\sqrt{3}-x}{\sqrt{-1+x^3}}}{28\sqrt[3]{2}b^3\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{a\sqrt[3]{x}+bx^{2}}}$$

4

$$= -\frac{45a^2\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}}{14\sqrt[3]{2}b^3\left(1-\sqrt{3}-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - \frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - -$$

**Mathematica [C]** time = 0.0185253, size = 61, normalized size = 0.06

$$\frac{9x\sqrt[3]{\frac{b\sqrt[3]{x}}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{11}{3};-\frac{b\sqrt[3]{x}}{a}\right)}{8\sqrt[3]{\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^(1/3) + b*x^(2/3))^(-1/3),x]
```

```
[Out] (9*(1 + (b*x^(1/3))/a)^(1/3)*x*Hypergeometric2F1[1/3, 8/3, 11/3, -((b*x^(1/
3))/a)])/(8*((a + b*x^(1/3))*x^(1/3))^(1/3))
```

---

**Maple [F]** time = 0.203, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x)
```

```
[Out] int(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-1/3), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**(1/3)+b*x**(2/3))**(1/3),x)
```

```
[Out] Integral((a*x**(1/3) + b*x**(2/3))**(-1/3), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{\frac{2}{3}} + ax^{\frac{1}{3}}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-1/3), x)
```

## 3.438 $\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx$

**Optimal**. Leaf size=487

$$\frac{6\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}-\sqrt{3}+1\right)^2}}\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}\right)^{2/3}\text{EllipticF}\left(\sin^{-}\right.}{5b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}-\sqrt{3}+1\right)^2}}\left(a+2b\sqrt[3]{x}\right)\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

```
[Out] (-18*a*(a + b*x^(1/3))*x^(1/3))/(5*b^2*(a*x^(1/3) + b*x^(2/3))^(2/3)) + (9*
(a + b*x^(1/3))*x^(2/3))/(5*b*(a*x^(1/3) + b*x^(2/3))^(2/3)) + (6*2^(1/3)*3
^(3/4)*Sqrt[2 - Sqrt[3]]*a^4*(1 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^
2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3) + 2
*2^(1/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)
*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2]*(-((b*(a*x^(1/3) + b*x^(2/3
)))/a^2))^(2/3)*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3
))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/
3))/a^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*b^3*Sqrt[-((1 - 2^(2/3)*(-((b*(a + b
*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3)
)*x^(1/3))/a^2))^(1/3))^2)]*(a + 2*b*x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(2/3)
)
```

---

**Rubi [A]** time = 0.973875, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2011, 341, 50, 61, 622, 619, 236, 219}

$$\frac{6\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}-\sqrt{3}+1\right)^2}}\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}\right)^{2/3}F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{\ }}{-2^{2/3}\sqrt[3]{\ }}\right.\right.}{5b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}-\sqrt{3}+1\right)^2}}\left(a+2b\sqrt[3]{x}\right)\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^(1/3) + b*x^(2/3))^(-2/3),x]
```

```
[Out] (-18*a*(a + b*x^(1/3))*x^(1/3))/(5*b^2*(a*x^(1/3) + b*x^(2/3))^(2/3)) + (9*
(a + b*x^(1/3))*x^(2/3))/(5*b*(a*x^(1/3) + b*x^(2/3))^(2/3)) + (6*2^(1/3)*3
^(3/4)*Sqrt[2 - Sqrt[3]]*a^4*(1 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^
2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3) + 2
*2^(1/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)
*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2]*(-((b*(a*x^(1/3) + b*x^(2/3
)))/a^2))^(2/3)*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3
))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/
3))/a^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*b^3*Sqrt[-((1 - 2^(2/3)*(-((b*(a + b
```

```
*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3)
)*x^(1/3))/a^2))^(1/3))^2)]*(a + 2*b*x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(2/3)
)
```

### Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

### Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

### Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 61

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/(a*c + (b*c + a*d)*x + b*d*x^2)^m, Int[(a*c + (b*c
 + a*d)*x + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d,
0] && LtQ[-1, m, 0] && LeQ[3, Denominator[m], 4] && AtomQ[b*c + a*d]
```

### Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

### Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

### Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

### Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
```

`] && NegQ[a]`

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx &= \frac{\left(\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}\right)\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}}\,dx}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\\
&= \frac{\left(3\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}\right)\mathrm{Subst}\left(\int \frac{x^{4/3}}{(a+bx)^{2/3}}\,dx,x,\sqrt[3]{x}\right)}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\\
&= \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}-\frac{\left(12a\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}\right)\mathrm{Subst}\left(\int \frac{\sqrt[3]{x}}{(a+bx)^{2/3}}\,dx,x,\sqrt[3]{x}\right)}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\\
&= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{\left(6a^2\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}\right)\mathrm{Subst}\left(\int \frac{1}{x^{2/3}(a+bx)^{2/3}}\right.}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\\
&= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{\left(6a^2\right)\mathrm{Subst}\left(\int \frac{1}{\left(ax+bx^2\right)^{2/3}}\,dx,x,\sqrt[3]{x}\right)}{5b^2}\\
&= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{\left(6a^2\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}\right)^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{\left(-\frac{bx}{a}-\frac{b^2x^2}{a^2}\right)^{2}}\right.}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\\
&= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}-\frac{\left(6\sqrt[3]{2}a^4\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}\right)^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{a^2x^2}{b^2}\right.}\right.}{5b^4\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\\
&= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}-\frac{\left(9\sqrt[3]{2}a^4\sqrt{-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}\right)^{2/3}\right)\mathrm{Sub}}{5b^3\left(a+2b\sqrt[3]{x}\right)\left(a\sqrt[3]{\right.}}\\
&= -\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{6\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}a^4\left(1-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)\sqrt{\phantom{x}}}{5b^3\sqrt{\phantom{x}}}
\end{aligned}
$$

**Mathematica [C]** time = 0.0159076, size = 61, normalized size = 0.13

$$\frac{9x\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{7}{3};\frac{10}{3};-\frac{b\sqrt[3]{x}}{a}\right)}{7\left(\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^(1/3) + b*x^(2/3))^(-2/3),x]
```

```
[Out] (9*(1 + (b*x^(1/3))/a)^(2/3)*x*Hypergeometric2F1[2/3, 7/3, 10/3, -((b*x^(1/
3))/a)])/(7*((a + b*x^(1/3))*x^(1/3))^(2/3))
```

---

**Maple [F]** time = 0.202, size = 0, normalized size = 0.

$$\int \left(a\sqrt[3]{x}+bx^{\frac{2}{3}}\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x)
```

```
[Out] int(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-2/3), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{\frac{2}{3}}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**(1/3)+b*x**(2/3))**(2/3),x)
```

```
[Out] Integral((a*x**(1/3) + b*x**(2/3))**(-2/3), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{\frac{2}{3}} + ax^{\frac{1}{3}}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-2/3), x)
```

## 3.439 $\int x^m \left(ax^j+bx^n\right)^p \, dx$

**Optimal**. Leaf size=89

$$\frac{x^{m+1}\left(a+bx^{n-j}\right)\left(ax^j+bx^n\right)^p {}_2F_1\left(1,p+\frac{m+jp+1}{n-j}+1;\frac{m+jp+1}{n-j}+1;-\frac{bx^{n-j}}{a}\right)}{a(jp+m+1)}$$

```
[Out] (x^(1 + m)*(a*x^j + b*x^n)^p*(a + b*x^(-j + n))*Hypergeometric2F1[1, 1 + p
+ (1 + m + j*p)/(-j + n), 1 + (1 + m + j*p)/(-j + n), -((b*x^(-j + n))/a)])
/(a*(1 + m + j*p))
```

---

**Rubi [A]** time = 0.0683331, antiderivative size = 92, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2032, 365, 364}

$$\frac{x^{m+1}\left(\frac{ax^{j-n}}{b}+1\right)^{-p}\left(ax^j+bx^n\right)^p {}_2F_1\left(-p,\frac{m+np+1}{j-n};\frac{m+np+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{m+np+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a*x^j + b*x^n)^p,x]
```

```
[Out] (x^(1 + m)*(a*x^j + b*x^n)^p*Hypergeometric2F1[-p, (1 + m + n*p)/(j - n), 1
 + (1 + m + n*p)/(j - n), -((a*x^(j - n))/b)])/((1 + m + n*p)*(1 + (a*x^(j
- n))/b)^p)
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^m \left(ax^j+bx^n\right)^p\,dx &= \left(x^{-np}\left(b+ax^{j-n}\right)^{-p}\left(ax^j+bx^n\right)^p\right)\int x^{m+np}\left(b+ax^{j-n}\right)^p\,dx\\ &= \left(x^{-np}\left(1+\frac{ax^{j-n}}{b}\right)^{-p}\left(ax^j+bx^n\right)^p\right)\int x^{m+np}\left(1+\frac{ax^{j-n}}{b}\right)^p\,dx\\ &= \frac{x^{1+m}\left(1+\frac{ax^{j-n}}{b}\right)^{-p}\left(ax^j+bx^n\right)^p\,{}_2F_1\left(-p,\frac{1+m+np}{j-n};1+\frac{1+m+np}{j-n};-\frac{ax^{j-n}}{b}\right)}{1+m+np}\end{aligned}$$

**Mathematica [A]** time = 0.103634, size = 92, normalized size = 1.03

$$\frac{x^{m+1}\left(\frac{ax^{j-n}}{b}+1\right)^{-p}\left(ax^j+bx^n\right)^p\,{}_2F_1\left(-p,\frac{m+np+1}{j-n};\frac{m+np+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{m+np+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a*x^j + b*x^n)^p,x]
```

```
[Out] (x^(1 + m)*(a*x^j + b*x^n)^p*Hypergeometric2F1[-p, (1 + m + n*p)/(j - n), 1
 + (1 + m + n*p)/(j - n), -((a*x^(j - n))/b)])/((1 + m + n*p)*(1 + (a*x^(j
- n))/b)^p)
```

---

**Maple [F]** time = 0.582, size = 0, normalized size = 0.

$$\int x^m\left(ax^j+bx^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*x^j+b*x^n)^p,x)
```

```
[Out] int(x^m*(a*x^j+b*x^n)^p,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax^j+bx^n\right)^p x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^p*x^m, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(ax^j+bx^n\right)^p x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x^j + b*x^n)^p*x^m, x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \left(ax^j + bx^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a*x**j+b*x**n)**p,x)
```

```
[Out] Integral(x**m*(a*x**j + b*x**n)**p, x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(ax^j + bx^n\right)^p x^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^p*x^m, x)
```

## 3.440 $\int x^{-1-pq}\left(bx^n+ax^q\right)^p\,dx$

**Optimal**. Leaf size=69

$$-\frac{x^{-pq}\left(a+bx^{n-q}\right)\left(ax^q+bx^n\right)^p\,{}_2F_1\left(1,p+1;p+2;\frac{bx^{n-q}}{a}+1\right)}{a(p+1)(n-q)}$$

```
[Out] -(((a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[1, 1 + p, 2 + p, 1
 + (b*x^(n - q))/a])/(a*(1 + p)*(n - q)*x^(p*q)))
```

---

**Rubi [A]** time = 0.0683911, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2032, 266, 65}

$$-\frac{x^{-pq}\left(a+bx^{n-q}\right)\left(ax^q+bx^n\right)^p\,{}_2F_1\left(1,p+1;p+2;\frac{bx^{n-q}}{a}+1\right)}{a(p+1)(n-q)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - p*q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] -(((a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[1, 1 + p, 2 + p, 1
 + (b*x^(n - q))/a])/(a*(1 + p)*(n - q)*x^(p*q)))
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$
\begin{aligned}
\int x^{-1-pq}\left(bx^n+ax^q\right)^p\,dx &= \left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\int\frac{\left(a+bx^{n-q}\right)^p}{x}\,dx\\
&=\frac{\left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\text{Subst}\left(\int\frac{(a+bx)^p}{x}\,dx,x,x^{n-q}\right)}{n-q}\\
&=-\frac{x^{-pq}\left(a+bx^{n-q}\right)\left(bx^n+ax^q\right)^p\,{}_2F_1\left(1,1+p;2+p;1+\frac{bx^{n-q}}{a}\right)}{a(1+p)(n-q)}
\end{aligned}
$$

**Mathematica [A]** time = 0.116537, size = 73, normalized size = 1.06

$$
\frac{x^{-pq}\left(ax^q+bx^n\right)^p\left(\frac{ax^{q-n}}{b}+1\right)^{-p}\,{}_2F_1\left(-p,-p;1-p;-\frac{ax^{q-n}}{b}\right)}{p(n-q)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - p*q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] ((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((a*x^(-n + q))/b)])/(
p*(n - q)*x^(p*q)*(1 + (a*x^(-n + q))/b)^p)
```

---

**Maple [F]** time = 0.632, size = 0, normalized size = 0.

$$
\int x^{-pq-1}\left(bx^n+ax^q\right)^p\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-p*q-1)*(b*x^n+a*x^q)^p,x)
```

```
[Out] int(x^(-p*q-1)*(b*x^n+a*x^q)^p,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$
\int\left(bx^n+ax^q\right)^p x^{-pq-1}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$
\text{integral}\left(\left(bx^n+ax^q\right)^p x^{-pq-1},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-p*q-1)*(b*x**n+a*x**q)**p,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + ax^q)^p x^{-pq-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

# 3.441 $\int x^{-1-np}\left(bx^n+ax^q\right)^p\,dx$

**Optimal**. Leaf size=66

$$-\frac{x^{-np}\left(a+bx^{n-q}\right)\left(ax^q+bx^n\right)^p\,{}_2F_1\left(1,1;1-p;-\frac{bx^{n-q}}{a}\right)}{ap(n-q)}$$

```
[Out] -(((a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[1, 1, 1 - p, -((b*
x^(n - q))/a)])/(a*p*(n - q)*x^(n*p)))
```

---

**Rubi [A]** time = 0.0675878, antiderivative size = 74, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2032, 365, 364}

$$-\frac{x^{-np}\left(\frac{bx^{n-q}}{a}+1\right)^{-p}\left(ax^q+bx^n\right)^p\,{}_2F_1\left(-p,-p;1-p;-\frac{bx^{n-q}}{a}\right)}{p(n-q)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*p)*(b*x^n + a*x^q)^p,x]
```

```
[Out] -(((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^(n - q))/a)])/
(p*(n - q)*x^(n*p)*(1 + (b*x^(n - q))/a)^p))
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-1-np}\left(bx^n+ax^q\right)^p\,dx&=\left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\int x^{-1-np+pq}\left(a+bx^{n-q}\right)^p\,dx\\&=\left(x^{-pq}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\int x^{-1-np+pq}\left(1+\frac{bx^{n-q}}{a}\right)^p\,dx\\&=-\frac{x^{-np}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}\left(bx^n+ax^q\right)^p\,{}_2F_1\left(-p,-p;1-p;-\frac{bx^{n-q}}{a}\right)}{p(n-q)}\end{aligned}$$

**Mathematica [A]** time = 0.0955013, size = 74, normalized size = 1.12

$$-\frac{x^{-np}\left(\frac{bx^{n-q}}{a}+1\right)^{-p}\left(ax^q+bx^n\right)^p\,{}_2F_1\left(-p,-p;1-p;-\frac{bx^{n-q}}{a}\right)}{p(n-q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*p)*(b*x^n + a*x^q)^p,x]
```

```
[Out] -(((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^(n - q))/a)])/
(p*(n - q)*x^(n*p)*(1 + (b*x^(n - q))/a)^p))
```

---

**Maple [F]** time = 0.511, size = 0, normalized size = 0.

$$\int x^{-np-1}\left(bx^n+ax^q\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-n*p-1)*(b*x^n+a*x^q)^p,x)
```

```
[Out] int(x^(-n*p-1)*(b*x^n+a*x^q)^p,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n+ax^q)^p x^{-np-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((bx^n+ax^q)^p x^{-np-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-n*p-1)*(b*x**n+a*x**q)**p,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + ax^q)^p x^{-np-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

## 3.442 $\int x^{-1-n-(-1+p)q}\left(bx^n+ax^q\right)^p\,dx$

**Optimal**. Leaf size=69

$$\frac{bx^{-pq}\left(a+bx^{n-q}\right)\left(ax^q+bx^n\right)^p\,{}_2F_1\left(2,p+1;p+2;\frac{bx^{n-q}}{a}+1\right)}{a^2(p+1)(n-q)}$$

```
[Out] (b*(a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[2, 1 + p, 2 + p, 1
 + (b*x^(n - q))/a])/(a^2*(1 + p)*(n - q)*x^(p*q))
```

---

**Rubi [A]** time = 0.077541, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2032, 266, 65}

$$\frac{bx^{-pq}\left(a+bx^{n-q}\right)\left(ax^q+bx^n\right)^p\,{}_2F_1\left(2,p+1;p+2;\frac{bx^{n-q}}{a}+1\right)}{a^2(p+1)(n-q)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n - (-1 + p)*q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] (b*(a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[2, 1 + p, 2 + p, 1
 + (b*x^(n - q))/a])/(a^2*(1 + p)*(n - q)*x^(p*q))
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int x^{-1-n-(-1+p)q}\left(bx^n+ax^q\right)^p\,dx &= \left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\int x^{-1-n-(-1+p)q+pq}\left(a+bx^{n-q}\right)^p\,dx\\ &= \frac{\left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\text{Subst}\left(\int\frac{(a+bx)^p}{x^2}\,dx,x,x^{n-q}\right)}{n-q}\\ &= \frac{bx^{-pq}\left(a+bx^{n-q}\right)\left(bx^n+ax^q\right)^p\,{}_2F_1\left(2,1+p;2+p;1+\frac{bx^{n-q}}{a}\right)}{a^2(1+p)(n-q)}\end{aligned}$$

**Mathematica [A]** time = 0.159438, size = 82, normalized size = 1.19

$$\frac{x^{-n-pq+q}\left(ax^q+bx^n\right)^p\left(\frac{ax^{q-n}}{b}+1\right)^{-p}{}_2F_1\left(1-p,-p;2-p;-\frac{ax^{q-n}}{b}\right)}{(p-1)(n-q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n - (-1 + p)*q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] (x^(-n + q - p*q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((
a*x^(-n + q))/b)])/((-1 + p)*(n - q)*(1 + (a*x^(-n + q))/b)^p)
```

---

**Maple [F]** time = 0.51, size = 0, normalized size = 0.

$$\int x^{-1-n-(p-1)q}\left(bx^n+ax^q\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n-(p-1)*q)*(b*x^n+a*x^q)^p,x)
```

```
[Out] int(x^(-1-n-(p-1)*q)*(b*x^n+a*x^q)^p,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^n+ax^q\right)^p x^{-(p-1)q-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^n+ax^q\right)^p x^{-(p-1)q-n-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n-(-1+p)*q)*(b*x**n+a*x**q)**p,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + ax^q)^p x^{-(p-1)q-n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

## 3.443 $\int x^{-1-n(-1+p)-q} (bx^n + ax^q)^p \, dx$

**Optimal**. Leaf size=84

$$\frac{x^{n(-p)+n-q}\left(\frac{bx^{n-q}}{a}+1\right)^{-p}(ax^q+bx^n)^p\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}$$

```
[Out] (x^(n - n*p - q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b
*x^(n - q))/a)])/((1 - p)*(n - q)*(1 + (b*x^(n - q))/a)^p)
```

---

**Rubi [A]** time = 0.081219, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2032, 365, 364}

$$\frac{x^{n(-p)+n-q}\left(\frac{bx^{n-q}}{a}+1\right)^{-p}(ax^q+bx^n)^p\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*(-1 + p) - q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] (x^(n - n*p - q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b
*x^(n - q))/a)])/((1 - p)*(n - q)*(1 + (b*x^(n - q))/a)^p)
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-1-n(-1+p)-q}\left(bx^n+ax^q\right)^p\,dx &= \left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\int x^{-1-n(-1+p)-q+pq}\left(a+bx^{n-q}\right)^p\,dx\\ &= \left(x^{-pq}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\int x^{-1-n(-1+p)-q+pq}\left(1+\frac{bx^{n-q}}{a}\right)^p\,dx\\ &= \frac{x^{n-np-q}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}\left(bx^n+ax^q\right)^p\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}\end{aligned}$$

**Mathematica [A]** time = 0.131903, size = 83, normalized size = 0.99

$$-\frac{x^{n(-p)+n-q}\left(\frac{bx^{n-q}}{a}+1\right)^{-p}\left(ax^q+bx^n\right)^p\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^{n-q}}{a}\right)}{(p-1)(n-q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*(-1 + p) - q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] -((x^(n - n*p - q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -(
(b*x^(n - q))/a)])/((-1 + p)*(n - q)*(1 + (b*x^(n - q))/a)^p))
```

---

**Maple [F]** time = 0.528, size = 0, normalized size = 0.

$$\int x^{-1-n(p-1)-q}\left(bx^n+ax^q\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n*(p-1)-q)*(b*x^n+a*x^q)^p,x)
```

```
[Out] int(x^(-1-n*(p-1)-q)*(b*x^n+a*x^q)^p,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^n+ax^q\right)^p x^{-n(p-1)-q-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*(p - 1) - q - 1), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(bx^n+ax^q\right)^p x^{-np+n-q-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a*x^q)^p*x^(-n*p + n - q - 1), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n*(-1+p)-q)*(b*x**n+a*x**q)**p,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx^n + ax^q)^p x^{-n(p-1)-q-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*(p - 1) - q - 1), x)
```

## 3.444 $\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx$

**Optimal**. Leaf size=44

$$\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

```
[Out] (a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

---

**Rubi [A]** time = 0.0154702, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2000}

$$\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m + b*x^(1 + m + m*p))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \left(ax^m+bx^{1+m+mp}\right)^p\,dx=\frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+mp}\right)^{1+p}}{b(1+p)(1+mp)}$$

**Mathematica [A]** time = 0.0350637, size = 43, normalized size = 0.98

$$\frac{x^{-m(p+1)}\left(x^m\left(a+bx^{mp+1}\right)\right)^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m + b*x^(1 + m + m*p))^p,x]
```

```
[Out] (x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

---

**Maple [F]** time = 0.547, size = 0, normalized size = 0.

$$\int \left(ax^m+bx^{mp+m+1}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m+b*x^(m*p+m+1))^p,x)
```

```
[Out] int((a*x^m+b*x^(m*p+m+1))^p,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^{mp+m+1} + ax^m\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)
```

---

**Fricas [A]** time = 0.754974, size = 147, normalized size = 3.34

$$\frac{\left(bxx^{mp+m+1} + axx^m\right)\left(bx^{mp+m+1} + ax^m\right)^p}{\left(bmp^2 + (bm + b)p + b\right)x^{mp+m+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^(m*p + m + 1) + a*x*x^m)*(b*x^(m*p + m + 1) + a*x^m)^p/((b*m*p^2 + (
b*m + b)*p + b)*x^(m*p + m + 1))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m+b*x**(m*p+m+1))**p,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(bx^{mp+m+1} + ax^m\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)
```

## 3.445 $\int \left(x^m \left(a+bx^{1+mp}\right)\right)^p \, dx$

**Optimal**. Leaf size=44

$$\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

```
[Out] (a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

---

**Rubi [A]** time = 0.0150384, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1979, 2000}

$$\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*x^(1 + m*p)))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(x^m \left(a+bx^{1+mp}\right)\right)^p \, dx &= \int \left(ax^m+bx^{1+m+mp}\right)^p \, dx\\ &= \frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+mp}\right)^{1+p}}{b(1+p)(1+mp)}\end{aligned}$$

**Mathematica [A]** time = 0.0025564, size = 43, normalized size = 0.98

$$\frac{x^{-m(p+1)}\left(x^m\left(a+bx^{mp+1}\right)\right)^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*x^(1 + m*p)))^p,x]
```

```
[Out] (x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

---

**Maple [F]** time = 0.388, size = 0, normalized size = 0.

$$\int \left(x^{m} \left(a+b x^{m p+1}\right)\right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a+b*x^(m*p+1)))^p,x)
```

```
[Out] int((x^m*(a+b*x^(m*p+1)))^p,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(\left(b x^{m p+1}+a\right) x^{m}\right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)
```

---

**Fricas [A]** time = 0.758676, size = 131, normalized size = 2.98

$$\frac{\left(b x x^{m p+1}+a x\right) \left(b x^{m p+1} x^{m}+a x^{m}\right)^{p}}{\left(b m p^{2}+(b m+b) p+b\right) x^{m p+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^(m*p + 1) + a*x)*(b*x^(m*p + 1)*x^m + a*x^m)^p/((b*m*p^2 + (b*m + b)
*p + b)*x^(m*p + 1))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**m*(a+b*x**(m*p+1)))**p,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(\left(b x^{m p+1}+a\right) x^{m}\right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="giac")
```

```
[Out] integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)
```

## 3.446 $\int x^n \left(x^m \left(a+bx^{1+n+mp}\right)\right)^p \, dx$

**Optimal**. Leaf size=46

$$\frac{x^{-m(p+1)} \left(ax^m+bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

```
[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 +
p)))
```

---

**Rubi [A]** time = 0.0753067, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1980, 2014}

$$\frac{x^{-m(p+1)} \left(ax^m+bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 +
p)))
```

Rule 1980

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] &&  !Generali
zedBinomialMatchQ[u, x]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^n \left(x^m \left(a+bx^{1+n+mp}\right)\right)^p \, dx &= \int x^n \left(ax^m+bx^{1+m+n+mp}\right)^p \, dx\\ &= \frac{x^{-m(1+p)} \left(ax^m+bx^{1+m+n+mp}\right)^{1+p}}{b(1+p)(1+n+mp)}\end{aligned}$$

**Mathematica [A]** time = 0.041979, size = 45, normalized size = 0.98

$$\frac{x^{-m(p+1)} \left(x^m \left(a+bx^{mp+n+1}\right)\right)^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]
```

```
[Out] (x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)
))
```

---

**Maple [F]** time = 0.542, size = 0, normalized size = 0.

$$\int x^{n} \left(x^{m} \left(a + b x^{m p + n + 1}\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)
```

```
[Out] int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(\left(b x^{m p + n + 1} + a\right) x^{m}\right)^{p} x^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)
```

---

**Fricas [A]** time = 0.767607, size = 174, normalized size = 3.78

$$\frac{\left(b x x^{m p + n + 1} x^{n} + a x x^{n}\right) \left(b x^{m p + n + 1} x^{m} + a x^{m}\right)^{p}}{\left(b m p^{2} + b n + (b m + b n + b) p + b\right) x^{m p + n + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^(m*p + n + 1)*x^n + a*x*x^n)*(b*x^(m*p + n + 1)*x^m + a*x^m)^p/((b*m
*p^2 + b*n + (b*m + b*n + b)*p + b)*x^(m*p + n + 1))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(x**m*(a+b*x**(m*p+n+1)))**p,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(\left(bx^{mp+n+1}+a\right)x^m\right)^p x^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="giac")
```

```
[Out] integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)
```

# 3.447 $\int x^n \left(ax^m+bx^{1+m+n+mp}\right)^p \, dx$

**Optimal**. Leaf size=46

$$\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

```
[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 +
p)))
```

---

**Rubi [A]** time = 0.0453976, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2014}

$$\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 +
p)))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^n \left(ax^m+bx^{1+m+n+mp}\right)^p \, dx=\frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+n+mp}\right)^{1+p}}{b(1+p)(1+n+mp)}$$

**Mathematica [A]** time = 0.0044973, size = 45, normalized size = 0.98

$$\frac{x^{-m(p+1)}\left(x^m\left(a+bx^{mp+n+1}\right)\right)^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]
```

```
[Out] (x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)
))
```

---

**Maple [F]** time = 0.542, size = 0, normalized size = 0.

$$\int x^{n} \left(a x^{m}+b x^{m p+m+n+1}\right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x)
```

```
[Out] int(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(b x^{m p+m+n+1}+a x^{m}\right)^{p} x^{n}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)
```

---

**Fricas [A]** time = 0.996988, size = 190, normalized size = 4.13

$$\frac{\left(b x x^{m p+m+n+1} x^{n}+a x x^{m} x^{n}\right) \left(b x^{m p+m+n+1}+a x^{m}\right)^{p}}{\left(b m p^{2}+b n+(b m+b n+b) p+b\right) x^{m p+m+n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^(m*p + m + n + 1)*x^n + a*x*x^m*x^n)*(b*x^(m*p + m + n + 1) + a*x^m)
^p/((b*m*p^2 + b*n + (b*m + b*n + b)*p + b)*x^(m*p + m + n + 1))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(a*x**m+b*x**(m*p+m+n+1))**p,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(b x^{m p+m+n+1}+a x^{m}\right)^{p} x^{n}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)
```

## 3.448 $\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx$

**Optimal**. Leaf size=44

$$\frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{3n-2}\right)^{3/2}}{3bn}$$

```
[Out] (2*x^(3*(1 - n))*(a/x^(2*(1 - n)) + b*x^(-2 + 3*n))^(3/2))/(3*b*n)
```

---

**Rubi [A]** time = 0.0181042, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1979, 2000}

$$\frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{3n-2}\right)^{3/2}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^(2*(-1 + n))*(a + b*x^n)],x]
```

```
[Out] (2*x^(3*(1 - n))*(a/x^(2*(1 - n)) + b*x^(-2 + 3*n))^(3/2))/(3*b*n)
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x^{2(-1+n)}\left(a+bx^n\right)}\,dx &= \int \sqrt{ax^{2(-1+n)}+bx^{2(-1+n)+n}}\,dx\\ &= \frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{-2+3n}\right)^{3/2}}{3bn}\end{aligned}$$

**Mathematica [A]** time = 0.0306752, size = 36, normalized size = 0.82

$$\frac{2x^{3-3n}\left(x^{2n-2}\left(a+bx^n\right)\right)^{3/2}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^(2*(-1 + n))*(a + b*x^n)],x]
```

```
[Out] (2*x^(3 - 3*n)*(x^(-2 + 2*n)*(a + b*x^n))^(3/2))/(3*b*n)
```

---

**Maple [A]** time = 0.028, size = 40, normalized size = 0.9

$$\frac{(2a+2bx^n)\,x}{3bx^n n}\sqrt{\frac{(x^n)^2\,(a+bx^n)}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(-2+2*n)*(a+b*x^n))^(1/2),x)
```

```
[Out] 2/3*(1/x^2*(x^n)^2*(a+b*x^n))^(1/2)*(a+b*x^n)/(x^n)*x/b/n
```

---

**Maxima [A]** time = 1.17945, size = 23, normalized size = 0.52

$$\frac{2\,(bx^n+a)^{\frac{3}{2}}}{3\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x^n + a)^(3/2)/(b*n)
```

---

**Fricas [A]** time = 0.775232, size = 88, normalized size = 2.

$$\frac{2\,(bxx^n+ax)\sqrt{\frac{bx^{3\,n}+ax^{2\,n}}{x^2}}}{3\,bnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x*x^n + a*x)*sqrt((b*x^(3*n) + a*x^(2*n))/x^2)/(b*n*x^n)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(-2+2*n)*(a+b*x**n))**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int\sqrt{(bx^n+a)x^{2\,n-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^n + a)*x^(2*n - 2)), x)
```

## 3.449 $\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx$

**Optimal**. Leaf size=44

$$\frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{4n-3}\right)^{4/3}}{4bn}$$

```
[Out] (3*x^(4*(1 - n))*(a/x^(3*(1 - n)) + b*x^(-3 + 4*n))^(4/3))/(4*b*n)
```

---

**Rubi [A]** time = 0.0172021, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1979, 2000}

$$\frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{4n-3}\right)^{4/3}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3*(-1 + n))*(a + b*x^n))^(1/3),x]
```

```
[Out] (3*x^(4*(1 - n))*(a/x^(3*(1 - n)) + b*x^(-3 + 4*n))^(4/3))/(4*b*n)
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x^{3(-1+n)}\left(a+bx^n\right)}\,dx &= \int \sqrt[3]{ax^{3(-1+n)}+bx^{3(-1+n)+n}}\,dx\\ &= \frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{-3+4n}\right)^{4/3}}{4bn}\end{aligned}$$

**Mathematica [A]** time = 0.0298387, size = 36, normalized size = 0.82

$$\frac{3x^{4-4n}\left(x^{3n-3}\left(a+bx^n\right)\right)^{4/3}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3*(-1 + n))*(a + b*x^n))^(1/3),x]
```

```
[Out] (3*x^(4 - 4*n)*(x^(-3 + 3*n)*(a + b*x^n))^(4/3))/(4*b*n)
```

**Maple [A]** time = 0.019, size = 40, normalized size = 0.9

$$\frac{3\,x\,(a+bx^{n})}{4\,bx^{n}n}\sqrt[3]{\frac{(x^{n})^{3}\,(a+bx^{n})}{x^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(-3+3*n)*(a+b*x^n))^(1/3),x)
```

```
[Out] 3/4*(1/x^3*(x^n)^3*(a+b*x^n))^(1/3)*x/(x^n)*(a+b*x^n)/b/n
```

---

**Maxima [A]** time = 1.19037, size = 23, normalized size = 0.52

$$\frac{3\,(bx^{n}+a)^{\frac{4}{3}}}{4\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*(b*x^n + a)^(4/3)/(b*n)
```

---

**Fricas [A]** time = 0.678657, size = 90, normalized size = 2.05

$$\frac{3\,(bxx^{n}+ax)\left(\frac{bx^{4\,n}+ax^{3\,n}}{x^{3}}\right)^{\frac{1}{3}}}{4\,bnx^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b*x*x^n + a*x)*((b*x^(4*n) + a*x^(3*n))/x^3)^(1/3)/(b*n*x^n)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(-3+3*n)*(a+b*x**n))**(1/3),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left((bx^{n}+a)x^{3\,n-3}\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(((b*x^n + a)*x^(3*n - 3))^(1/3), x)
```

# 3.450 $\int \sqrt[4]{x^{4(-1+n)}\,(a+bx^n)}\,dx$

**Optimal**. Leaf size=44

$$\frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{5n-4}\right)^{5/4}}{5bn}$$

```
[Out] (4*x^(5*(1 - n))*(a/x^(4*(1 - n)) + b*x^(-4 + 5*n))^(5/4))/(5*b*n)
```

---

**Rubi [A]** time = 0.0183645, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1979, 2000}

$$\frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{5n-4}\right)^{5/4}}{5bn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(4*(-1 + n))*(a + b*x^n))^(1/4),x]
```

```
[Out] (4*x^(5*(1 - n))*(a/x^(4*(1 - n)) + b*x^(-4 + 5*n))^(5/4))/(5*b*n)
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt[4]{x^{4(-1+n)}\,(a+bx^n)}\,dx &= \int \sqrt[4]{ax^{4(-1+n)}+bx^{4(-1+n)+n}}\,dx\\ &= \frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{-4+5n}\right)^{5/4}}{5bn}\end{aligned}$$

**Mathematica [A]** time = 0.030217, size = 36, normalized size = 0.82

$$\frac{4x^{5-5n}\left(x^{4n-4}\,(a+bx^n)\right)^{5/4}}{5bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(4*(-1 + n))*(a + b*x^n))^(1/4),x]
```

```
[Out] (4*x^(5 - 5*n)*(x^(-4 + 4*n)*(a + b*x^n))^(5/4))/(5*b*n)
```

---

**Maple [A]** time = 0.018, size = 40, normalized size = 0.9

$$\frac{4\,x\,(a+b x^{n})}{5\,b x^{n} n}\sqrt[4]{\frac{(x^{n})^{4}\,(a+b x^{n})}{x^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(-4+4*n)*(a+b*x^n))^(1/4),x)
```

```
[Out] 4/5*(1/x^4*(x^n)^4*(a+b*x^n))^(1/4)*x/(x^n)*(a+b*x^n)/b/n
```

---

**Maxima [A]** time = 1.06904, size = 23, normalized size = 0.52

$$\frac{4\,(b x^{n}+a)^{\frac{5}{4}}}{5\,b n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="maxima")
```

```
[Out] 4/5*(b*x^n + a)^(5/4)/(b*n)
```

---

**Fricas [A]** time = 0.93438, size = 90, normalized size = 2.05

$$\frac{4\,(b x x^{n}+a x)\left(\frac{b x^{5\,n}+a x^{4\,n}}{x^{4}}\right)^{\frac{1}{4}}}{5\,b n x^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="fricas")
```

```
[Out] 4/5*(b*x*x^n + a*x)*((b*x^(5*n) + a*x^(4*n))/x^4)^(1/4)/(b*n*x^n)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(-4+4*n)*(a+b*x**n))**(1/4),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left((b x^{n}+a) x^{4\,n-4}\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="giac")
```

```
[Out] integrate(((b*x^n + a)*x^(4*n - 4))^(1/4), x)
```

## 3.451 $\int \left(x^{(-1+n)p}\left(a+bx^n\right)\right)^{\frac{1}{p}}\,dx$

**Optimal**. Leaf size=57

$$\frac{px^{(1-n)(p+1)}\left(ax^{-(1-n)p}+bx^{n-(1-n)p}\right)^{\frac{1}{p}+1}}{bn(p+1)}$$

```
[Out] (p*x^((1 - n)*(1 + p))*(a/x^((1 - n)*p) + b*x^(n - (1 - n)*p))^(1 + p^(-1))
)/(b*n*(1 + p))
```

---

**Rubi [A]** time = 0.0277425, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1979, 2000}

$$\frac{px^{(1-n)(p+1)}\left(ax^{-(1-n)p}+bx^{n-(1-n)p}\right)^{\frac{1}{p}+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1),x]
```

```
[Out] (p*x^((1 - n)*(1 + p))*(a/x^((1 - n)*p) + b*x^(n - (1 - n)*p))^(1 + p^(-1))
)/(b*n*(1 + p))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(x^{(-1+n)p}\left(a+bx^n\right)\right)^{\frac{1}{p}}\,dx &= \int \left(ax^{(-1+n)p}+bx^{n+(-1+n)p}\right)^{\frac{1}{p}}\,dx\\ &= \frac{px^{(1-n)(1+p)}\left(ax^{-(1-n)p}+bx^{n-(1-n)p}\right)^{1+\frac{1}{p}}}{bn(1+p)}\end{aligned}$$

**Mathematica [A]** time = 0.021195, size = 47, normalized size = 0.82

$$\frac{x^{1-n}\left(a+bx^n\right)\left(x^{(n-1)p}\left(a+bx^n\right)\right)^{\frac{1}{p}}}{bn\left(\frac{1}{p}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1),x]
```

```
[Out] (x^(1 - n)*(a + b*x^n)*(x^((-1 + n)*p)*(a + b*x^n))^p^(-1))/(b*n*(1 + p^(-1
)))
```

---

**Maple [F]** time = 0.435, size = 0, normalized size = 0.

$$\int \sqrt[p]{x^{(-1+n)p}\left(a+bx^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^((-1+n)*p)*(a+b*x^n))^(1/p),x)
```

```
[Out] int((x^((-1+n)*p)*(a+b*x^n))^(1/p),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left((bx^{n}+a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)
```

---

**Fricas [A]** time = 0.918842, size = 101, normalized size = 1.77

$$\frac{\left(bpxx^{n}+apx\right)\left((bx^{n}+a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}}{\left(bnp+bn\right)x^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="fricas")
```

```
[Out] (b*p*x*x^n + a*p*x)*((b*x^n + a)*x^((n - 1)*p))^(1/p)/((b*n*p + b*n)*x^n)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**((-1+n)*p)*(a+b*x**n))**(1/p),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left((bx^n + a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="giac")
```

```
[Out] integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)
```

## 3.452 $\int \left(x^{\frac{-1+n}{p}} (a+bx^n)\right)^p \, dx$

**Optimal**. Leaf size=61

$$\frac{x^{\frac{(1-n)(p+1)}{p}} \left(ax^{-\frac{1-n}{p}} + bx^{n-\frac{1-n}{p}}\right)^{p+1}}{bn(p+1)}$$

```
[Out] (x^(((1 - n)*(1 + p))/p)*(b*x^(n - (1 - n)/p) + a/x^((1 - n)/p))^(1 + p))/(
b*n*(1 + p))
```

---

**Rubi [A]** time = 0.0248845, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1979, 2000}

$$\frac{x^{\frac{(1-n)(p+1)}{p}} \left(ax^{-\frac{1-n}{p}} + bx^{n-\frac{1-n}{p}}\right)^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^((-1 + n)/p)*(a + b*x^n))^p,x]
```

```
[Out] (x^(((1 - n)*(1 + p))/p)*(b*x^(n - (1 - n)/p) + a/x^((1 - n)/p))^(1 + p))/(
b*n*(1 + p))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned} \int \left(x^{\frac{-1+n}{p}} (a+bx^n)\right)^p \, dx &= \int \left(bx^{n+\frac{-1+n}{p}} + ax^{\frac{-1+n}{p}}\right)^p \, dx \\ &= \frac{x^{\frac{(1-n)(1+p)}{p}} \left(bx^{n-\frac{1-n}{p}} + ax^{-\frac{1-n}{p}}\right)^{1+p}}{bn(1+p)} \end{aligned}$$

**Mathematica [A]** time = 0.0250275, size = 45, normalized size = 0.74

$$\frac{x^{1-n} (a+bx^n) \left(x^{\frac{n-1}{p}} (a+bx^n)\right)^p}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^((-1 + n)/p)*(a + b*x^n))^p,x]
```

```
[Out] (x^(1 - n)*(a + b*x^n)*(x^((-1 + n)/p)*(a + b*x^n))^p)/(b*n*(1 + p))
```

---

**Maple [F]** time = 0.395, size = 0, normalized size = 0.

$$\int \left(x^{\frac{-1+n}{p}} \left(a + b x^{n}\right)\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^((-1+n)/p)*(a+b*x^n))^p,x)
```

```
[Out] int((x^((-1+n)/p)*(a+b*x^n))^p,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(\left(b x^{n} + a\right) x^{\frac{n-1}{p}}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)
```

---

**Fricas [A]** time = 0.907711, size = 107, normalized size = 1.75

$$\frac{\left(b x x^{n} + a x\right) \left(b x^{n} x^{\frac{n-1}{p}} + a x^{\frac{n-1}{p}}\right)^{p}}{\left(b n p + b n\right) x^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^n + a*x)*(b*x^n*x^((n - 1)/p) + a*x^((n - 1)/p))^p/((b*n*p + b*n)*x^
n)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**((-1+n)/p)*(a+b*x**n))**p,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left((bx^n + a)x^{\frac{n-1}{p}}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)
```

## 3.453 $\int x^{-1+n-p(1+q)} (ax^n + bx^p)^q \, dx$

**Optimal**. Leaf size=39

$$\frac{x^{-p(q+1)} (ax^n + bx^p)^{q+1}}{a(q+1)(n-p)}$$

```
[Out] (a*x^n + b*x^p)^(1 + q)/(a*(n - p)*(1 + q)*x^(p*(1 + q)))
```

---

**Rubi [A]** time = 0.0498496, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2014}

$$\frac{x^{-p(q+1)} (ax^n + bx^p)^{q+1}}{a(q+1)(n-p)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q,x]
```

```
[Out] (a*x^n + b*x^p)^(1 + q)/(a*(n - p)*(1 + q)*x^(p*(1 + q)))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1+n-p(1+q)} (ax^n + bx^p)^q \, dx = \frac{x^{-p(1+q)} (ax^n + bx^p)^{1+q}}{a(n-p)(1+q)}$$

**Mathematica [A]** time = 0.0240465, size = 40, normalized size = 1.03

$$-\frac{x^{-p(q+1)} (ax^n + bx^p)^{q+1}}{a(q+1)(p-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q,x]
```

```
[Out] -((a*x^n + b*x^p)^(1 + q)/(a*(-n + p)*(1 + q)*x^(p*(1 + q))))
```

---

**Maple [F]** time = 0.53, size = 0, normalized size = 0.

$$\int x^{-1+n-p(1+q)} (ax^n + bx^p)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x)
```

```
[Out] int(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ax^n + bx^p)^q x^{-p(q+1)+n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="maxima")
```

```
[Out] integrate((a*x^n + b*x^p)^q*x^(-p*(q + 1) + n - 1), x)
```

---

**Fricas [A]** time = 0.87875, size = 155, normalized size = 3.97

$$\frac{\left(axx^{-pq+n-p-1}x^n + bxx^{-pq+n-p-1}x^p\right)(ax^n + bx^p)^q}{\left(an - ap + \left(an - ap\right)q\right)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="fricas")
```

```
[Out] (a*x*x^(-p*q + n - p - 1)*x^n + b*x*x^(-p*q + n - p - 1)*x^p)*(a*x^n + b*x^
p)^q/((a*n - a*p + (a*n - a*p)*q)*x^n)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n-p*(1+q))*(a*x**n+b*x**p)**q,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (ax^n + bx^p)^q x^{-p(q+1)+n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="giac")
```

```
[Out] integrate((a*x^n + b*x^p)^q*x^(-p*(q + 1) + n - 1), x)
```

## 3.454 $\int x^{-1-nq-p(1+q)} \left(x^n \left(a+bx^p\right)\right)^q \, dx$

**Optimal**. Leaf size=40

$$-\frac{x^{(q+1)(-(n+p))} \left(ax^n+bx^{n+p}\right)^{q+1}}{ap(q+1)}$$

```
[Out] -((a*x^n + b*x^(n + p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))
```

---

**Rubi [A]** time = 0.0738037, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1980, 2014}

$$-\frac{x^{(q+1)(-(n+p))} \left(ax^n+bx^{n+p}\right)^{q+1}}{ap(q+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]
```

```
[Out] -((a*x^n + b*x^(n + p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))
```

Rule 1980

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] &&  !Generali
zedBinomialMatchQ[u, x]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int x^{-1-nq-p(1+q)} \left(x^n \left(a+bx^p\right)\right)^q \, dx &= \int x^{-1-nq-p(1+q)} \left(ax^n+bx^{n+p}\right)^q \, dx \\ &= -\frac{x^{-(n+p)(1+q)} \left(ax^n+bx^{n+p}\right)^{1+q}}{ap(1+q)} \end{aligned}$$

**Mathematica [A]** time = 0.0397407, size = 38, normalized size = 0.95

$$-\frac{x^{(q+1)(-(n+p))} \left(x^n \left(a+bx^p\right)\right)^{q+1}}{ap(q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]
```

```
[Out] -((x^n*(a + b*x^p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))
```

---

**Maple [F]** time = 0.47, size = 0, normalized size = 0.

$$\int x^{-1-nq-p(1+q)}\left(x^n\left(a+bx^p\right)\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x)
```

```
[Out] int(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(\left(bx^p+a\right)x^n\right)^q x^{-p(q+1)-nq-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)
```

---

**Fricas [A]** time = 0.858222, size = 135, normalized size = 3.38

$$-\frac{\left(bxx^{-(n+p)q-p-1}x^p+axx^{-(n+p)q-p-1}\right)\left(bx^nx^p+ax^n\right)^q}{apq+ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="fricas")
```

```
[Out] -(b*x*x^(-(n + p)*q - p - 1)*x^p + a*x*x^(-(n + p)*q - p - 1))*(b*x^n*x^p +
 a*x^n)^q/(a*p*q + a*p)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n*q-p*(1+q))*(x**n*(a+b*x**p))**q,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(\left(bx^p+a\right)x^n\right)^q x^{-p(q+1)-nq-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="giac")
```

```
[Out] integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)
```

# Chapter 4

# Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

### 4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

### 4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```